The Incremental Cooperative Design of Preventive Healthcare Networks

Soheil Davari*
University of Hertfordshire, Hertfordshire Business School
United Kingdom

Abstract. In the Preventive Healthcare Network Design Problem (PHNDP), one seeks to locate facilities in a way that the uptake of services is maximised given certain constraints such as congestion considerations. We introduce the incremental and cooperative version of the problem, IC-PHNDP for short, in which facilities are added incrementally to the network (one at a time), contributing to the service levels. We first develop a general non-linear model of this problem and then present a method to make it linear. As the problem is of a combinatorial nature, an efficient *Variable Neighbourhood Search* (VNS) algorithm is proposed to solve it. In order to gain insight into the problem, the computational studies were performed with randomly generated instances of different settings. Results clearly show that VNS performs well in solving IC-PHNDP with errors not more than 1.54%.

Keywords. Preventive healthcare, Facility location, Cooperative covering, Variable neighbourhood search, Network design.

1 Introduction

Limited resources and ageing population are two of the major challenges of the health industry in the 21st century. The problem of managing healthcare supply chains becomes more complicated by increased customer expectations, shortage of healthcare workers, and the need to invest on new technologies. This has brought about increased healthcare expenditure in many countries. For instance, according to the Office of National Statistics [1], the total healthcare expenditure in the UK as a percentage of gross domestic product had increased from 6.2% in 1997 to 8.8% in 2013.

Preventive healthcare aims at saving lives and improving health through early detection of diseases. It is comprised of programmes such as cholesterol screening, HIV screenings, immunisation vaccination, and diet counseling services. These programmes can prevent a wide range of chronic diseases such as heart disease, cancer, and diabetes which are responsible for seven out of ten deaths and account for 75% of nation's health spending among Americans [2]. Although most of these services are offered for no cost in most countries, the participation rates are low and improving the uptake rates of these services is a concern to governments all over the world. The uptake rate can be different among various groups of education and occupation (Damiani et al. [3]), income (Fox and Shaw [4]), and gender (Meissner et al. [5]) and a variety of qualitative and quantitative factors can influence it. For instance, the proximity of the service centres, congestion in facilities, and even closeness of these centres to other facilities such as shopping malls can all influence the uptake rates (Refer to Santos et al. [6] and references therein for further information). Among these, proximity to the facilities has been known to be the most significant factor (refer to Muller et al. [7], Varkevisser et al. [8], and Haynes et al. [9]).

The current paper deals with a discrete, incremental, and cooperative version of Preventive Healthcare

*Corresponding author Email: s.davari@herts.ac.uk

Network Design problem (IC-PHNDP) which concerns with finding the optimal number and location of facilities among a set of potential nodes in order to maximise the uptake of services. We assume that the network is built gradually and over the periods (incrementally) and once a facility is opened, it should remain operational until the last period. Similar to any other real-world optimisation problem, it has a set of constraints such as the congestion constraint. There is an analogy between our problem and the *competitive facility location problem* in the sense that both make use of the idea of gravity-based attraction (Hotelling [10]) to model the attraction of clients to facilities. In this model, the probability of a customer patronizing a facility is proportional to the attractiveness of the facility and inversely proportional with the distance. This idea was later extended by many scientists such as Nakanishi and Cooper [11]. A review of these contributions can be found in Bell et al. [12].

We assume that facilities cooperate in providing services to the clients (in line with the seminal paper of Berman et al. [13]). In such a setting, the coverage of a demand node is not determined by only the closest facility, but all the facilities in its vicinity. In other words, each facility j emits signals decaying over distance based on a known non-negative and non-increasing function of distance $\phi(d(i,j))$ (e.g. $\phi(d(i,j)) = \frac{1}{d_{ij}^2}$ or $\phi(d(i,j)) = exp(-d_{ij})$) and each demand point i is affected by an aggregation of all the signals received. This aggregation operator can take different forms such as summation, maximum, and truncated sum (**Figure (1)**). Regardless of the coverage type used, a demand point is covered if the aggregated signal exceeds a certain threshold Θ. For instance, in the case of a summation operator and assuming p established facilities, demand point i is covered if and only if:

$$\sum_{j=1}^{p} \phi(d(i,j)) \geq \Theta \tag{1}$$

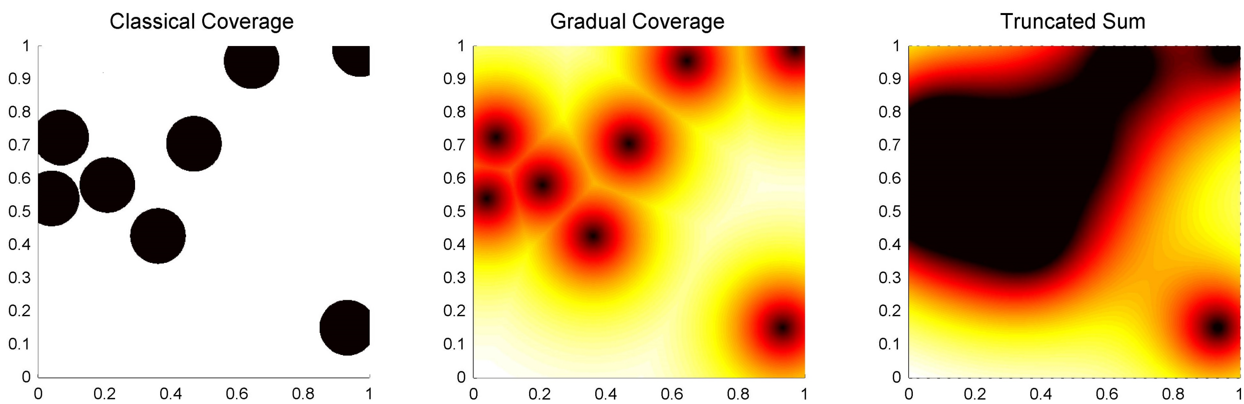

Figure 1: Heat map of different aggregation operators

In this paper, we model IC-PHNDP as a mixed-integer programming model and provide a method to linearize it. Then, we propose an efficient *Variable Neighbourhood Search* heuristic to solve it with optimality gaps of not more than 1.54% while the average optimality gap is 0.71%. The contributions of this paper are as follows:

- To the best of our knowledge, there is no publication in the literature for modelling the incremental or cooperative version of a preventive healthcare network design. This paper contributes to the literature by modelling this problem.
- We propose an efficient heuristic procedure to solve the problem and analyse its performance with a set of hypothetically generated test problems.

The outline of this paper is as follows: It proceeds with a literature review of relevant publications in section (2). The mathematical model of the paper is presented in section (3). In section (4), our proposed solution procedure is elaborated. Numerical experiments and some analysis appear in section (5), and finally, conclusions and some future research avenues are provided in section (6).

2 Background and Literature Review

To the best of our knowledge, Hakimi [14] was the first scholar addressing the problem of healthcare network design in the literature. Later, a lot of research has been carried out on different problems in the area of healthcare network design such as public healthcare facility location (Kim and Kim [15]), health care facility location-allocation (Syam and Cote [16]), and healthcare facility location/vehicle routing (Veenstra et al. [17]). Interested readers can refer to a recent survey by Ahmadi et al. [18] for further information about the healthcare facility location literature.

One of the relatively less studied variants of the healthcare network design is the problem of designing preventive healthcare networks. As far as we know, Verter and LaPierre [19] was the first publication in the literature addressing this problem where case studies in Georgia, USA and Montreal, Canada were given. Following on from that, Zhang et al. [20] presented the problem of preventive healthcare network design on a graph with optimal choice allocation aiming at maximising service uptake and compared the performance of four heuristics in terms of their accuracy and computational requirements. Zhang et al. [21] studied a different version of the problem where a bi-level non-linear optimisation model was developed with equilibrium constraints and a tabu search heuristic was proposed to solve it. Another study in the area of PHNDP is Gu et al. [22] in which the impact of client choice behaviour on the network was studied as a bi-objective model which was solved using an interchange algorithm. Zhang et al. [23] was another study of the client choice behaviour on the network presenting both an optimal choice model and a probabilistic choice model. The problem was formulated as a mixed integer programming problem and a genetic algorithm was presented to solve it. The bi-objective fuzzy variant of the problem was studied in Davari et al. [24] where a fuzzy goal programming and a chance constrained solution procedure were proposed to solve the problem. Haas and Muller [25] employed a multinomial logit model to model the client choice behaviour and solved the problem with instances up to 20 potential nodes and 400 demand zones. They presented a procedure for finding lower bounds for larger sizes and a definition of clients' utility. Davari et al. [26] considered PHNDP with impatient clients and budget constraints and proposed an efficient VNS heuristic to solve it.

The problem of multi-period facility location is not new to the literature and there has been numerous publications dealing with this problem. **Table (1)** gives an overview of the recent research on multi-period models. Although this table does not cover an exhaustive list of features of each paper, it mainly aims at linking our study to the literature and providing an overview of the solution procedures used in the literature.

Table 1: Literature review of some recent multi-period facility location models

Paper	Year	Problem	Solution approach
Gen and Syarif [27]	2005	Production/distribution planning	Genetic algorithm
McKendall and Shang [28]	2006	Dynamic facility layout	Simulated annealing
Ko and Evans [29]	2007	Integrated forward/reverse logistics network for 3PLs	Genetic algorithm
Yi and Ozdamar [30]	2007	Evacuation and support in disaster response	Exact method
Ndiaye and Alfares [31]	2008	Health services for moving population groups	Exact method
Wang et al. [32]	2008	Two-echelon integrated competitive/Uncompetitive facility location problem	Genetic algorithm
Rajagopalan et al. [33]	2008	Dynamic redeployment of ambulances	Tabu search
Manzini et al. [34]	2008	Multi-stage, multi-commodity location allocation	Exact method
Gourdin and Klopfenstein [35]	2008	Capacitated location with modular equipments	Polyhedral properties
Hinojosa et al. [36]	2008	Dynamic supply chain design with inventory	Lagrangian relaxation
Albareda-Sambola et al. [37]	2009	Multi-period incremental service facility location problem	Lagrangian relaxation
Lee and Dong [38]	2009	Dynamic location and allocation models	Heuristic method
Mahar et al. [39]	2009	On-line fulfilment assignment problem	Branch & bound, Dynamic programming
Schmid and Doerner [40]	2010	Ambulance location-relocation problems with time-dependent travel times	Variable neighbourhood search
Basar et al. [41]	2011	Emergency medical stations	Tabu search
Fazel Zarandi et al. [42]	2011	Large-scale dynamic maximal covering location problem	Simulated annealing
Torres and Uster [43]	2011	Capacitated facility location with relocations and changing demand	Lagrangian relaxation
Beneyyan et al. [44]	2012	Single and multi-period location-allocation models in the health sector	Exact method
Sha and Huang [45]	2012	Emergency blood supply scheduling model	Heuristic based on Lagrangian relaxation
Rottkemper et al. [46]	2012	Inventory relocation and distribution in humanitarian logistics	Rolling horizon solution method
Schmid [47]	2012	Dynamic ambulance relocation and dispatching problem	Approximate dynamic programming
Albareda-Sambola et al. [48]	2012	Multi-period Location-Routing with Decoupled Time Scales	Heuristic
Albareda-Sambola et al. [49]	2013	Multi-period location-allocation problem under uncertainty	Fix-and-Relax-Coordination
Ghaderi and Jabalameli [50]	2013	Budget-constrained dynamic uncapacitated facility location network design	Exact method and simulated annealing
Correia et al. [51]	2013	Two-echelon supply chain network design problem with sizing decisions	Valid inequalities
Zhen et al. [52]	2014	Emergency medical stations	Genetic algorithm
Gelareh et al. [53]	2015	Multi-period hub location problem	Benders decomposition
Chung and Kwon [54]	2015	Location of electric car charging station	Heuristic (myopic methods)
Elbek and Wohlk [55]	2016	Scheduling of recyclable materials collection	Constructive variable neighbourhood search
Duhamel et al. [56]	2016	Location-allocation problem for post-disaster operations	Decomposition approach
Correia and Melo [57]	2016	Facility location under delayed demand satisfaction	Valid inequalities
Markovic et al. [58]	2016	Stochastic facility location problem with independent demand	Lagrangian relaxation
Vatsa and Jayaswal [59]	2016	Multi-period maximal covering facility location problem with server uncertainty	Benders decomposition

From the above and to the best of our knowledge, there is not any attempt in the literature towards modelling the multi-period design of a preventive healthcare network. Another gap in the literature of PHNDP is the realistic assumption of cooperativeness in covering nodes in a preventive helathcare setting. In line with some of the existing literature, our paper aims at filling these two gaps by proposing an efficient integer programming model and presenting an efficient variable neighbourhood search procedure which is capable of solving large instances with errors not worse than 1.54% to the best known solutions.

3 Mathematical model

Consider a region (say a city) with a set of nodes representing demands in different sub-regions. The decision maker is interested in minimising the costs while ensuring that a minimum level of coverage is guaranteed for all the nodes. There are candidate locations to establish facilities and the proximity of clients to facilities is the key factor in making facilities more attractive. Besides, there are congestion considerations in the problem. We assume that there is no existing facility in the network; however, the model is easily generalizable for the case where facilities exist. Moreover, facilities cooperate in providing service to the population centres and the network is established incrementally.

The model is studied in a discrete space with N as the demand nodes ($|N| = m$) and a set of $V \subset N$ to be the set of potential nodes to establish a facility. The demand associated with node $i \in N$ at time $t \in T$ is denoted as p_{it}. The shortest path between each demand zone $i \in N$ and a potential facility at node $j \in V$ is represented as d_{ij}. Moreover, we follow a similar approach to Pastor [60] in defining the attractiveness. Like that, the attractiveness of each facility $j \in V$ to clients living at node $i \in N$ is shown as ϕ_{ij} and found using a negative exponential function as $\phi_{ij} = e^{-\eta d_{ij}}$ where η is an empirically defined value (different decay functions can be used such as the power function $\phi_{ij} = d^{-\eta}$. However, based on the empirical study of Drezner [61], we use the exponential function in this study). It should be noted that this attractiveness measure can be modified in order to address other attractiveness parameters such as the appearance of a facility, its size, and other factors (see Drezner [62] and references therein). Moreover, in each period $t \in T$, a minimum of π_t people should be covered. The π function is defined as a non-decreasing function of t (linear, piecewise, etc.) to gradually increase the service level in the network.

We assume that the demand of node $j \in V$ is partially met by each opened facility and inversely proportional to the distance between the demand node and the facility (based on the basic concept of gravity rule by Reily [63]). Last but not least, the cooperative aggregation operator is shown as Φ_{it} which can take different forms. In this paper, we assume that the aggregate coverage of a node i at time t (Φ_{it}) is found as:

$$\Phi_{it} = min\{1, \sum_{j \in V} \phi_{ij} x_{jt}\} \tag{2}$$

where x_{jt} is defined as follows:

$$x_{jt} = \begin{cases} 1 & \text{If there is a facility at node } j \in V \text{ at time } t \in T \\ 0 & \text{Otherwise} \end{cases}$$

The other parameters of the problem are as follows:
Parameters

a_{ij}	The attractiveness of facility at node $j \in V$ to the demand node at node $i \in N$
d_{ij}	The distance between nodes $i \in N$ and $j \in V$
p_{it}	The population of node $i \in N$ at time $t \in T$
f_{jt}	Cost of establishing a facility at node $j \in V$ at time $t \in T$
o_{jt}	Operation cost of node $j \in V$ at time $t \in T$
π_t	The total population to be covered at time $t \in T$
$\bar{\lambda}$	The maximum number of clients each server can serve
T_{max}	Number of periods of the study

Let ζ_{ijt} to be the share of the facility at node $j \in V$ from the demand at node $i \in N$ at time $t \in T$. Spatial interaction models assume that this ratio equals the relative utility of facility at node $j \in V$ compared to other facilities available on the network which can be represented as follows.

$$\zeta_{ijt} = \frac{\phi_{ij} x_{jt}}{\sum\limits_{l \in V} \phi_{il} x_{lt} + \epsilon} \tag{3}$$

where a sufficiently small ϵ is added to the denominator to avoid undefined values for ζ. Now, the problem can be formulated as follows.

$$\min \sum_{j \in V} \sum_{t \in T} f_{jt}(x_{jt} - x_{j(t-1)}) + \sum_{j \in V} \sum_{t \in T} o_{jt} x_{jt} \tag{4}$$

$$\Phi_{it} \leq 1 \qquad \forall i \in N, \forall t \in T \tag{5}$$

$$\Phi_{it} \leq \sum_{j \in V} \phi_{ij} x_{jt} \qquad \forall i \in N, \forall t \in T \tag{6}$$

$$x_{jt} \leq x_{j(t+1)} \qquad \forall j \in V, t \in T \setminus \{T_{max}\} \tag{7}$$

$$\sum_{i \in N} \frac{\phi_{ij} x_{jt}}{\sum\limits_{l \in V} \phi_{il} x_{lt} + \epsilon} p_{it} \leq \bar{\lambda} x_{jt} \qquad \forall j \in V, \forall t \in T \tag{8}$$

$$\sum_{i \in N} \Phi_{it} p_{it} \geq \pi_t \qquad \forall t \in T \tag{9}$$

$$x_{jt} \in \{0, 1\} \qquad \forall j \in V, \forall t \in T \tag{10}$$

The **objective function (4)** minimises the total cost of the network which is a function of fixed establishment costs and the variable server costs. **Constraints (5)** and **(6)** linearize the **equation (2)**. **Constraint set (7)** ensures that while a facility is opened, it should operate for the remaining periods. In other words, the problem is modelled as an *uninterrupted* facility location problem which is a more realistic one in practice. The congestion constraint is enforced to the model through **Constraint (8)**. **Constraint (9)** states that in each period $t \in T$, a minimum population of π_t should be covered. As stated earlier, the π function can be defined as a non-decreasing function of $t \in T$ to gradually improve the service level in the network. Finally, **constraint (10)** is the integrality constraint for the variable x_{jt}. The model is a non-linear one owing to **constraint (8)** which can be linearized using **Proposition (1)**.

Proposition 1. ***Constraint (8)*** *can be rewritten as:*

$$\sum_{i \in N} p_{it} \phi_{ij} z_{ijt} \leq \bar{\lambda} x_{jt} \qquad \forall j \in V, \forall t \in T \tag{11}$$

Proof. Define the auxiliary variable w_{it} to represent the following component of constraint (8):

$$w_{it} = \frac{1}{\sum\limits_{l \in V} \phi_{il} x_{lt} + \epsilon} \tag{12}$$

and the variable z_{ijt} to represent the following component of **constraint (8)**.

$$z_{ijt} = w_{it} x_{jt} \tag{13}$$

Note that there will be a need to add the following constraints to the model.

$$z_{ijt} \leq M x_{jt} \qquad \forall i \in N, \forall j \in V, \forall t \in T \tag{14}$$

$$z_{ijt} \leq w_{it} \qquad \forall i \in N, \forall j \in V, \forall t \in T \tag{15}$$

$$w_{it} - z_{ijt} \leq M(1 - x_{jt}) \qquad \forall i \in N, \forall j \in V, \forall t \in T \tag{16}$$

$$z_{ijt} \geq 0 \qquad \forall i \in N, \forall j \in V, \forall t \in T \tag{17}$$

$$\sum_{j \in V} z_{ijt} \phi_{ij} = 1 \qquad \forall i \in N, \forall t \in T \tag{18}$$

in which M is a sufficiently large positive number. □

Now, the linearized mathematical model can be rewritten as follows.

$$\min \sum_{j \in V} \sum_{t \in T} f_{jt}(x_{jt} - x_{j(t-1)}) + \sum_{j \in V} \sum_{t \in T} o_{jt} x_{jt} \tag{19}$$

$$\sum_{i \in N} \Phi_{it} p_{it} \geq \pi_t \qquad \forall t \in T \tag{20}$$

$$\Phi_{it} \leq 1 \qquad \forall i \in N, \forall t \in T \tag{21}$$

$$\Phi_{it} \leq \sum_{j \in V} \phi_{ij} x_{jt} \qquad \forall i \in N, \forall t \in T \tag{22}$$

$$\sum_{i \in N} p_{it} \phi_{ij} z_{ijt} \leq \bar{\lambda} x_{jt} \qquad \forall j \in V, \forall t \in T \tag{23}$$

$$x_{jt} \leq x_{j(t+1)} \qquad \forall j \in V, t \in T \backslash \{T_{max}\} \tag{24}$$

$$\sum_{j \in V} z_{ijt} \phi_{ij} = 1 \qquad \forall i \in N, \forall t \in T \tag{25}$$

$$z_{ijt} \leq M x_{jt} \qquad \forall i \in N, \forall j \in V, \forall t \in T \tag{26}$$

$$z_{ijt} \leq w_{it} \qquad \forall i \in N, \forall j \in V, \forall t \in T \tag{27}$$

$$w_{it} - z_{ijt} \leq M(1 - x_{jt}) \qquad \forall i \in N, \forall j \in V, \forall t \in T \tag{28}$$

$$z_{ijt} \geq 0 \qquad \forall i \in N, \forall j \in V, \forall t \in T \tag{29}$$

$$x_{jt} \in \{0, 1\} \qquad \forall j \in V, \forall t \in T \tag{30}$$

IC-PHNDP belongs to the class of NP-hard problems since its relaxation makes it an uncapaciated facility location problem which has been proven to be an NP-hard problem. Therefore, owing to its combinatorial optimization nature, we propose an efficient VNS to solve it.

4 Solution procedure

Variable Neighbourhood Search (Mladenović and Hansen [64]) is a local search procedure based on systematically improving the incumbent solution by applying a set of neighbourhood search structures. VNS has been a popular heuristic in a variety of problems from scheduling (Karimi et al. [65]) and vehicle routing (Belhaiza et al. [66]) to facility location (Davari et al. [67]). Interested readers can refer to Hansen et al. [68] and references therein for further applications of VNS.

Similar to other heuristics, one of the main issues in designing a VNS is to keep a balance between the intensification and diversification of the algorithm. Since its introduction, different scholars have proposed various mechanisms to improve this balance. In this paper, we will apply a general skewed version of VNS as given in **Algorithm (1)**.

Algorithm 1 Skewed Variable Neighbourhood Search

procedure VNS
Initialization;
Define neighbourhood structures N_k; $k = 1, ..., k_{max}$
Find an initial solution $s \in S$
Choose stopping criteria
while stopping criteria is not met **do**:
    Set $k = 1$
    while $k \leq k_{max}$ **do**:
        Shaking:
        Generate a point $s' \in N_k(s)$ at random;
        Local search:
        Obtain the local optima s'' by applying some local search to s';
        Move or not:
        If $f(s'') < f(s)(1 + \kappa\rho(s, s''))$ **then**
            $s = s''$;
            $k = 1$;
        else
            $k = k + 1$;
        end if
    end while
end while

For the distance function (ρ), we propose a function to find the dissimilarity between s and $\bar{s}$. Considering θ_{it} as a binary variable taking a value of 1 if a facility at node i is opened at time t and 0 otherwise (the same for $\bar{s}$), **Equation (31)** finds the distance between the two solutions s and $\bar{s}$.

$$\rho(s, \bar{s}) = \frac{\sum_{i=1}^{N} \sum_{t=1}^{T} |\theta_{it} - \bar{\theta}_{it}|}{|N||T|} \tag{31}$$

In order to increase the diversification ability of the proposed procedure, we allow infeasible solutions to be explored as well. There are two types of infeasibilities in our problem as the violation of the coverage (Constraint (20)) and violation of the congestion (Constraint (23)). We add two penalty terms to the objective function as φ and ψ for each unit of violation for the two constraints respectively. These two parameters are updated dynamically during the run to have an optimal trade-off between the intensification and diversification of the procedure. **Algorithm (2)** presents the procedure to update these parameters.

Algorithm 2 Updating the penalization parameters

procedure ParamUpdate
if $\sum_{i \in N} \Phi_{it} p_{it} \geq \pi_t$ **then**
  $\varphi = \varphi(1 - \epsilon_\varphi)$
else
  $\varphi = \varphi(1 + \epsilon_\varphi)$
if $\sum_{i \in N} p_{it} \phi_{ij} z_{ijt} \leq \bar{\lambda} x_{jkt}$ **then**
  $\psi = \psi(1 - \epsilon_\psi)$
else
  $\psi = \psi(1 + \epsilon_\psi)$

In this section, we will elaborate the encoding scheme, initialisation procedure, and neighbourhood search structures.

4.1 Solution Representation

Solution representation plays a crucial role in success of any heuristic method and VNS is not an exception. In this paper, we have used a vector to represent a solution. Assuming n potential facilities to locate and T_{max} as the number of periods, each vector is composed of n elements each with a value in the range of [0, T_{max}] showing the index of the period the facility starts to operate. Since the problem is studied in an uninterrupted settings, this compact representation can be easily transferred to a vector/matrix showing the location of each facility at each time period. The sample solution in **Figure (2)** shows a problem with eight facilities where a facility is located at node two in the second period and another facility is established at node four in the first period.

0	2	0	1	0	0	0	0

Figure 2: Solution representation

This representation facilitates carrying out neighbourhood search structures quickly which leads to a fast and efficient heuristic. Moreover, the sparse nature of the vector enables the algorithm to benefit from massive memory savings. Hence, we believe that this solution representation is efficient.

4.2 Initial solution construction

We employed a fuzzy c-means procedure to generate initial solutions which are feasible (interested readers can refer to Sato and Jain [69] for further information about fuzzy c-means algorithm and its variants). A sketch of this procedure is given in **Algorithm (3)**.

Algorithm 3 FCM-based Initialisation

procedure FCM-BASED INITIALISATION
$t = 1$;
while $t \leq T_{max}$ **do**:
    Cluster the data using a fuzzy c-means procedure
    Find the closest nodes to each cluster centre and locate a facility there
    If solution is feasible **then**
        $t = t + 1$;
    else
        while there is an infeasibility **do**:
            Find the cluster(s) in which an infeasibility occurs
            Locate the facility in that cluster which has the lowest cost of establishment
        $t = t + 1$;
    end if
end while

4.3 Neighbourhood Operators

We defined a set of five different neighbourhood structures to guide the search and to maintain a balance between intensification and diversification as $\mathcal{N}$={$\mathcal{N}_1$, $\mathcal{N}_2$, $\mathcal{N}_3$, $\mathcal{N}_4$, $\mathcal{N}_5$}. The moves $\mathcal{N}_3$-$\mathcal{N}_5$ have a nested relation as $\mathcal{N}_3 \subseteq \mathcal{N}_4 \subseteq \mathcal{N}_5$. The following sections explain each structure using an example.

4.3.1 Backward Shift ($\mathcal{N}_1$)

The neighbourhood structure shifts back the establishment period of an existing facility. **Figure (3)** depicts a sample solution where the establishment of facility (2) has been rescheduled to period (1). Please note that this neighbourhood structure can improve the solution by closing a facility or by shifting its establishment period to an earlier one and also make a solution feasible by providing higher service levels for the earlier periods. This operator is invoked repeatedly for all the possible values of $t \in T$ in order to find a better solution.

0	2	0	0	5	0	3	3	0	0	1	0	0	0	2

0	1	0	0	5	0	3	3	0	0	1	0	0	0	2

Figure 3: Pushing Back

4.3.2 Forward Shift ($\mathcal{N}_2$)

The neighbourhood structure performs in an opposite way to $\mathcal{N}_1$ by shifting forward the establishment period of an existing facility. The move is applicable for non-existing facilities as well by locating them in a consequent period. **Figure (4)** shows a sample solution where the establishment period of facility (7) has been rescheduled to period (4) from period (3). This operator is used repeatedly for all the possible values and the best one is opted for.

0	2	0	0	5	0	3	3	0	0	1	0	0	0	2

0	2	0	0	5	0	4	3	0	0	1	0	0	0	2

Figure 4: Pulling Forward

4.3.3 Swap ($\mathcal{N}_3 - \mathcal{N}_5$)

A neighbour of a solution s is obtained by exchanging the values of q pairs of it where $1 \leq q \leq 3$. In other words, the values of a set of q bits are exchanged with the values of a different set of q bits in the solution. **Figure (5)** presents a sample solution obtained using $q = 2$ where the values of two pairs of bits are exchanged (second bit with the eleventh and the sixth bit with the eighth). Our preliminary analysis showed that using the values of more than three for q perturbs the solution to a level that might negatively affect the quality of solutions.

0	2	0	0	5	0	3	3	0	0	1	0	0	0	2
0	1	0	0	5	3	3	0	0	0	2	0	0	0	2

Figure 5: Swap(2)

We will use operators $\mathcal{N}_3 - \mathcal{N}_5$ for the shaking phase and the structures $\mathcal{N}_1 - \mathcal{N}_2$ as the local search procedures. The shaking structures are able to perturb the local optima strongly to increase the diversification of the algorithm and the local search procedures are intensification operators exploring the neighbourhood of a solution to find a better one by slightly changing it.

4.3.4 Stopping Criteria

Our experiments showed that for small-scale problems ($|N| \leq 25$), running the procedure for more than 60 seconds rarely brings about improvement in the solution quality. Hence, the proposed procedure stops after running for 60 seconds regardless of the problem size and its parameters. However, for large-scale problems, we adopted a different stopping criterion based on the number of nodes ($|N|$) and time periods ($|T|$) as stopping after $60\frac{|N||T|}{50}$ seconds. This time corresponds to the wall-clock time of the algorithm including the pre-processing tasks and the reporting time. It should be noted that these stopping criteria are dependent on the hardware specification and the programming language.

5 Numerical experiments

Since no benchmark instances exists in the literature for IC-PHNDP, we generated a hypothetical set of 216 test problems with different settings[1]. In particular, we considered three dimensions of $|N| \in \{15, 20, 25\}$. We assumed that there is a direct link between any two nodes enabling the demand to access services in the shortest time.The other settings are shown in **Table (2)**. It should be noted that the third option of $\bar{\lambda}$ in **Table (2)** is basically the uncapcitated version of the problem where there is no congestion in facilities. For the node distributions, we used the Beta distribution with different (α,β) values to have symmetric/non-symmetric and also dense/sparse distributions. **Figure (6)** depicts the four types of node distributions on the plane with different values of (α, β). For the sake of reading the table easier, the total population over the periods is denoted as Δ as is shown in **Equation (32)**. Finally, the π vector was generated using two different parameters imposing different rates of constraint on the problem. In order to address the test problems throughout the paper, we adopt the $|N|/|T|/(\alpha, \beta)/\bar{\lambda}/\pi$ notation.

$$\Delta = \sum_{i \in N} \sum_{t \in T} p_{it} \tag{32}$$

[1]Test problems can be shared upon request via email.

Table 2: Parameter settings for the test problems

Parameter	Levels			
$\lvert N \rvert$	15	20	25	
$\lvert T \rvert$	1	3	6	
(α,β)	(1,1)	(2,5)	(5,5)	(1,5)
$\bar{\lambda}$	$\frac{2\Delta}{\lvert N\rvert\lvert T\rvert}$	$\frac{5\Delta}{\lvert N\rvert\lvert T\rvert}$	∞	
π_t	$(1-0.7)^t$	$(1-0.95)^t$		

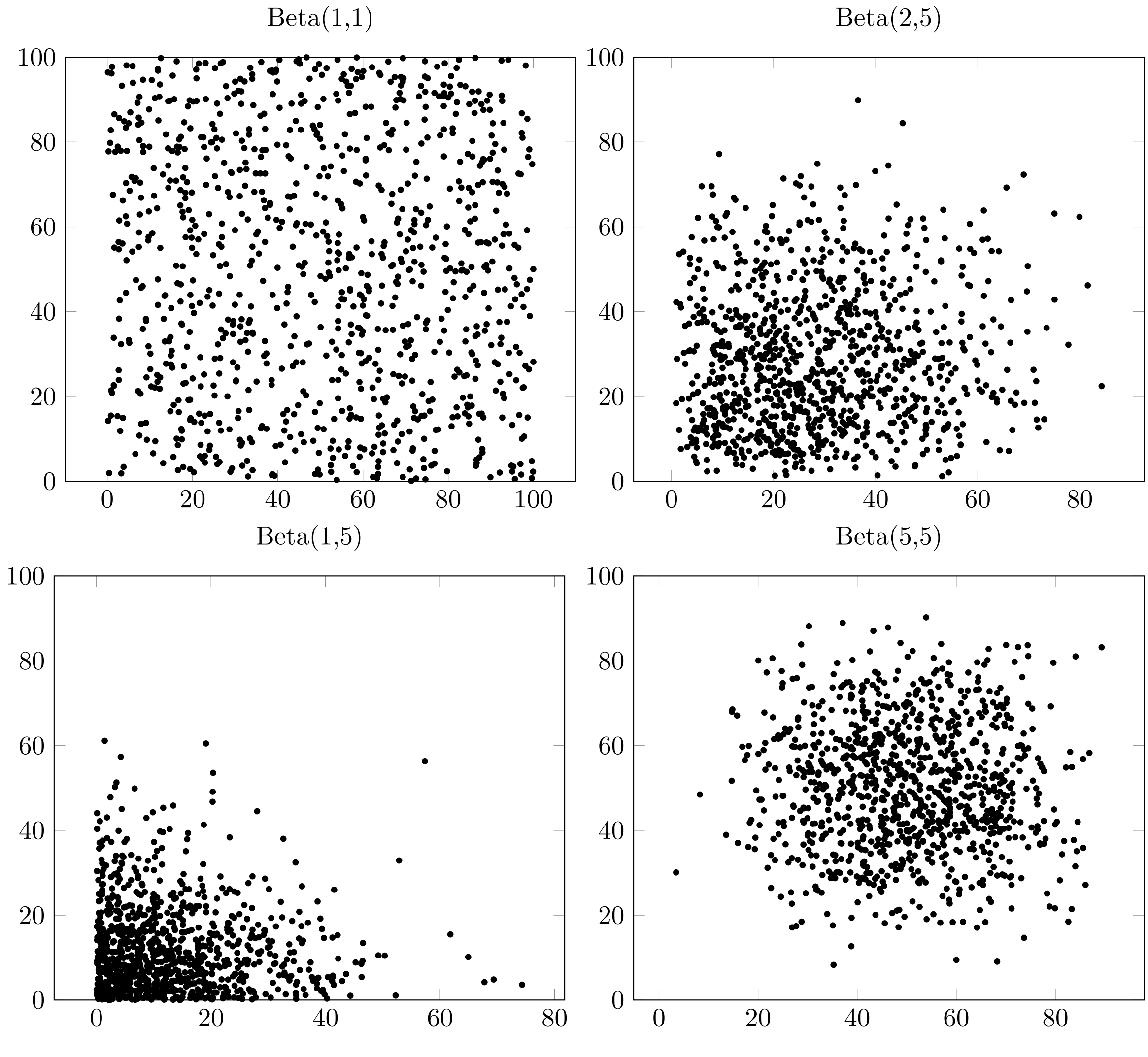

Figure 6: Four demand profiles

All experiments described in this section have been carried out on a laptop with Intel i5 2.30 GHz CPU and 16 gigabyte RAM memory. The proposed VNS algorithm was coded in C++ on Visual Studio 2015 and the Gurobi Optimizer was used to solve the MILP model. We allowed the optimizer to run for at most five hours regardless of the problem size. For those instances that Gurobi failed to find the optimal fitness, the lower bound has been reported for the sake of comparison with the proposed VNS. Moreover, we ran our heuristic ten times on each test problem and reported the worst, average, and best performances in terms of the gap to the best found solution.

5.1 Parameter tuning

There are four parameters to be optimised before running the proposed heuristic: the time limit, the maximum neighbourhood size (k_{max}), and the values of ϵ_φ and ϵ_ψ. We found that running the proposed VNS for more than 60 seconds is rarely effective in improving the solution (marginal improvements in seven out of 500 random instances). Hence, we opted for a termination criterion of reaching 60 seconds. Needless to say, running the heuristic for a longer time still improves the solution, but the pace of improvements slows down. It should be mentioned that plotting the solutions shows that the heuristic does not need the whole 60 seconds to reach an optimal solution for the small-scale problems.

The value of k_{max} has been already tuned as explained earlier to be five. This can be attributed to the fact that for the values of k above five, there is occasionally an improvement seen and the search becomes almost random, negatively affecting the structure of current solution.

In order to find the optimal value of ϵ_φ and ϵ_ψ, we ran the algorithm ten times with values in the range of [0-0.1] with increments of 0.01 to solve 36 test problems with different number of nodes, number of periods, and distribution of demand nodes and reported the average gaps to the optimal solution. The results for ϵ_ψ were inconclusive and its value has been set to zero. However, as **Figure (7)** shows, a value of 0.04 for the ϵ_φ leads to the best results in terms of the error.

In order to test the superiority of using $\epsilon_\varphi = 0.04$ over the other values, we performed a Wilcoxin signed-rank test on top of the visual test which showed that the results obtained using $\epsilon_\varphi = 0.04$ leads to better results compared to the other values.

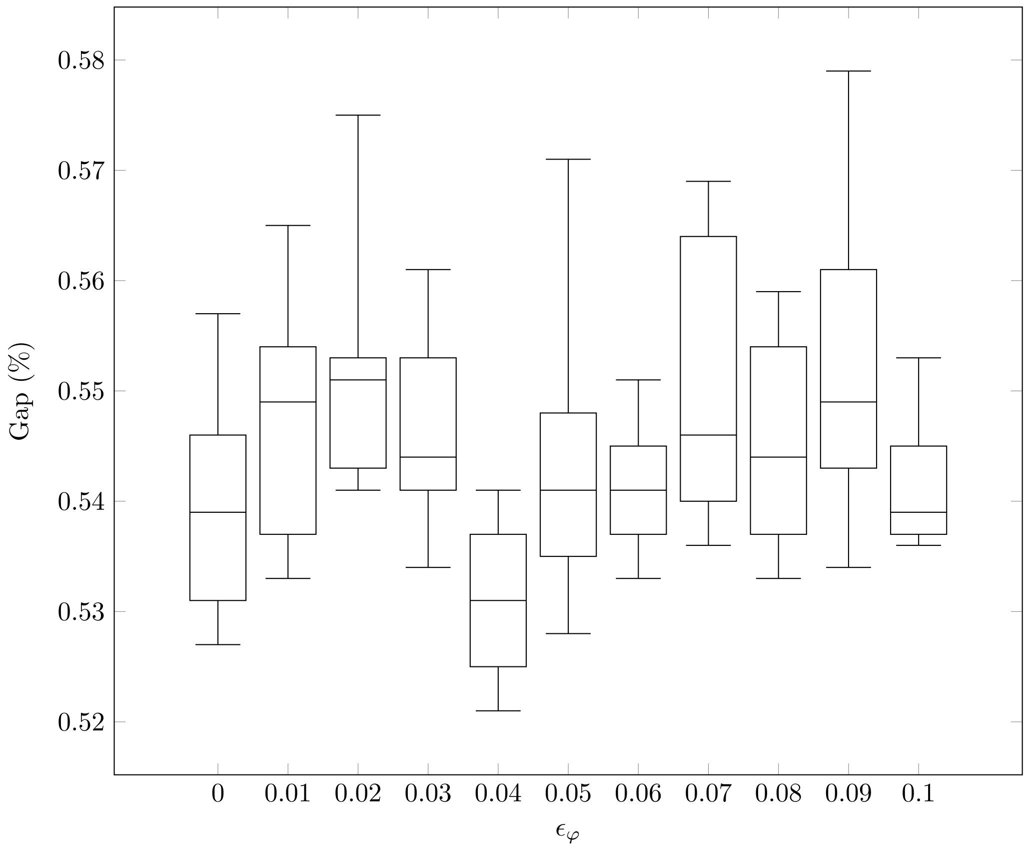

Figure 7: Box-plot of errors using eleven values of ϵ_φ

6 Results and discussion

Each of the 216 instances were solved to optimality using Gurobi Optimizer Version 6.5.1 and also ten times using the proposed algorithm. **Table (3)** summarise the obtained results for both the Gurobi and heuristic runs. The table contains the results for the average performance of the proposed heuristic compared to the Gurobi outputs. While the first four columns represent the problem parameters, the last four columns report on the average gap of the proposed heuristic for each of the demand distribution types.

A detailed summary of the results for each distribution can be found in **Tables (4)-(7)** in the appendix. While the first six columns of these tables show the problem parameters, the two columns under "Gurobi" give the optimal solution and runtime for each instance respectively whereas the three columns under "Heuristic" give the best, average, and worst results found of the ten runs of the heuristic. Finally, the last three columns report the gaps found in the ten runs of the heuristic. We did not report the runtime needed for heuristics as they were set as 60 seconds for all the instances regardless of the problem settings. Please note that for those instances Gurobi was unable to reach an optimal solution in five hours, the lower bound has been reported and the corresponding row has been made boldfaced. Besides, the rows of those instances for which the heuristic reached the optimal solution at least once have been highlighted.

Table 3: Summary of the heuristic performance

Parameters				Average Gap			
$\|N\|$	$\|T\|$	$\grave{\lambda}$	π	(1,1)	(1,5)	(2,5)	(5,5)
15	1	2	0.7	0.00%	0.00%	0.16%	0.15%
15	1	2	0.95	0.00%	0.00%	0.11%	0.21%
15	1	5	0.7	0.00%	0.00%	0.17%	0.16%
15	1	5	0.95	0.00%	0.00%	0.21%	0.29%
15	1	∞	0.7	0.00%	0.00%	0.32%	0.18%
15	1	∞	0.95	0.00%	0.00%	0.16%	0.15%
15	3	2	0.7	0.82%	0.75%	0.52%	0.24%
15	3	2	0.95	0.58%	0.35%	0.35%	0.82%
15	3	5	0.7	0.51%	0.83%	0.64%	0.35%
15	3	5	0.95	0.56%	0.58%	0.30%	0.94%
15	3	∞	0.7	0.45%	0.36%	0.46%	0.17%
15	3	∞	0.95	0.31%	0.29%	0.55%	0.24%
15	6	2	0.7	0.53%	0.91%	1.13%	0.95%
15	6	2	0.95	1.07%	0.65%	0.96%	1.33%
15	6	5	0.7	0.51%	0.85%	0.86%	1.22%
15	6	5	0.95	0.59%	0.39%	1.14%	0.40%
15	6	∞	0.7	0.07%	0.35%	0.33%	0.38%
15	6	∞	0.95	0.35%	0.08%	0.63%	0.40%
20	1	2	0.7	0.12%	0.13%	0.26%	0.27%
20	1	2	0.95	0.21%	0.36%	0.22%	0.47%
20	1	5	0.7	0.04%	0.29%	0.34%	0.51%
20	1	5	0.95	0.12%	0.19%	0.12%	0.41%
20	1	∞	0.7	0.00%	0.09%	0.18%	0.22%
20	1	∞	0.95	0.00%	0.16%	0.09%	0.14%
20	3	2	0.7	0.88%	0.84%	1.33%	1.12%
20	3	2	0.95	1.15%	1.08%	0.88%	1.36%
20	3	5	0.7	0.85%	1.08%	0.97%	1.58%
20	3	5	0.95	0.81%	0.96%	0.72%	1.44%
20	3	∞	0.7	0.74%	0.77%	0.77%	1.43%
20	3	∞	0.95	0.56%	0.76%	0.29%	0.94%
20	6	2	0.7	1.05%	1.39%	1.62%	1.39%
20	6	2	0.95	1.06%	1.20%	1.32%	1.29%
20	6	5	0.7	0.99%	0.95%	1.18%	1.47%
20	6	5	0.95	1.32%	1.18%	1.13%	1.33%
20	6	∞	0.7	0.78%	0.86%	1.02%	1.34%
20	6	∞	0.95	0.32%	0.40%	0.90%	0.96%
25	1	2	0.7	0.37%	0.42%	1.01%	0.74%
25	1	2	0.95	0.47%	0.45%	0.85%	0.46%
25	1	5	0.7	0.49%	0.44%	0.44%	0.64%
25	1	5	0.95	0.05%	0.51%	0.69%	0.66%
25	1	∞	0.7	0.12%	0.62%	0.82%	0.35%
25	1	∞	0.95	0.05%	0.49%	0.42%	0.34%
25	3	2	0.7	1.77%	0.73%	1.07%	1.63%
25	3	2	0.95	1.23%	0.89%	1.18%	1.51%
25	3	5	0.7	1.12%	1.08%	1.04%	1.68%
25	3	5	0.95	1.76%	0.70%	1.47%	2.26%
25	3	∞	0.7	0.85%	0.53%	0.91%	1.76%
25	3	∞	0.95	1.25%	0.38%	0.77%	1.64%
25	6	2	0.7	2.31%	1.65%	1.56%	1.79%
25	6	2	0.95	4.25%	2.03%	1.08%	1.21%
25	6	5	0.7	2.32%	1.22%	2.40%	1.26%
25	6	5	0.95	1.67%	1.26%	1.29%	1.50%
25	6	∞	0.7	1.20%	0.94%	1.11%	1.41%
25	6	∞	0.95	1.52%	0.84%	0.85%	0.92%

Results show that while for the instances with 15 nodes, Gurobi was able to solve all the instances to optimality within 11 minutes, it needed more than two hours to solve some instances with $|N| = 20$, and failed to reach an optimal solution for some of the instances with $|N| = 25$ within five hours. Our experiments showed that for larger node sizes, Gurobi failed to reach an optimal solution in much longer times. However, the proposed heuristic consumed considerably lower times (60 seconds) to reach solutions which are not worse than 0.55% from the optimal solution on average. In particular, for the large-scale problems, the proposed heuristic reached solutions which are 0.81% higher than the optimal solutions on average. However, as **Figure (8)** shows, for a case with 25 nodes and six periods, Gurobi was unable to reach an optimal solution with less than 15% gap to the optimal within two hours. It should be emphasised that there is no guarantee with the proposed heuristic to reach an optimal solution. However, for larger instances in which Gurobi is unable to find the optimal solution, the heuristic can offer a near-optimal solution as shown in the numerical experiments.

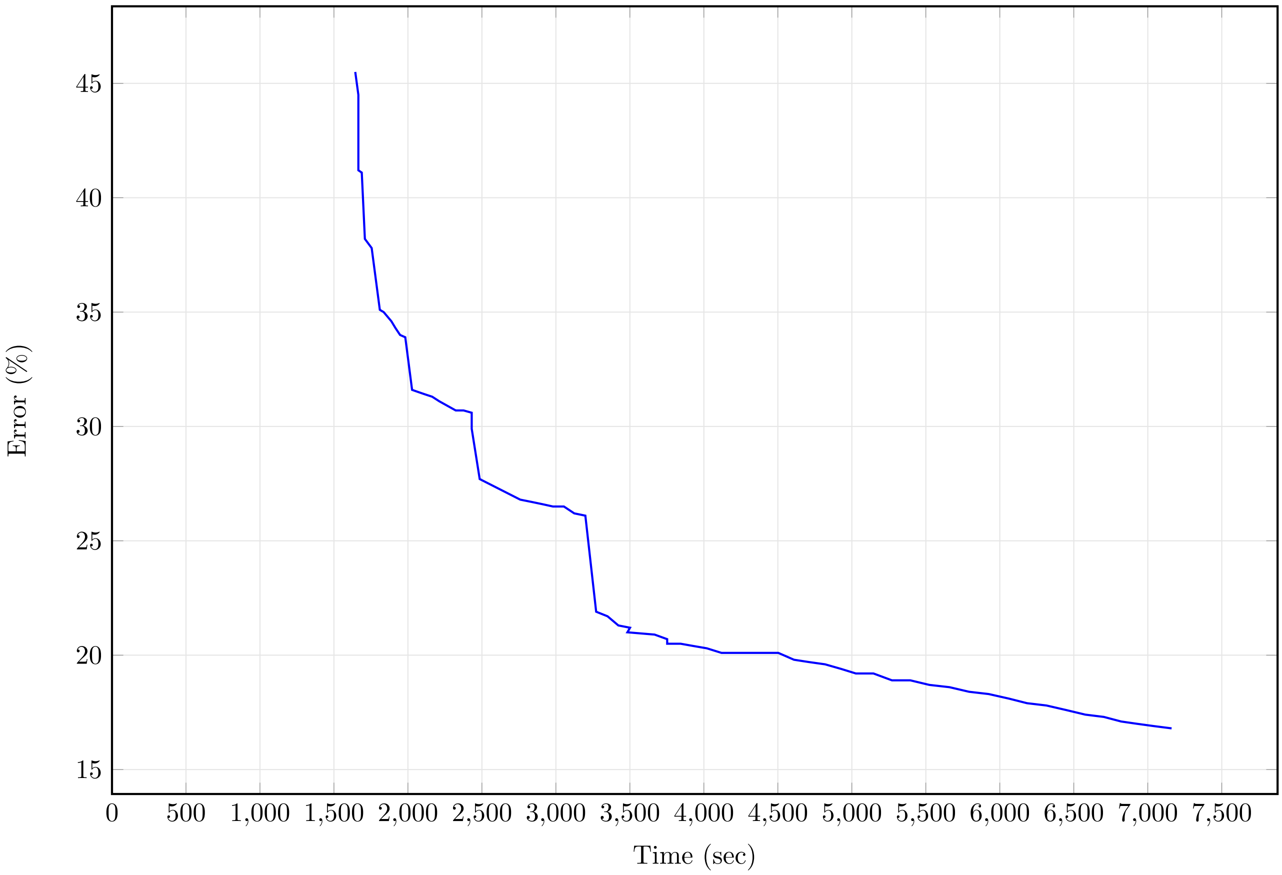

Figure 8: Performance of Gurobi on an instance of 25/6/(1,5)/2/0.7

One of the observations is that the computational requirement grows exponentially by an increase in the number of nodes and periods, whereas an increase in the value of congestion parameter ($\bar{\lambda}$) is strongly correlated with a decrease in the runtime. While all the instances with a $\lambda = \infty$ were solved in less than a minute, only 24 out of 48 with $\lambda = 2$ or $\lambda = 5$ and $|N| = 25$ were solved optimally in less than ten minutes. Moreover, the heuristic reached the optimal solution in 36 and resulted in solutions within an error margin of one percent in 186 out of 216 instances which are clear indications that the proposed heuristic performs significantly better in terms of the runtime while its solution quality is comparable to Gurobi.

Another observation in the running of the heuristic was the ability of the proposed heuristic to avoid getting stuck in local optima. For instance, **Figure (9)** depicts a sample run of the heuristic for 60 seconds where the algorithm reduced the gap gradually without being stuck in local optima.

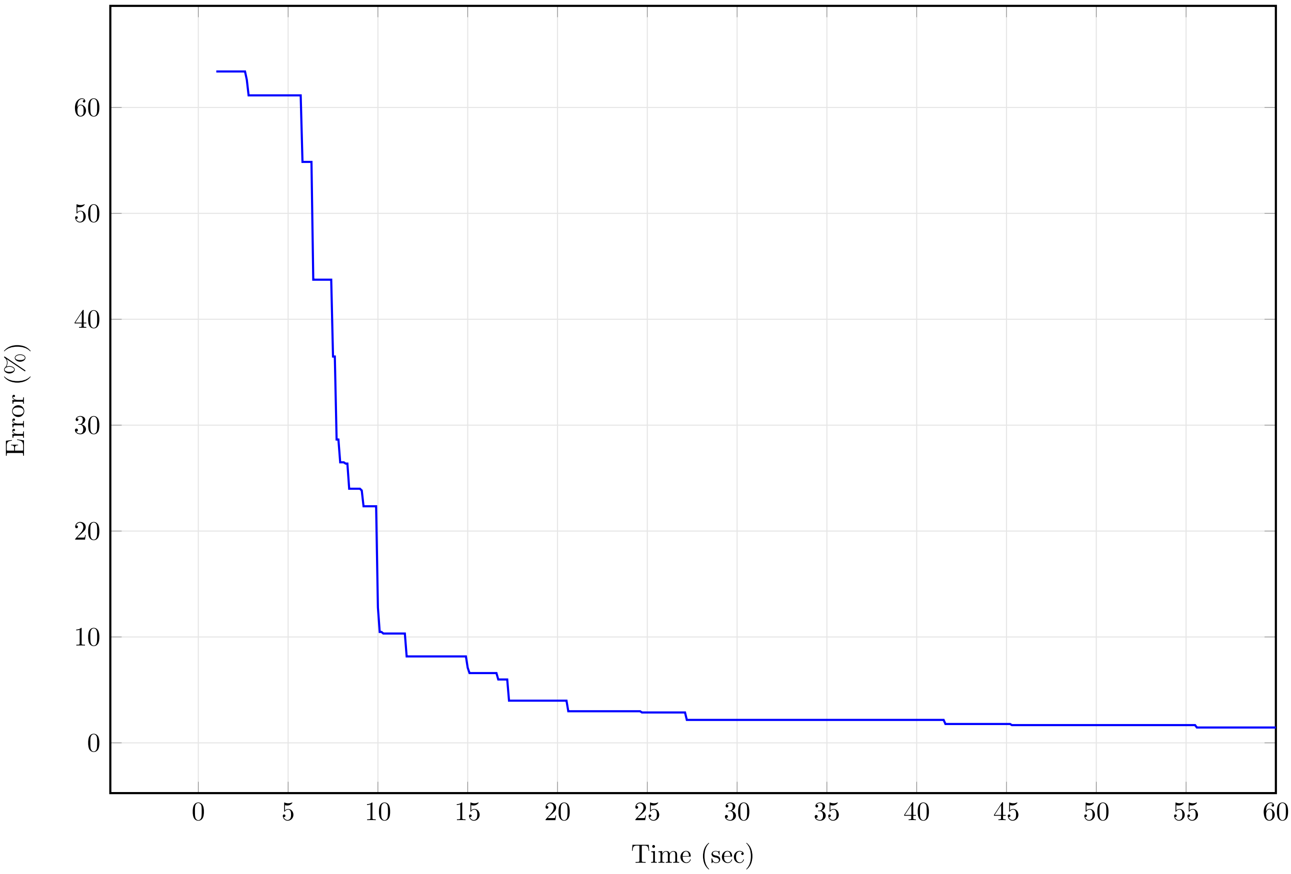

Figure 9: Performance of the algorithm on an 25/6/(1,1)/5/0.7

In order to analyse the efficiency of the proposed heuristic, we used two measures simultaneously, namely the runtime and also the Relative Percentage Deviation (RPD) as **Equation (33)**:

$$RPD = \frac{z - z^*}{z^*} \tag{33}$$

where z and z^* are the best found fitness using the proposed VNS and the optimal value found using Gurobi respectively. While for small-sized problems, we compared the performance of the proposed procedure with the optimal solution found with Gurobi, the performance for the larger ones, for which Gurobi was unable to find the optimal solution within five hours, was compared with the lower bounds.

We observe that the performance of the proposed heuristic is not far from the Gurobi results with an average error of 0.81% for those instances Gurobi managed to solve. However, in terms of the runtime, VNS is better by an order of magnitude. The gap of VNS is higher for those instances with a lower value of $\bar{\lambda}$ which can be attributed to the fact that these instances are more difficult to solve. However, this issue is offset considering the fact that the time needed to solve these instances to optimality is high as Gurobi fails to reach an optimal even in five hours (For some instances, no optimal solution is found within even twelve hours). Furthermore, in 50% of test instances, the proposed general SVNS could reach the optimal solution in at least one of the runs and in almost 20% of the cases, it gets the optimal results in all the ten runs. Therefore, the proposed algorithm is capable in reaching high quality solutions in considerably less time than Gurobi.

In order to examine the performance of the proposed heuristic for larger datasets, we have carried out a set of complimentary analysis for larger datasets of $|N| \in \{100, 1000\}$ with increments of 100 and the results are given in **Tables (8)-(10)**. Since Gurobi fails to find the optimal solutions for these instances in a reasonable time (even a week), we were unable to compare the performance of heuristic with the optimal results. However, the best, average, and worst solutions of the ten runs are reported along with the standard deviation of the results in the last column which has been shown as σ.

We have done a set of additional tests to find out if the performance of the heuristic is affected by using incomplete graphs. To this end, we used the standard p-median test problems of Beasley [70] for $|N| \in \{100, 200, 300, 400, 500\}$, $|T| \in \{1, 3, 6\}$, $\bar{\lambda} = 50$ and $\pi = 0.7$. Results of these test problems are given in Table (11) showing that the heuristic performs well for the incomplete graphs as well, leading to consistent results.

We carried out an additional test to examine the efficiency of the skewed VNS and if it makes a significant difference compared to the general VNS. To do so, we ran the same test problems with a general VNS ($\kappa = 0$) and compared the means using the unpaired t-test. These analyses have led to p-values equal to 0.093, 0.066, 0.041, and 0.085 for $(\alpha, \beta) = (1, 1)$, $(\alpha, \beta) = (1, 5)$, $(\alpha, \beta) = (2, 5)$, and $(\alpha, \beta) = (5, 5)$ respectively. Therefore, we can conclude that there is a significant difference between the general VNS and the proposed skewed VNS at ten percent level of confidence.

7 Conclusion and future research

The literature of facility location models has relatively less publications for multi-period models than single-period ones which can be attributed to the complexity of these problems and the high computational effort needed to solve them. Moreover, the cooperative facility location problem is relatively new. In this paper, we tried to fill this gap by considering a multi-period cooperative facility location problem for preventive health care centres. We proposed a linear integer programming model and developed an efficient heuristic to solve it. Our experiments using a set of randomly generated data showed that the proposed heuristic is able to reach near-optimal solutions in considerably less runtimes compared to the optimal solutions found by Gurobi Optimizer.

We believe that future research stems from considering probabilistic choice environment, assuming uncertain travel times or developing other heuristics. Another appealing future research is to assume a case in which preventive facilities are dynamic, like immunisation programs. Then, the problem is to locate facilities dynamically and to decide on the time the locations should be changed. As another possible extension to the IC-PHNDP, other qualitative factors such as quality of the healthcare facility, availability of amenities near the facility, etc. which also influence the attractiveness of the healthcare facility besides the proximity, can also be incorporated while modelling participation to preventive programs. Another interesting area of research would be to compare the performance of Gurobi with other solvers to find out if any solver shows a better performance to solve IC-PHNDP. Last but not least, one can model the participation by means of fuzzy numbers rather than crisp numbers and to utilise fuzzy mathematical programming approaches.

Appendix. Numerical Results

Table 4: Results for the instance with (α,β)=(1,1)

$\lvert N\rvert$	$\lvert T\rvert$	α	β	$\bar{\lambda}$	π	Gurobi		Heuristic			Gap		
						Objective	Time (s)	Best	Average	Worst	Best	Average	Worst
15	1	1	1	2	0.7	334,687	2.0	334,687	334,687	334,687	0.00%	0.00%	0.00%
15	1	1	1	2	0.95	334,687	2.1	334,687	334,687	334,687	0.00%	0.00%	0.00%
15	1	1	1	5	0.7	86,914	1	86,914	86,914	86,914	0.00%	0.00%	0.00%
15	1	1	1	5	0.95	86,914	1.1	86,914	86,914	86,914	0.00%	0.00%	0.00%
15	1	1	1	∞	0.7	29,626	<1	29,626	29,626	29,626	0.00%	0.00%	0.00%
15	1	1	1	∞	0.95	29,626	<1	29,626	29,626	29,626	0.00%	0.00%	0.00%
15	3	1	1	2	0.7	412,718	<1	414,823	416,086	416,423	0.51%	0.82%	0.90%
15	3	1	1	2	0.95	412,718	<1	414,905	415,124	416,568	0.53%	0.58%	0.93%
15	3	1	1	5	0.7	141,895	4.5	142,548	142,613	142,900	0.46%	0.51%	0.71%
15	3	1	1	5	0.95	107,187	4.2	107,734	107,788	108,330	0.51%	0.56%	1.07%
15	3	1	1	∞	0.7	107,929	1.0	108,231	108,413	108,509	0.28%	0.45%	0.54%
15	3	1	1	∞	0.95	44,809	<1	44,926	44,949	45,047	0.26%	0.31%	0.53%
15	6	1	1	2	0.7	607,902	283.1	610,394	611,142	611,790	0.41%	0.53%	0.64%
15	6	1	1	2	0.95	607,902	250.0	611,975	614,419	616,374	0.67%	1.07%	1.39%
15	6	1	1	5	0.7	263,743	6.5	264,640	265,088	265,626	0.34%	0.51%	0.71%
15	6	1	1	5	0.95	163,405	47.0	164,042	164,361	165,221	0.39%	0.59%	1.11%
15	6	1	1	∞	0.7	246,927	2.4	247,075	247,090	247,204	0.06%	0.07%	0.11%
15	6	1	1	∞	0.95	79,688	<1	79,919	79,965	80,159	0.29%	0.35%	0.59%
20	1	1	1	2	0.7	452,865	19.9	452,865	453,408	453,898	0.00%	0.12%	0.23%
20	1	1	1	2	0.95	452,865	21.1	452,865	453,816	454,006	0.00%	0.21%	0.25%
20	1	1	1	5	0.7	107,534	1.6	107,534	107,577	107,620	0.00%	0.04%	0.08%
20	1	1	1	5	0.95	107,534	1.5	107,534	107,663	107,740	0.00%	0.12%	0.19%
20	1	1	1	∞	0.7	29,626	<1	29,626	29,626	29,626	0.00%	0.00%	0.00%
20	1	1	1	∞	0.95	29,626	<1	29,626	29,626	29,626	0.00%	0.00%	0.00%
20	3	1	1	2	0.7	581,278	496.4	585,521	586,370	588,916	0.73%	0.88%	1.31%
20	3	1	1	2	0.95	581,278	446.3	585,463	587,974	594,671	0.72%	1.15%	2.30%
20	3	1	1	5	0.7	165,260	17.2	166,334	166,656	167,355	0.65%	0.85%	1.27%
20	3	1	1	5	0.95	150,619	20.8	151,432	151,839	152,937	0.54%	0.81%	1.54%
20	3	1	1	∞	0.7	122,044	2.4	122,642	122,941	123,031	0.49%	0.74%	0.81%
20	3	1	1	∞	0.95	44,809	<1	44,975	45,058	45,306	0.37%	0.56%	1.11%
20	6	1	1	2	0.7	703,727	6274.2	710,412	711,081	716,964	0.95%	1.05%	1.88%
20	6	1	1	2	0.95	737,308	1,937.2	743,796	745,094	752,880	0.88%	1.06%	2.11%
20	6	1	1	5	0.7	342,645	315.6	344,769	346,044	347,744	0.62%	0.99%	1.49%
20	6	1	1	5	0.95	219,152	705.9	221,081	222,045	224,359	0.88%	1.32%	2.38%
20	6	1	1	∞	0.7	298,106	7.6	300,223	300,434	301,831	0.71%	0.78%	1.25%
20	6	1	1	∞	0.95	81,201	2.1	81,436	81,460	81,719	0.29%	0.32%	0.64%
25	1	1	1	2	0.7	427,843	42.3	428,827	429,417	430,834	0.23%	0.37%	0.70%
25	1	1	1	2	0.95	427,843	41.1	429,683	429,867	431,688	0.43%	0.47%	0.90%
25	1	1	1	5	0.7	122,433	6.8	122,862	123,033	123,153	0.35%	0.49%	0.59%
25	1	1	1	5	0.95	122,433	7.0	122,666	122,735	123,008	0.19%	0.25%	0.47%
25	1	1	1	∞	0.7	29,626	<1	29,659	29,662	29,680	0.11%	0.12%	0.18%
25	1	1	1	∞	0.95	29,626	<1	29,626	29,641	29,647	0.00%	0.05%	0.07%
25	3	1	1	2	0.7	589,540	1,808.7	596,497	599,975	608,323	1.18%	1.77%	3.19%
25	3	1	1	2	0.95	589,540	2,200.0	594,728	596,803	598,256	0.88%	1.23%	1.48%
25	3	1	1	5	0.7	197,395	326.0	199,408	199,610	200,496	1.02%	1.12%	1.57%
25	3	1	1	5	0.95	173,639	186.1	175,983	176,686	179,429	1.35%	1.76%	3.33%
25	3	1	1	∞	0.7	127,596	<1	128,591	128,691	128,800	0.78%	0.86%	0.94%
25	3	1	1	∞	0.95	43,970	<1	44,335	44,517	44,846	0.83%	1.25%	1.99%
25	6	1	1	2	0.7	838,646	9472.7	851,561	858,019	867,705	1.54%	2.31%	3.47%
25	**6**	**1**	**1**	**2**	**0.95**	**727,461**	**18,000**	**740,046**	**743,821**	**745,457**	**3.73%**	**4.25%**	**4.47%**
25	6	1	1	5	0.7	382,977	889.7	388,530	391,862	395,416	1.45%	2.32%	3.25%
25	6	1	1	5	0.95	246,099	2802.2	249,520	250,204	251,025	1.39%	1.67%	2.00%
25	6	1	1	∞	0.7	322,585	15.6	325,553	326,443	329,530	0.92%	1.20%	2.15%
25	6	1	1	∞	0.95	79688	2.5	80,493	80,895	81,257	1.01%	1.52%	1.97%

Table 5: Results for the instance with (α,β)=(1,5)

$\|N\|$	$\|T\|$	α	β	$\bar{\lambda}$	π	Gurobi		Heuristic			Gap		
						Objective	Time (s)	Best	Average	Worst	Best	Average	Worst
15	1	1	5	2	0.7	276,585	1.9	276,585	27,6585	276,585	0.00%	0.00%	0.00%
15	1	1	5	2	0.95	276,585	1.9	276,585	276,585	276,585	0.00%	0.00%	0.00%
15	1	1	5	5	0.7	87,198	<1	87,198	87,198	87,198	0.00%	0.00%	0.00%
15	1	1	5	5	0.95	87,198	<1	87,198	87,198	87,198	0.00%	0.00%	0.00%
15	1	1	5	∞	0.7	13,661	<1	13,661	13,661	13,661	0.00%	0.00%	0.00%
15	1	1	5	∞	0.95	13,661	<1	13,661	13,661	13,661	0.00%	0.00%	0.00%
15	3	1	5	2	0.7	368,519	72.1	370,656	371,298	371,575	0.58%	0.75%	0.83%
15	3	1	5	2	0.95	368,519	56.1	369,588	369,801	369,930	0.29%	0.35%	0.38%
15	3	1	5	5	0.7	115592	4.2	116,228	116,546	116,927	0.55%	0.83%	1.16%
15	3	1	5	5	0.95	115592	3.6	116,008	116,258	116,591	0.36%	0.58%	0.86%
15	3	1	5	∞	0.7	37,090	<1	37,194	37,225	37,266	0.28%	0.36%	0.47%
15	3	1	5	∞	0.95	22250	<1	22,297	22,315	22,322	0.21%	0.29%	0.32%
15	6	1	5	2	0.7	449,711	189.8	452,634	453,803	454,622	0.65%	0.91%	1.09%
15	6	1	5	2	0.95	449,711	176.3	452,364	452,630	455,548	0.59%	0.65%	1.30%
15	6	1	5	5	0.7	149,682	16.2	150,745	150,957	151,722	0.71%	0.85%	1.36%
15	6	1	5	5	0.95	143,061	10.0	143,562	143,612	143,667	0.35%	0.39%	0.42%
15	6	1	5	∞	0.7	100,754	1.4	101,046	101,105	101,350	0.29%	0.35%	0.59%
15	6	1	5	∞	0.95	31,885	<1	31,901	31,909	31,918	0.05%	0.08%	0.11%
20	1	1	5	2	0.7	355,451	6.5	355,842	355,920	356,061	0.11%	0.13%	0.17%
20	1	1	5	2	0.95	355,451	6.4	356,304	356,731	358,010	0.24%	0.36%	0.72%
20	1	1	5	5	0.7	113,597	2.5	113,801	113,924	114,251	0.18%	0.29%	0.58%
20	1	1	5	5	0.95	113597	2.4	113,733	113,815	114,033	0.12%	0.19%	0.38%
20	1	1	5	∞	0.7	13661	<1	13,672	13,673	13,680	0.08%	0.09%	0.14%
20	1	1	5	∞	0.95	13661	<1	13,661	13,683	13,703	0.00%	0.16%	0.30%
20	3	1	5	2	0.7	510207	238.8	514,085	514,472	514,899	0.76%	0.84%	0.92%
20	3	1	5	2	0.95	510,207	214.3	515,207	515,707	518,457	0.98%	1.08%	1.62%
20	3	1	5	5	0.7	155,869	29.4	157,163	157,551	158,728	0.83%	1.08%	1.83%
20	3	1	5	5	0.95	155,869	31.3	157,225	157,361	158,554	0.87%	0.96%	1.72%
20	3	1	5	∞	0.7	37,090	<1	37,327	37,375	37,460	0.64%	0.77%	1.00%
20	3	1	5	∞	0.95	22,250	<1	22,370	22,418	22,452	0.54%	0.76%	0.91%
20	6	1	5	2	0.7	602,010	1,968.5	607,247	610,390	615,418	0.87%	1.39%	2.23%
20	6	1	5	2	0.95	602,010	1,299.4	606,826	609,234	612,124	0.80%	1.20%	1.68%
20	6	1	5	5	0.7	200,509	214.1	201,772	202,404	204,299	0.63%	0.95%	1.89%
20	6	1	5	5	0.95	200,509	238.7	202,334	202,881	205,253	0.91%	1.18%	2.37%
20	6	1	5	∞	0.7	95,198	3.1	95,712	96,021	96,350	0.54%	0.86%	1.21%
20	6	1	5	∞	0.95	31,885	<1	31,965	32,013	32,038	0.25%	0.40%	0.48%
25	1	1	5	2	0.7	471,281	35.4	472,930	473,260	474,052	0.35%	0.42%	0.59%
25	1	1	5	2	0.95	471,281	33.5	473,213	473,406	473,619	0.41%	0.45%	0.50%
25	1	1	5	5	0.7	127,433	21.1	127,803	127,987	128,542	0.29%	0.44%	0.87%
25	1	1	5	5	0.95	127,433	18.7	127,930	128,079	128,531	0.39%	0.51%	0.86%
25	1	1	5	∞	0.7	13661	<1	13,717	13,745	13,795	0.41%	0.62%	0.98%
25	1	1	5	∞	0.95	13,661	<1	13,709	13,728	13,741	0.35%	0.49%	0.59%
25	3	1	5	2	0.7	585,441	771.1	589,305	589,691	592,667	0.66%	0.73%	1.23%
25	3	1	5	2	0.95	585,441	601.9	590,183	590,657	595,352	0.81%	0.89%	1.69%
25	3	1	5	5	0.7	172,858	101.1	174,189	174,721	176,585	0.77%	1.08%	2.16%
25	3	1	5	5	0.95	172,858	88.0	173,791	174,071	174,557	0.54%	0.70%	0.98%
25	3	1	5	∞	0.7	37,090	<1	37,220	37,285	37,460	0.35%	0.53%	1.00%
25	3	1	5	∞	0.95	21,720	<1	21,783	21,802	21,859	0.29%	0.38%	0.64%
25	6	1	5	2	0.7	748,191	15,628.0	757,020	760,551	767,967	1.18%	1.65%	2.64%
25	6	1	5	2	0.95	74,8191	6,351.3	757,693	763,394	772,516	1.27%	2.03%	3.25%
25	6	1	5	5	0.7	246,657	1,579.4	249,395	249,669	250,271	1.11%	1.22%	1.47%
25	6	1	5	5	0.95	246,657	802.0	248,877	249,765	251,319	0.90%	1.26%	1.89%
25	6	1	5	∞	0.7	95,964	3.9	96,713	96,862	97,491	0.78%	0.94%	1.59%
25	6	1	5	∞	0.95	31,885	<1	32,064	32,153	32,314	0.56%	0.84%	1.34%

Table 6: Results for the instance with (α,β)=(2,5)

$\|N\|$	$\|T\|$	α	β	$\bar{\lambda}$	π	Gurobi		Heuristic			Gap		
						Objective	Time (s)	Best	Average	Worst	Best	Average	Worst
15	1	2	5	2	0.7	334,751	3.2	334,751	335,287	335,769	0.00%	0.16%	0.30%
15	1	2	5	2	0.95	334751	3.2	334,952	335,119	335,340	0.06%	0.11%	0.18%
15	1	2	5	5	0.7	97552	<1	97,552	97,718	97,884	0.00%	0.17%	0.34%
15	1	2	5	5	0.95	97,552	<1	97,581	97,757	97,798	0.03%	0.21%	0.25%
15	1	2	5	∞	0.7	13,661	<1	13,661	13,705	13,718	0.00%	0.32%	0.42%
15	1	2	5	∞	0.95	13,661	<1	13,661	13,683	13,692	0.00%	0.16%	0.22%
15	3	2	5	2	0.7	418,084	77.0	419,756	420,258	422,215	0.40%	0.52%	0.99%
15	3	2	5	2	0.95	418084	74.6	419,046	419,526	420,680	0.23%	0.35%	0.62%
15	3	2	5	5	0.7	129,164	6.7	129,913	129,988	130,730	0.58%	0.64%	1.21%
15	3	2	5	5	0.95	129,164	8.8	129,513	129,548	129,855	0.27%	0.30%	0.53%
15	3	2	5	∞	0.7	52,040	<1	52,212	52,280	52,521	0.33%	0.46%	0.92%
15	3	2	5	∞	0.95	22559	<1	22,654	22,682	22,719	0.42%	0.55%	0.71%
15	6	2	5	2	0.7	504,970	652.1	509,717	510,666	512,944	0.94%	1.13%	1.58%
15	6	2	5	2	0.95	504,970	518.6	509,363	509,803	512,702	0.87%	0.96%	1.53%
15	6	2	5	5	0.7	206,586	39.0	207,949	208,359	208,890	0.66%	0.86%	1.12%
15	6	2	5	5	0.95	175,112	62.8	176,355	177,101	178,295	0.71%	1.14%	1.82%
15	6	2	5	∞	0.7	155,336	4.2	155,724	155,841	156,144	0.25%	0.33%	0.52%
15	6	2	5	∞	0.95	47,475	<1	47,689	47,774	47,924	0.45%	0.63%	0.95%
20	1	2	5	2	0.7	457,064	20.3	457,795	458,234	458,936	0.16%	0.26%	0.41%
20	1	2	5	2	0.95	457,064	19.0	457,064	458,070	458,572	0.00%	0.22%	0.33%
20	1	2	5	5	0.7	133,322	4.0	133,735	133,777	134,095	0.31%	0.34%	0.58%
20	1	2	5	5	0.95	133,322	3.1	133,469	133,483	133,516	0.11%	0.12%	0.15%
20	1	2	5	∞	0.7	13,661	<1	13,661	13,686	13,710	0.00%	0.18%	0.36%
20	1	2	5	∞	0.95	13,661	<1	13,661	13,673	13,676	0.00%	0.09%	0.11%
20	3	2	5	2	0.7	562,675	1,128.3	568,921	570,170	576,915	1.11%	1.33%	2.53%
20	3	2	5	2	0.95	562,675	1,042.1	566,783	567,604	569,576	0.73%	0.88%	1.23%
20	3	2	5	5	0.7	163,021	48.5	164,456	164,599	164,757	0.88%	0.97%	1.06%
20	3	2	5	5	0.95	163,021	39.2	164,081	164,187	164,536	0.65%	0.72%	0.93%
20	3	2	5	∞	0.7	63,447	1.6	63,821	63,934	64,420	0.59%	0.77%	1.53%
20	3	2	5	∞	0.95	22,559	<1	22,609	22,624	22,662	0.22%	0.29%	0.46%
20	6	2	5	2	0.7	563,738	12,028.7	569,432	572,848	574,670	1.01%	1.62%	1.94%
20	6	2	5	2	0.95	579,115	10,571.0	584,559	586,737	592,071	0.94%	1.32%	2.24%
20	6	2	5	5	0.7	248,071	498.2	250,328	251,006	251,593	0.91%	1.18%	1.42%
20	6	2	5	5	0.95	225,091	933.5	227,049	227,637	228,146	0.87%	1.13%	1.36%
20	6	2	5	∞	0.7	169325	4.5	170,561	171,056	171,402	0.73%	1.02%	1.23%
20	6	2	5	∞	0.95	53805	1.4	54,106	54,287	54,335	0.56%	0.90%	0.99%
25	1	2	5	2	0.7	544,986	101.8	549,237	550,512	554933	0.78%	1.01%	1.83%
25	1	2	5	2	0.95	544,986	96.9	548,310	549,640	550,571	0.61%	0.85%	1.02%
25	1	2	5	5	0.7	161,749	32.7	162,218	162,453	162,945	0.29%	0.44%	0.74%
25	1	2	5	5	0.95	161,749	21.1	162,445	162,862	163,196	0.43%	0.69%	0.89%
25	1	2	5	∞	0.7	13,661	<1	13,731	13,772	13,806	0.51%	0.82%	1.06%
25	1	2	5	∞	0.95	13,661	<1	13,697	13,718	13,746	0.26%	0.42%	0.62%
25	3	2	5	2	0.7	652,828	973.9	658,638	659,800	665,378	0.89%	1.07%	1.92%
25	3	2	5	2	0.95	652,828	2358.6	659,813	660,512	661,280	1.07%	1.18%	1.29%
25	3	2	5	5	0.7	197,837	230.2	199,420	199,895	200,718	0.80%	1.04%	1.46%
25	3	2	5	5	0.95	197,837	212.0	199,776	200,745	202,490	0.98%	1.47%	2.35%
25	3	2	5	∞	0.7	56,428	1.1	56,857	56,943	57,046	0.76%	0.91%	1.09%
25	3	2	5	∞	0.95	21720	<1	21,824	21,887	21,987	0.48%	0.77%	1.23%
25	**6**	**2**	**5**	**2**	**0.7**	**761,326**	**18,000.0**	**770,462**	**773,203**	**783,892**	**1.20%**	**1.56%**	**2.96%**
25	**6**	**2**	**5**	**2**	**0.95**	**799570**	**18,000.0**	**807,406**	**808,189**	**815,947**	**0.98%**	**1.08%**	**2.05%**
25	6	2	5	5	0.7	278,346	2,220.0	282,521	285,026	287,030	1.50%	2.40%	3.12%
25	6	2	5	5	0.95	272,862	3,222.0	275,209	276,382	279,198	0.86%	1.29%	2.32%
25	6	2	5	∞	0.7	165,051	13.5	166,355	166,876	167,059	0.79%	1.11%	1.22%
25	6	2	5	∞	0.95	48,569	1.7	48,865	48,984	49,274	0.61%	0.85%	1.45%

Table 7: Results for the instance with (α,β)=(5,5)

$\|N\|$	$\|T\|$	α	β	$\bar{\lambda}$	π	Gurobi		Heuristic			Gap		
						Objective	Time (s)	Best	Average	Worst	Best	Average	Worst
15	1	5	5	2	0.7	285,263	3.6	285,263	285,691	285,905	0.00%	0.15%	0.23%
15	1	5	5	2	0.95	285,263	3.5	285,263	285,862	286,102	0.00%	0.21%	0.29%
15	1	5	5	5	0.7	87,198	<1	87,198	87,338	87,463	0.00%	0.16%	0.30%
15	1	5	5	5	0.95	87,198	<1	87,198	87,451	87,577	0.00%	0.29%	0.44%
15	1	5	5	∞	0.7	13,661	<1	13,661	13,686	13,700	0.00%	0.18%	0.29%
15	1	5	5	∞	0.95	13,661	<1	13,661	13,681	13,684	0.00%	0.15%	0.17%
15	3	5	5	2	0.7	364,987	50.1	365,790	365,870	366,224	0.22%	0.24%	0.34%
15	3	5	5	2	0.95	364,987	47.2	366,848	367,965	368,859	0.51%	0.82%	1.06%
15	3	5	5	5	0.7	113,337	3.7	113,700	113,736	113,856	0.32%	0.35%	0.46%
15	3	5	5	5	0.95	113,337	4.5	114,006	114,407	115,263	0.59%	0.94%	1.70%
15	3	5	5	∞	0.7	51,731	<1	51,731	51,819	51,898	0.00%	0.17%	0.32%
15	3	5	5	∞	0.95	22,250	<1	22,299	22,304	22,352	0.22%	0.24%	0.46%
15	6	5	5	2	0.7	455,523	275.6	458,621	459,860	463,329	0.68%	0.95%	1.71%
15	6	5	5	2	0.95	455,523	244.7	460,169	461,563	465,791	1.02%	1.33%	2.25%
15	6	5	5	5	0.7	179,837	19.9	181,204	182,024	183,336	0.76%	1.22%	1.95%
15	6	5	5	5	0.95	157,942	40.5	158,511	158,567	158,630	0.36%	0.40%	0.44%
15	6	5	5	∞	0.7	138,651	3.3	139,053	139,174	139,383	0.29%	0.38%	0.53%
15	6	5	5	∞	0.95	47,475	1.3	47,632	47,663	47,851	0.33%	0.40%	0.79%
20	1	5	5	2	0.7	359,325	23.6	359,936	360,302	361,084	0.17%	0.27%	0.49%
20	1	5	5	2	0.95	359,325	21.9	360,439	360,996	361,330	0.31%	0.47%	0.56%
20	1	5	5	5	0.7	106,636	2.3	106,636	107,180	107,343	0.00%	0.51%	0.66%
20	1	5	5	5	0.95	106,636	2.2	106,945	107,069	107,156	0.29%	0.41%	0.49%
20	1	5	5	∞	0.7	13,661	<1	13,661	13,691	13,697	0.00%	0.22%	0.26%
20	1	5	5	∞	0.95	13,661	<1	13,676	13,681	13,700	0.11%	0.14%	0.29%
20	3	5	5	2	0.7	469,628	415.5	473,996	474,869	479,586	0.93%	1.12%	2.12%
20	3	5	5	2	0.95	469,628	454.4	474,935	475,996	479,817	1.13%	1.36%	2.17%
20	3	5	5	5	0.7	156,222	43.0	157,769	158,697	160,429	0.99%	1.58%	2.69%
20	3	5	5	5	0.95	156,222	44.0	157,831	158,475	160,277	1.03%	1.44%	2.60%
20	3	5	5	∞	0.7	56,571	<1	57,148	57,379	57,460	1.02%	1.43%	1.57%
20	3	5	5	∞	0.95	22,250	<1	22,399	22,459	22,584	0.67%	0.94%	1.50%
20	6	5	5	2	0.7	576,945	10,812.0	583,118	584,970	589,785	1.07%	1.39%	2.23%
20	6	5	5	2	0.95	619,430	9,358.8	625,129	627,408	630,600	0.92%	1.29%	1.80%
20	6	5	5	5	0.7	205,960	136.0	207,855	208,992	210,508	0.92%	1.47%	2.21%
20	6	5	5	5	0.95	196,848	268.4	199,033	199,470	201,568	1.11%	1.33%	2.40%
20	6	5	5	∞	0.7	145,360	4.9	146,755	147,314	148,290	0.96%	1.34%	2.02%
20	6	5	5	∞	0.95	37,114	<1	37,337	37,470	37,684	0.60%	0.96%	1.54%
25	1	5	5	2	0.7	451,850	28.2	454,245	455,203	455,538	0.53%	0.74%	0.82%
25	1	5	5	2	0.95	451,850	27.8	453,567	453,910	455,971	0.38%	0.46%	0.91%
25	1	5	5	5	0.7	122,717	6.7	123,318	123,499	123,655	0.49%	0.64%	0.76%
25	1	5	5	5	0.95	122,717	6.9	123,257	123,527	123,851	0.44%	0.66%	0.92%
25	1	5	5	∞	0.7	13,661	<1	13,692	13,708	13,727	0.23%	0.35%	0.48%
25	1	5	5	∞	0.95	13661	<1	13,661	13,707	13,740	0.00%	0.34%	0.58%
25	3	5	5	2	0.7	573124	759.6	578970	582477	583,413	1.02%	1.63%	1.80%
25	3	5	5	2	0.95	573,124	706.8	580,976	581,761	585,216	1.37%	1.51%	2.11%
25	3	5	5	5	0.7	164,402	87.0	166,128	167,164	169,374	1.05%	1.68%	3.02%
25	3	5	5	5	0.95	164,402	108.7	166,720	168,111	170,173	1.41%	2.26%	3.51%
25	3	5	5	∞	0.7	51,201	<1	51,892	52,100	52,639	1.35%	1.76%	2.81%
25	3	5	5	∞	0.95	21,720	<1	21,957	22,075	22,288	1.09%	1.64%	2.62%
25	**6**	**5**	**5**	**2**	**0.7**	**698,295**	**18,000.0**	**707,233**	**710,809**	**723,322**	**1.28%**	**1.79%**	**3.58%**
25	**6**	**5**	**5**	**2**	**0.95**	**695,576**	**18,000.0**	**702,045**	**703,985**	**707,349**	**0.93%**	**1.21%**	**1.69%**
25	6	5	5	5	0.7	245,205	1,905.9	247,583	248,297	250,771	0.97%	1.26%	2.27%
25	6	5	5	5	0.95	225,201	629.8	227,791	228,568	231,598	1.15%	1.50%	2.84%
25	6	5	5	∞	0.7	150,292	9.6	151,705	152,411	154,106	0.94%	1.41%	2.54%
25	6	5	5	∞	0.95	37,114	1.2	37,426	37,457	37,697	0.84%	0.92%	1.57%

Table 8: Results of $|T| = 1$

Parameters				(1,1)				(1,5)				(2,5)				(5,5)			
$\|N\|$	$\bar{\lambda}$	π	Time	Best	Average	Worst	σ	Best	Average	Worst	σ	Best	Average	Worst	σ	Best	Average	Worst	σ
100	50	0.7	120	30,614	30,882	31,001	129	30,692	30,845	31,061	113	31,234	31,390	31,578	111	29,956	30,166	30,407	128
100	50	0.95	120	30,363	30,651	30,822	168	29,944	30,064	30,244	100	30,734	30,918	30,949	71	29,956	30,226	30,438	150
200	50	0.7	240	63,056	63,687	63,687	284	62,522	63,085	63,211	256	65,146	65,146	65,667	165	62,526	62,588	63,151	174
200	50	0.95	240	64,619	64,656	65,001	144	64,606	64,864	65,513	251	63,582	64,101	64,333	223	62,526	62,963	63,467	283
300	50	0.7	360	96,350	97,314	97,314	520	94,748	95,222	95,793	348	93,979	94,073	94,073	127	93,979	94,073	94,637	208
300	50	0.95	360	97,133	98,104	99,085	607	93,965	94,529	94,718	296	94,762	95,236	95,998	387	93,979	94,355	94,449	239
400	50	0.7	480	130,900	131,011	131,090	289	131,885	132,808	134,003	623	128,692	129,593	130,889	703	127,628	128,649	129,935	688
400	50	0.95	480	127,707	128,984	130,274	911	128,694	129,209	130,243	506	128,692	129,722	130,371	505	127,628	128,649	129,550	690
500	50	0.7	600	162,766	162,991	163,114	386	162,733	163,547	165,019	674	161,390	162,358	162,845	453	161,393	162,684	164,148	822
500	50	0.95	600	162,766	162,766	164,394	483	165,423	165,423	166,581	366	168,115	169,796	169,796	492	161,393	161,715	162,685	456
600	50	0.7	720	201,469	201,554	201,615	395	196,575	197,754	197,952	707	194,945	194,945	194,945	312	194,949	195,339	196,120	461
600	50	0.95	720	196,595	196,595	198,561	591	199,824	201,423	201,423	864	194,945	195,140	195,335	271	194,949	195,728	196,315	518
700	50	0.7	840	233,666	233,666	236,003	721	237,480	239,142	241,533	1,220	239,398	239,637	241,314	573	229,821	231,200	232,356	837
700	50	0.95	840	237,497	239,872	239,872	1,274	229,819	231,658	231,658	932	231,737	231,969	233,825	617	229,821	230,510	231,202	593
800	50	0.7	960	265,438	266,019	266,991	618	261,040	261,823	262,870	701	265,391	267,249	267,783	721	261,040	261,301	262,346	528
800	50	0.95	960	269,789	272,487	275,212	1,648	271,917	272,461	273,278	621	271,917	272,461	272,461	325	261,040	261,040	263,389	730
900	50	0.7	1,080	301,066	301,412	301,879	589	303,467	306,502	306,502	1,514	293,678	295,440	296,622	943	293,678	294,853	296,327	820
900	50	0.95	1,080	305,962	309,022	309,022	1,550	301,020	301,923	302,225	487	303,467	304,377	304,681	457	293,678	293,678	293,972	491
1,000	50	0.7	1,200	337,543	337,810	338,004	691	324,042	326,958	327,285	1,780	334,843	336,852	338,199	1,073	324,042	325,338	328,266	1,304
1,000	50	0.95	1,200	337,543	337,819	338,111	682	324,042	324,690	325,664	693	329,442	331,089	333,407	1,248	324,042	325,986	327,290	1,296
100	∞	0.7	120	15,541	15,541	15,696	39	14,927	15,017	15,122	64	14,563	14,650	14,767	63	14,563	14,665	14,738	56
100	∞	0.95	120	15,417	15,417	15,571	45	14,927	15,002	15,107	53	15,170	15,185	15,276	32	14,563	14,651	14,768	57
200	∞	0.7	240	30,838	30,922	31,004	68	31,191	31,472	31,598	152	30,735	30,766	31,012	87	29,955	30,015	30,015	65
200	∞	0.95	240	31,339	31,652	31,652	151	30,692	30,753	30,968	89	31,234	31,484	31,704	153	29,955	30,105	30,286	105
300	∞	0.7	360	45,467	45,922	45,922	201	45,772	46,092	46,507	226	46,181	46,366	46,644	148	44,663	45,065	45,200	213
300	∞	0.95	360	46,213	46,675	47,142	302	44,656	44,835	45,104	161	45,809	46,002	47,443	550	44,663	44,797	45,111	143
400	∞	0.7	480	62,428	62,428	63,052	194	61,409	61,962	62,458	355	61,433	61,740	61,863	133	60,904	61,209	61,821	272
400	∞	0.95	480	62,428	62,428	63,052	185	61,916	62,226	62,848	283	60,925	61,412	61,535	195	60,904	61,392	61,699	313
500	∞	0.7	600	80,454	81,259	81,259	401	79,163	79,717	80,275	385	79,830	80,548	80,951	358	77,237	77,701	78,167	311
500	∞	0.95	600	77,236	78,008	78,008	374	77,876	78,032	78,344	135	77,255	77,564	78,262	318	77,237	77,778	78,556	415
600	∞	0.7	720	96,948	96,948	97,917	284	93,822	93,916	93,916	197	97,739	98,032	99,012	406	93,815	93,815	94,566	234
600	∞	0.95	720	94,603	95,549	96,504	600	93,822	94,291	94,574	339	95,393	96,252	96,926	198	93,815	94,097	95,038	346
700	∞	0.7	840	114,250	115,393	115,393	565	113,317	114,450	115,022	546	112,401	112,738	113,076	337	110,557	111,331	111,776	518
700	∞	0.95	840	111,486	112,601	113,727	741	111,474	112,031	112,255	329	110,558	110,669	111,776	700	110,557	111,331	111,554	434
800	∞	0.7	960	125,077	125,077	126,328	369	128,165	128,421	129,192	335	130,264	130,525	131,569	251	125,045	125,420	125,545	336
800	∞	0.95	960	126,119	127,380	127,380	572	129,207	129,724	130,892	536	125,053	125,303	125,554	633	125,045	125,170	126,297	393
900	∞	0.7	1,080	145,849	146,144	146,221	344	145,800	145,800	147,258	447	144,641	145,653	145,799	665	139,970	141,370	141,511	756
900	∞	0.95	1,080	145,849	147,307	148,780	925	141,134	141,416	142,547	437	139,975	141,375	142,223	724	139,970	140,810	142,218	823
1,000	∞	0.7	1,200	157,831	159,409	161,003	1,022	155,182	155,337	155,648	328	159,043	159,202	159,361	204	153,902	154,364	155,908	560
1,000	∞	0.95	1,200	153,981	155,521	155,521	718	159,030	159,984	160,784	619	156,477	157,572	158,517	639	153,902	155,441	156,840	957

Table 9: Results of $|T| = 3$

Parameters				(1,1)				(1,5)				(2,5)				(5,5)			
$\|N\|$	$\bar{\lambda}$	π	Time	Best	Average	Worst	σ	Best	Average	Worst	σ	Best	Average	Worst	σ	Best	Average	Worst	σ
100	50	0.7	360	177,495	177,691	177,995	378	42,511	42,639	42,852	111	59,880	60,359	60,419	166	60,212	60,573	60,815	211
100	50	0.95	360	44,017	44,718	45,002	348	43,208	43,251	43,554	110	42,870	43,170	43,386	159	41,814	42,148	42,232	140
200	50	0.7	720	195,994	196,006	196,054	258	89,951	90,041	90,941	285	93,747	94,403	94,875	344	89,953	90,492	91,306	420
200	50	0.95	720	89,968	90,868	90,868	392	93,699	94,542	95,109	474	90,747	91,654	92,021	400	89,953	90,043	90,673	229
300	50	0.7	1,080	201,084	203,095	205,126	1,240	132,860	133,657	134,994	742	135,083	135,488	136,843	539	131,767	132,952	133,484	620
300	50	0.95	1,080	132,904	134,233	134,233	735	131,762	132,948	134,012	727	136,181	136,181	136,317	179	131,767	132,821	132,954	614
400	50	0.7	1,440	207,841	209,919	212,018	1,357	179,017	179,554	180,811	610	181,958	183,232	183,415	457	176,073	177,481	178,546	1,017
400	50	0.95	1,440	179,097	180,888	182,697	1,107	176,082	176,258	176,963	308	181,958	183,596	184,698	861	176,073	177,481	179,078	973
500	50	0.7	1,800	243,900	244,151	244,226	332	235,533	236,946	239,079	1,132	226,115	227,019	228,835	855	226,093	227,223	227,223	627
500	50	0.95	1,800	228,068	228,068	230,349	669	229,880	230,340	232,643	787	233,653	234,120	234,822	363	226,093	226,997	227,678	574
600	50	0.7	2,160	277,407	280,181	282,983	1,475	288,885	290,329	290,329	810	286,572	286,859	288,293	540	277,307	279,526	281,762	1,522
600	50	0.95	2,160	279,719	280,009	281,445	560	281,951	283,361	284,778	900	277,328	278,160	279,273	534	277,307	278,971	281,203	1,272
700	50	0.7	2,520	333,356	336,690	340,057	2,047	330,569	332,883	334,547	1,455	327,828	327,828	328,811	461	327,811	328,467	330,438	970
700	50	0.95	2,520	327,891	331,170	334,482	1,889	327,837	328,493	330,792	871	338,756	341,466	342,149	992	327,811	328,139	330,108	733
800	50	0.7	2,880	385,240	389,092	392,983	2,685	372,706	376,060	377,940	1,705	382,024	382,944	383,003	490	372,706	374,570	378,316	1,627
800	50	0.95	2,880	385,240	386,104	386,267	974	375,812	376,564	379,200	1,189	372,706	372,706	374,942	744	372,706	375,315	377,942	1,871
900	50	0.7	3,240	428,295	432,578	432,578	2,287	421,262	423,368	426,755	1,698	421,262	422,105	423,793	795	417,780	418,198	420,289	892
900	50	0.95	3,240	424,813	429,061	433,352	2,720	428,225	432,079	435,104	2,623	417,780	419,869	421,129	1,071	417,780	420,705	421,126	1,810
1,000	50	0.7	3,600	458,629	458,629	463,215	1,349	466,273	466,273	469,537	896	466,273	470,003	473,293	2,237	458,629	462,757	465,534	2,049
1,000	50	0.95	3,600	462,451	462,881	463,014	1,124	462,451	462,913	466,153	1,159	470,095	473,386	477,173	2,228	458,629	460,005	462,765	1,438
100	∞	0.7	360	178,962	180,752	182,560	1,141	36,982	37,315	37,688	206	55,441	55,829	56,052	199	54,428	54,972	55,137	224
100	∞	0.95	360	32,981	33,311	33,644	202	21,217	21,387	21,473	85	21,371	21,521	21,672	98	20,533	20,697	20,863	109
200	∞	0.7	720	183,653	183,910	184,005	257	44,434	44,656	44,969	180	57,748	58,037	58,559	246	57,205	57,720	57,893	230
200	∞	0.95	720	43,536	43,781	44,001	181	42,732	42,817	42,903	79	42,798	42,884	43,013	72	41,365	41,614	41,739	147
300	∞	0.7	1,080	183,093	184,924	186,773	1,106	63,593	64,165	64,486	299	67,244	67,782	67,850	190	63,749	64,195	64,644	251
300	∞	0.95	1,080	65,171	65,823	66,481	382	65,170	65,561	65,561	191	63,105	63,484	63,611	146	63,077	63,266	63,899	240
400	∞	0.7	1,440	181,499	183,314	185,147	1,076	87,312	87,574	88,362	324	88,141	88,758	89,024	281	84,517	85,109	85,109	283
400	∞	0.95	1,440	85,917	86,776	87,644	511	87,312	87,399	87,923	201	86,631	87,044	80,101	2,403	84,502	84,840	84,925	226
500	∞	0.7	1,800	178,678	179,004	179,412	327	110,178	111,059	111,836	627	110,196	111,188	111,744	483	107,495	107,925	108,788	443
500	∞	0.95	1,800	111,085	112,196	112,196	467	110,178	110,729	110,950	413	111,988	112,996	113,561	504	107,495	108,140	108,248	329
600	∞	0.7	2,160	194,695	194,695	196,642	556	135,264	136,211	137,028	569	134,171	134,708	135,920	553	130,876	131,007	131,400	269
600	∞	0.95	2,160	130,951	132,261	133,584	738	131,992	132,388	133,712	513	133,080	134,145	134,145	347	130,876	131,662	132,847	623
700	∞	0.7	2,520	199,823	200,005	200,112	448	155,346	155,346	156,744	419	157,918	158,708	159,978	645	154,049	155,589	156,678	859
700	∞	0.95	2,520	160,545	162,150	163,772	1,067	156,630	156,630	156,943	247	154,066	154,066	155,607	443	154,049	155,435	156,368	790
800	∞	0.7	2,880	214,751	216,899	216,899	1,139	184,061	184,981	185,906	686	182,595	183,508	184,426	552	176,690	177,221	178,816	562
800	∞	0.95	2,880	182,668	182,921	183,056	364	181,116	181,297	182,747	504	182,595	182,960	184,424	590	176,690	176,867	177,044	324
900	∞	0.7	3,240	223,364	225,598	227,854	1,443	198,950	199,149	199,348	338	202,230	203,848	204,256	640	197,292	199,265	199,464	889
900	∞	0.95	3,240	199,002	199,002	200,992	584	200,594	202,600	204,221	1,092	205,518	206,340	206,546	330	197,292	197,489	197,489	391
1,000	∞	0.7	3,600	229,802	232,100	232,100	1,133	177,634	178,522	179,593	593	179,090	180,523	180,704	447	174,722	174,896	175,596	319
1,000	∞	0.95	3,600	218,400	218,400	220,584	603	177,634	177,812	179,412	535	176,178	176,883	177,944	551	174,722	174,722	176,120	422

Table 10: Results of $|T| = 6$

Parameters				(1,1)				(1,5)				(2,5)				(5,5)			
$\|N\|$	$\bar{\lambda}$	π	Time	Best	Average	Worst	σ	Best	Average	Worst	σ	Best	Average	Worst	σ	Best	Average	Worst	σ
100	50	0.7	720	463,597	468,233	468,233	1,907	122,803	123,908	123,908	505	167,433	167,768	168,104	275	156,238	157,332	157,961	651
100	50	0.95	720	102,487	102,487	103,512	324	64,582	64,969	65,229	222	64,056	64,056	64,633	171	63,528	63,972	64,612	321
200	50	0.7	1,440	461,913	466,532	471,197	2,871	160,358	161,641	163,096	786	184,818	185,557	185,928	324	177,408	179,005	180,795	971
200	50	0.95	1,440	131,786	131,919	132,102	220	137,235	137,235	137,784	213	134,015	135,087	136,168	693	131,763	132,949	134,146	773
300	50	0.7	2,160	431,989	431,989	436,309	1,345	202,787	203,801	205,024	727	213,900	215,397	217,120	1,038	197,314	199,090	200,285	1,087
300	50	0.95	2,160	191,956	193,876	193,876	924	195,072	195,072	195,852	297	188,906	190,417	190,607	519	187,285	188,221	189,915	726
400	50	0.7	2,880	460,056	464,657	469,304	2,543	262,665	264,241	264,241	825	262,480	263,005	264,846	717	253,873	255,650	257,951	1,177
400	50	0.95	2,880	256,059	258,620	258,620	1,517	258,115	258,373	258,631	418	264,498	266,349	268,213	1,169	253,873	253,873	255,142	514
500	50	0.7	3,600	483,055	487,886	487,886	2,219	323,750	324,398	326,669	941	331,721	335,038	337,048	1,725	318,416	318,734	320,965	849
500	50	0.95	3,600	329,085	329,085	332,376	944	318,443	319,398	321,314	897	318,452	318,452	321,637	962	318,416	319,690	322,887	1,342
600	50	0.7	4,320	510,566	510,566	515,672	1,480	397,019	399,004	401,797	1,769	406,765	410,019	410,839	1,265	390,462	391,243	391,634	902
600	50	0.95	4,320	390,548	390,548	394,453	1,154	393,764	394,552	397,708	1,192	397,003	400,973	404,582	2,439	390,462	393,195	395,161	1,526
700	50	0.7	5,040	543,160	545,171	548,101	1,650	483,361	484,328	486,750	1,071	464,026	465,418	468,676	1,449	464,026	465,418	466,349	1,216
700	50	0.95	5,040	471,436	471,436	476,150	1,402	483,029	486,893	489,327	2,193	483,029	484,478	486,900	1,178	463,707	464,635	465,100	958
800	50	0.7	5,760	583,223	589,055	589,055	2,744	526,936	531,678	536,463	2,697	514,290	514,290	516,861	825	505,859	509,400	509,400	1,368
800	50	0.95	5,760	546,874	552,343	557,866	3,179	546,874	551,796	553,451	2,821	555,767	555,767	560,213	1,407	533,536	537,271	541,569	2,695
900	50	0.7	6,480	626,602	632,868	632,868	3,195	555,384	560,382	561,503	2,189	555,384	557,050	557,050	727	555,384	558,716	562,627	2,519
900	50	0.95	6,480	612,787	612,996	613,199	805	607,845	608,229	609,005	907	602,903	607,726	610,765	2,505	593,019	597,763	600,154	2,733
1,000	50	0.7	7,200	636,238	638,110	638,544	1,592	611,917	612,154	612,228	935	616,975	623,145	627,507	3,353	606,860	611,715	613,550	2,520
1,000	50	0.95	7,200	645,270	645,661	645,910	1,258	650,559	657,065	657,065	2,680	639,981	641,901	647,036	2,180	634,692	637,231	640,417	2,192
100	∞	0.7	720	459,797	464,395	464,395	1,711	117,446	117,798	118,976	442	161,263	161,263	161,586	182	154,391	154,855	155,165	389
100	∞	0.95	720	98,910	99,002	99,441	215	30,735	30,981	31,074	153	32,195	32,195	32,292	44	31,937	31,937	32,129	57
200	∞	0.7	1,440	457,000	457,000	461,570	1,264	123,393	124,504	125,500	681	156,439	157,534	158,322	587	152,148	152,452	153,214	348
200	∞	0.95	1,440	98,278	98,278	99,261	293	64,507	65,088	65,348	307	63,479	63,923	64,434	305	61,930	62,116	62,675	212
300	∞	0.7	2,160	432,017	432,017	436,337	1,177	133,352	133,485	134,152	224	174,664	174,918	175,198	198	151,615	152,221	152,373	357
300	∞	0.95	2,160	106,361	106,555	106,918	204	91,344	91,801	92,627	446	89,119	89,565	90,013	266	89,118	89,118	89,653	160
400	∞	0.7	2,880	424,819	429,067	429,067	2,387	146,487	147,512	147,955	618	177,867	177,867	179,646	569	162,779	162,942	163,594	269
400	∞	0.95	2,880	125,443	126,697	126,697	609	122,272	123,372	123,742	594	120,271	120,752	121,477	378	120,278	120,278	120,879	223
500	∞	0.7	3,600	429,503	433,798	438,136	2,876	169,866	169,866	170,206	234	194,674	194,998	195,503	277	179,597	180,495	181,578	720
500	∞	0.95	3,600	154,056	154,056	155,597	444	155,238	155,238	156,169	296	157,766	159,186	159,345	485	151,460	152,823	152,823	722
600	∞	0.7	4,320	429,579	429,888	430,006	781	200,486	200,686	200,887	320	205,160	205,570	207,626	732	198,542	199,137	200,133	458
600	∞	0.95	4,320	190,981	192,891	192,891	744	189,426	189,426	190,752	436	187,886	189,389	191,094	1,033	184,806	185,545	185,545	416
700	∞	0.7	5,040	446,239	450,701	450,701	1,834	207,728	208,351	208,976	387	199,419	200,017	200,817	452	199,419	200,615	200,615	742
700	∞	0.95	5,040	218,945	221,134	221,134	923	200,001	201,401	203,012	1,095	201,614	203,630	204,445	901	193,549	195,098	196,464	927
800	∞	0.7	5,760	445,925	446,111	446,925	788	212,389	212,601	214,514	659	214,102	215,815	217,757	1,143	205,538	205,538	206,360	346
800	∞	0.95	5,760	236,510	238,875	241,264	1,329	206,102	206,102	207,751	464	199,453	200,450	200,450	354	199,453	201,049	201,853	783
900	∞	0.7	6,480	436,947	441,316	441,316	2,195	208,593	209,427	210,893	800	208,593	209,219	209,428	330	206,869	208,731	209,148	1,029
900	∞	0.95	6,480	246,977	246,977	249,447	722	206,401	207,020	207,434	534	208,093	209,342	209,761	472	203,017	203,423	204,847	530
1,000	∞	0.7	7,200	442,057	446,478	450,943	2,943	210,676	212,361	214,060	1,086	208,935	209,562	210,610	475	208,935	209,771	209,771	482
1,000	∞	0.95	7,200	265,695	265,891	266,012	536	214,036	215,962	216,178	1,013	208,900	209,109	210,782	589	205,475	207,324	208,153	885

Table 11: Results of the Beasley [70] instances

Instance	$\|N\|$	$\|T\| = 1$					$\|T\| = 3$					$\|T\| = 6$				
		Time	Best	Average	Worst	σ	Time	Best	Average	Worst	σ	Time	Best	Average	Worst	σ
pmed1	100	120	113,777	114,573	115,490	547	360	1,113,922	1,115,036	1,120,611	1,829	720	2,621,262	2,626,505	2,629,132	2,608
pmed2	100	120	107,862	108,941	109,921	878	360	1,055,433	1,056,488	1,061,770	1,866	720	2,433,800	2,436,234	2,441,106	2,515
pmed3	100	120	112,493	113,280	114,413	714	360	1,132,450	1,134,715	1,139,254	2,029	720	2,679,723	2,682,403	2,687,768	2,950
pmed4	100	120	110,465	110,686	111,682	400	360	1,126,159	1,129,537	1,134,055	2,485	720	2,714,670	2,720,099	2,725,539	3,633
pmed5	100	120	98,460	98,755	99,644	378	360	852,209	853,061	856,473	1,244	720	2,278,011	2,282,567	2,284,850	2,162
pmed6	200	240	165,888	167,547	168,050	755	720	1,630,259	1,635,150	1,640,055	2,912	1,440	3,737,267	3,741,004	3,748,486	3,718
pmed7	200	240	141,719	143,136	143,852	876	720	1,512,401	1,515,426	1,519,972	2,459	1,440	3,761,620	3,769,143	3,776,681	4,628
pmed8	200	240	137,589	138,965	139,660	736	720	1,588,046	1,591,222	1,594,404	2,227	1,440	3,964,318	3,968,282	3,976,219	4,096
pmed9	200	240	162,155	162,804	163,130	390	720	1,480,788	1,483,750	1,488,201	2,356	1,440	3,762,916	3,766,679	3,770,446	3,073
pmed10	200	240	127,704	128,215	129,369	612	720	1,140,789	1,141,930	1,145,356	1,518	1,440	3,194,135	3,200,523	3,206,924	3,930
pmed11	300	360	141,719	143,136	143,422	526	1,080	1,455,797	1,457,253	1,460,168	1,421	2,160	3,932,233	3,940,097	3,947,977	4,977
pmed12	300	360	146,026	146,902	148,224	774	1,080	1,386,920	1,391,081	1,395,254	2,560	2,160	3,823,420	3,827,243	3,834,897	3,515
pmed13	300	360	148,690	149,285	149,584	266	1,080	1,363,244	1,367,334	1,370,069	1,892	2,160	4,040,619	4,044,660	4,052,749	4,020
pmed14	300	360	163,332	164,475	165,133	626	1,080	1,375,588	1,378,339	1,381,096	1,839	2,160	3,906,544	3,910,451	3,914,361	4,118
pmed15	300	360	151,639	152,397	153,159	504	1,080	1,337,956	1,339,294	1,343,312	1,730	2,160	3,849,205	3,853,054	3,860,760	3,539
pmed16	400	480	156,984	158,397	159,031	668	1,440	925,736	926,662	928,515	863	2,880	2,967,213	2,973,147	2,979,093	3,909
pmed17	400	480	145,961	147,275	148,306	715	1,440	859,030	859,889	864,188	1,546	2,880	2,810,868	2,813,679	2,819,306	3,095
pmed18	400	480	155,194	155,660	157,217	636	1,440	1,247,759	1,250,255	1,255,256	2,287	2,880	3,499,936	3,503,436	3,506,939	2,882
pmed19	400	480	151,259	151,562	152,623	449	1,440	940,258	943,079	945,908	1,542	2,880	3,075,437	3,078,512	3,084,669	3,177
pmed20	400	480	159,126	160,399	161,843	895	1,440	1,016,720	1,017,737	1,020,790	1,475	2,880	3,054,630	3,057,685	3,060,743	2,495
pmed21	500	600	178,223	179,114	180,547	405	1,800	631,814	632,446	634,976	918	3,600	2,137,946	2,140,084	2,144,364	2,254
pmed22	500	600	182,250	183,708	185,361	889	1,800	809,235	811,663	815,721	1,893	3,600	2,629,539	2,634,798	2,640,068	3,329
pmed23	500	600	176,327	177,914	179,337	918	1,800	772,260	773,032	774,578	823	3,600	2,572,788	2,575,361	2,580,512	2,560
pmed24	500	600	177,973	179,575	181,191	961	1,800	724,235	725,683	727,860	1,051	3,600	2,296,600	2,298,897	2,301,196	1,937
pmed25	500	600	184,306	184,675	186,522	818	1,800	765,699	767,996	769,532	1,125	3,600	2,552,448	2,557,553	2,560,111	2,467

References

[1] Office of National Statistics. Healthcare expenditure in the UK, 2015.

[2] Preventive health care, what's the problem. `http://www.cdc.gov/healthcommunication/toolstemplates/entertainmented/tips/preventivehealth.html`, 2013.

[3] Gianfranco Damiani, Bruno Federico, Danila Basso, Alessandra Ronconi, Caterina Bianca Neve Aurora Bianchi, Gian Marco Anzellotti, Gabriella Nasi, Franco Sassi, and Walter Ricciardi. Socioeconomic disparities in the uptake of breast and cervical cancer screening in Italy: a cross sectional study. *BMC Public Health*, 12(1):1, 2012.

[4] Jared B Fox and Frederic E Shaw. Clinical preventive services coverage and the affordable care act. *American Journal of Public Health*, 105(1):e7–e10, 2015.

[5] Helen I Meissner, Nancy Breen, Carrie N Klabunde, and Sally W Vernon. Patterns of colorectal cancer screening uptake among men and women in the United States. *Cancer Epidemiology Biomarkers & Prevention*, 15(2):389–394, 2006.

[6] Rita Santos, Hugh Gravelle, and Carol Propper. Does quality affect patients choice of doctor? evidence from England. *The Economic Journal*, 127(600):445–494, 2016.

[7] Ivo Muler, Tom Smith, Steve Mellor, Lawrence Rare, and Blaise Genton. The effect of distance from home on attendance at a small rural health centre in Papua New Guinea. *International Journal of Epidemiology*, 27(5):878–884, 1998.

[8] Marco Varkevisser, Stéphanie A van der Geest, and Frederik T Schut. Do patients choose hospitals with high quality ratings? empirical evidence from the market for angioplasty in the Netherlands. *Journal of Health Economics*, 31(2):371–378, 2012.

[9] Robin Haynes, Andrew Lovett, and Gisela Sunnenberg. Potential accessibility, travel time, and consumer choice: geographical variations in general medical practice registrations in eastern England. *Environment and Planning A*, 35(10):1733–1750, 2003.

[10] Harold Hotelling. Stability in competition. In *The Collected Economics Articles of Harold Hotelling*, pages 50–63. Springer, 1990.

[11] Masao Nakanishi and Lee G Cooper. Parameter estimation for a multiplicative competitive interaction model: least squares approach. *Journal of Marketing Research*, 11(3):303–311, 1974.

[12] David R Bell, Teck-Hua Ho, and Christopher S Tang. Determining where to shop: Fixed and variable costs of shopping. *Journal of Marketing Research*, 35(3):352–369, 1998.

[13] Oded Berman, Zvi Drezner, and Dmitry Krass. Cooperative cover location problems: the planar case. *IIE Transactions*, 42(3):232–246, 2009.

[14] S Louis Hakimi. Optimum locations of switching centers and the absolute centers and medians of a graph. *Operations Research*, 12(3):450–459, 1964.

[15] Dong-Guen Kim and Yeong-Dae Kim. A lagrangian heuristic algorithm for a public healthcare facility location problem. *Annals of Operations Research*, 206(1):221–240, 2013.

[16] Siddhartha S Syam and Murray J Côté. A location-allocation model for service providers with application to not-for-profit health care organizations. *Omega*, 38(3):157–166, 2010.

[17] Marjolein Veenstra, Kees Jan Roodbergen, Leandro C Coelho, and Stuart X Zhu. A simultaneous facility location and vehicle routing problem arising in health care logistics in the Netherlands. August 2016(CIRRELT-2016-44).

[18] Amir Ahmadi-Javid, Pardis Seyedi, and Siddhartha S Syam. A survey of healthcare facility location. *Computers & Operations Research*, 79:223–263, 2017.

[19] Vedat Verter and Sophie D Lapierre. Location of preventive health care facilities. *Annals of Operations Research*, 110(1-4):123–132, 2002.

[20] Yue Zhang, Oded Berman, and Vedat Verter. Incorporating congestion in preventive healthcare facility network design. *European Journal of Operational Research*, 198(3):922–935, 2009.

[21] Yue Zhang, Oded Berman, Patrice Marcotte, and Vedat Verter. A bilevel model for preventive healthcare facility network design with congestion. *IIE Transactions*, 42(12):865–880, 2010.

[22] Wei Gu, Xin Wang, and Elizabeth S. McGregor . Optimization of preventive health care facility locations. *International Journal of Health Geographics*, 9(1):17, 2010.

[23] Yue Zhang, Oded Berman, and Vedat Verter. The impact of client choice on preventive healthcare facility network design. *OR Spectrum*, 34(2):349–370, 2012.

[24] Soheil Davari, Kemal Kilic, and Gurdal Ertek. Fuzzy bi-objective preventive health care network design. *Health Care Management Science*, 18(3):303–317, 2015.

[25] Knut Haase and Sven Müller. Insights into clients choice in preventive health care facility location planning. *OR Spectrum*, 37(1):273–291, 2015.

[26] Soheil Davari, Kemal Kilic, and Siamak Naderi. A heuristic approach to solve the preventive health care problem with budget and congestion constraints. *Applied Mathematics and Computation*, 276:442–453, 2016.

[27] Mitsuo Gen and Admi Syarif. Hybrid genetic algorithm for multi-time period production/distribution planning. *Computers & Industrial Engineering*, 48(4):799–809, 2005.

[28] Alan R McKendall Jr and Jin Shang. Hybrid ant systems for the dynamic facility layout problem. *Computers & Operations Research*, 33(3):790–803, 2006.

[29] Hyun Jeung Ko and Gerald W Evans. A genetic algorithm-based heuristic for the dynamic integrated forward/reverse logistics network for 3PLs. *Computers & Operations Research*, 34(2):346–366, 2007.

[30] Wei Yi and Linet Özdamar. A dynamic logistics coordination model for evacuation and support in disaster response activities. *European Journal of Operational Research*, 179(3):1177–1193, 2007.

[31] Malick Ndiaye and Hesham Alfares. Modeling health care facility location for moving population groups. *Computers & Operations Research*, 35(7):2154–2161, 2008.

[32] Xue-Feng Wang, Xiao-Ming Sun, and Yang Fang. Genetic algorithm solution for multi-period two-echelon integrated competitive/uncompetitive facility location problem. *Asia-Pacific Journal of Operational Research*, 25(01):33–56, 2008.

[33] Hari K Rajagopalan, Cem Saydam, and Jing Xiao. A multiperiod set covering location model for dynamic redeployment of ambulances. *Computers & Operations Research*, 35(3):814–826, 2008.

[34] Riccardo Manzini and Elisa Gebennini. Optimization models for the dynamic facility location and allocation problem. *International Journal of Production Research*, 46(8):2061–2086, 2008.

[35] Éric Gourdin and Olivier Klopfenstein. Multi-period capacitated location with modular equipments. *Computers & Operations Research*, 35(3):661–682, 2008.

[36] Yolanda Hinojosa, Jörg Kalcsics, Stefan Nickel, Justo Puerto, and Sebastian Velten. Dynamic supply chain design with inventory. *Computers & Operations Research*, 35(2):373–391, 2008.

[37] Maria Albareda-Sambola, Elena Fernandez, Yolanda Hinojosa, and Justo Puerto. The multi-period incremental service facility location problem. *Computers & Operations Research*, 36(5):1356–1375, 2009.

[38] Der-Horng Lee and Meng Dong. Dynamic network design for reverse logistics operations under uncertainty. *Transportation Research Part E: Logistics and Transportation Review*, 45(1):61–71, 2009.

[39] Stephen Mahar, Kurt M Bretthauer, and MA Venkataramanan. An algorithm for solving the multi-period online fulfillment assignment problem. *Mathematical and Computer Modelling*, 50(9):1294–1304, 2009.

[40] Verena Schmid and Karl F Doerner. Ambulance location and relocation problems with time-dependent travel times. *European Journal of Operational Research*, 207(3):1293–1303, 2010.

[41] Ayfer Basar, Bulent Catay, and Tonguc Unluyurt. A multi-period double coverage approach for locating the emergency medical service stations in Istanbul. *Journal of the Operational Research Society*, 62(4):627–637, 2011.

[42] Mohammad Hossein Fazel Zarandi, Soheil Davari, and Ali Haddad Sisakht. The large scale maximal covering location problem. *Scientia Iranica*, 18(6):1564–1570, 2011.

[43] Joaquin E Torres-Soto and Halit Üster. Dynamic-demand capacitated facility location problems with and without relocation. *International Journal of Production Research*, 49(13):3979–4005, 2011.

[44] James C Benneyan, Hande Musdal, Mehmet Erkan Ceyhan, Brian Shiner, and Bradley V Watts. Specialty care single and multi-period location-allocation models within the veterans health administration. *Socio-economic Planning Sciences*, 46(2):136–148, 2012.

[45] Yue Sha and Jun Huang. The multi-period location-allocation problem of engineering emergency blood supply systems. *Systems Engineering Procedia*, 5:21–28, 2012.

[46] Beate Rottkemper, Kathrin Fischer, and Alexander Blecken. A transshipment model for distribution and inventory relocation under uncertainty in humanitarian operations. *Socio-Economic Planning Sciences*, 46(1):98 – 109, 2012.

[47] Verena Schmid. Solving the dynamic ambulance relocation and dispatching problem using approximate dynamic programming. *European Journal of Operational Research*, 219(3):611–621, 2012.

[48] Maria Albareda-Sambola, Elena Fernández, and Stefan Nickel. Multiperiod location-routing with decoupled time scales. *European Journal of Operational Research*, 217(2):248–258, 2012.

[49] Maria Albareda-Sambola, Antonio Alonso-Ayuso, Laureano F Escudero, Elena Fernández, and Celeste Pizarro. Fix-and-relax-coordination for a multi-period location-allocation problem under uncertainty. *Computers & Operations Research*, 40(12):2878–2892, 2013.

[50] Abdolsalam Ghaderi and Mohammad Saeed Jabalameli. Modeling the budget-constrained dynamic uncapacitated facility location-network design problem and solving it via two efficient heuristics: a case study of health care. *Mathematical and Computer Modelling*, 57(3):382–400, 2013.

[51] Francisco Saldanha-da-Gama Isabel Correia, Teresa Melo. Comparing classical performance measures for a multi-period, two-echelon supply chain network design problem with sizing decisions. *Computers & Industrial Engineering*, 64:366–380, 2012.

[52] Lu Zhen, Kai Wang, Hongtao Hu, and Daofang Chang. A simulation optimization framework for ambulance deployment and relocation problems. *Computers & Industrial Engineering*, 72(0):12 – 23, 2014.

[53] Shahin Gelareh, Rahimeh Neamatian Monemi, and Stefan Nickel. Multi-period hub location problems in transportation. *Transportation Research Part E: Logistics and Transportation Review*, 75:67 – 94, 2015.

[54] Sung Hoon Chung and Changhyun Kwon. Multi-period planning for electric car charging station locations: A case of Korean expressways. *European Journal of Operational Research*, 242(2):677–687, 2015.

[55] Maria Elbek and Sanne Wøhlk. A variable neighborhood search for the multi-period collection of recyclable materials. *European Journal of Operational Research*, 249(2):540–550, 2016.

[56] Christophe Duhamel, Andréa Cynthia Santos, Daniel Brasil, Eric Châtelet, and Babiga Birregah. Connecting a population dynamic model with a multi-period location-allocation problem for post-disaster relief operations. *Annals of Operations Research*, 247(2):1–21, 2016.

[57] Isabel Correia and Teresa Melo. Multi-period capacitated facility location under delayed demand satisfaction. *European Journal of Operational Research*, 255(3):729 – 746, 2016.

[58] Nikola Markovic, Ilya O. Ryzhov, and Paul Schonfeld. Evasive flow capture: A multi-period stochastic facility location problem with independent demand. *European Journal of Operational Research*, 257(2):687 – 703, 2017.

[59] Amit Kumar Vatsa and Sachin Jayaswal. A new formulation and benders decomposition for the multi-period maximal covering facility location problem with server uncertainty. *European Journal of Operational Research*, 251(2):404 – 418, 2016.

[60] Jesus T Pastor. Bicriterion programs and managerial location decisions: application to the banking sector. *Journal of the Operational Research Society*, 45(12):1351–1362, 1994.

[61] Tammy Drezner. Derived attractiveness of shopping malls. *IMA Journal of Management Mathematics*, 17(4):349–358, 2006.

[62] Tammy Drezner. A review of competitive facility location in the plane. *Logistics Research*, 7(1):1–12, 2014.

[63] William John Reilly. *The law of retail gravitation*. New York: Knickerbocker Press, 1931.

[64] Nenad Mladenović and Pierre Hansen. Variable neighborhood search. *Computers & Operations Research*, 24(11):1097–1100, 1997.

[65] Hamid Karimi, Seyed Habib A Rahmati, and M Zandieh. An efficient knowledge-based algorithm for the flexible job shop scheduling problem. *Knowledge-Based Systems*, 36:236–244, 2012.

[66] Slim Belhaiza, Pierre Hansen, and Gilbert Laporte. A hybrid variable neighborhood tabu search heuristic for the vehicle routing problem with multiple time windows. *Computers & Operations Research*, 52:269–281, 2014.

[67] Soheil Davari, Mohammad Hossein Fazel Zarandi, and I Burhan Turksen. A greedy variable neighborhood search heuristic for the maximal covering location problem with fuzzy coverage radii. *Knowledge-Based Systems*, 41:68–76, 2013.

[68] Pierre Hansen, Nenad Mladenović, and José A Moreno Pérez. Variable neighbourhood search: methods and applications. *4OR*, 6(4):319–360, 2008.

[69] Mika Sato and Lakhmi C Jain. *Innovations in Fuzzy Clustering: Theory and Applications (Studies in Fuzziness and Soft Computing)*. Springer-Verlag, 2006.

[70] John E Beasley. A note on solving large p-median problems. *European Journal of Operational Research*, 21(2):270–273, 1985.