Quantum Statistics Can Suppress Classical Interference

O. Steuernagel
Dept. of Physical Sciences, University of Hertfordshire, College Lane, Hatfield, AL10 9AB, UK
(July 31, 2007)

Classical optical interference experiments correspond to a measurement of the first-order correlation function of the electromagnetic field. The converse of this statement: experiments that measure the first order correlation functions do not distinguish between the quantum and classical theories of light, does not always hold. A counterexample is given.

42.50.Ar, 42.25.Hz, 42.25.Kb

I. INTRODUCTION

Dirac's somewhat unfortunate statement about 'interference between two (or more) photons never occurring' has led to fruitful discussions clarifying the understanding of interference from a quantum mechanical point of view. These days textbook presentations elucidate that single photon light fields behave just like classical states of light [1] when used in a Young's double-slit interference experiment [2–4]. To exhibit non-classical features, measurements which detect two- or more photon *coincidences*, phase sensitive measurements such as homodyning and quantum tomography [5], and waiting-time distribution (anti-bunching) measurements can be performed [2–6].

Single photon detection, however, i.e. intensity or first-order coherence measurements [7], are often held to be classical in character [2,3]. This conclusion stems from the analysis of Young's interference patterns for single photon and Glauber-coherent states of light [2], single photons emitted from two atoms [3] and thermal fields [3,4]. In these examples quantum and classical predictions for the form and visibility of interference patterns are identical [2–4]. Some physicists have therefore concluded that "Experiments of this kind which measure the first-order correlation functions of the electromagnetic field do not distinguish between the quantum and classical theories of light" [2]. One can even find more generalized statements such as "For fields with identical spectral properties it is not possible to distinguish the nature of the light source from only the first-order coherence properties" [3]. Differences between classical and quantum expressions for first-order coherence phenomena are commonly attributed to different frequency modes only [3,4], but we will see that they can occur for monochromatic fields as well [8].

To explain the perceived equivalence between the classical and quantum behaviour in first-order coherence phenomena some books refer to Dirac's assertion regarding interference between two photons not occurring [2,4]. Let us seek a more detailed explanation instead.

The purpose of this paper is twofold. Firstly I will give a simple general proof for the equivalence between the classical and quantum behaviour in first-order coherence phenomena for the conventional Young's double-slit setup. And then, I will derive the simplest possible example based on photon statistics which should show maximal violation of the classically expected behaviour: whereas in the classical case an interference pattern with perfect visibility is observed, one should find the same in one but zero visibility in another corresponding quantum case.

II. YOUNG'S DOUBLE-SLIT

Firstly, let us briefly reexamine Young's double-slit: it is, assuming very narrow slits, a pedagogically appealing choice for explaining classical and quantum interference effects since it is an intuitive, geometrically simple, pure two-mode system. In its classic form it has the drawback, though, of not representing the most general case since both slits are illuminated from the same source. This limits the choice of states inside the interferometer to binomially distributed states of light which indeed cannot show a behaviour different from classical states [1,8]. The proof is given below.

Giving up on this restriction allows us to tailor the quantum states needed for the non-classical behaviour of the first-order coherence we want to demonstrate here.

A Young's double-slit setup can be cast into the general form of a two-mode interferometer illuminated with two *different* fields at its two input ports by being illuminated through a semitransparent beam-splitter positioned right between its two slits. Such a modified Young's setup would correspond to a spatially constricted Mach-Zehnder interferometer. In order to avoid all issues regarding the spatial mode arrangement and parameterization of slit and screen locations [8] we will therefore, from now on, consider a general two-mode (Mach-Zehnder) setup as displayed in FIG. 1.

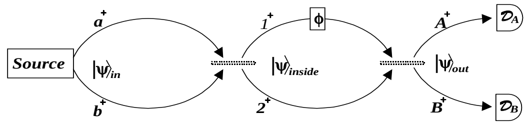

FIG. 1. Sketch of the interferometer: dotted lines outline balanced beam splitters, $\mathcal{D}$ stands for detectors and $a, b, \mathit{1}, \mathit{2}, A$ and B label the modes before, inside and beyond the interferometer. ϕ is the phase shifter in the upper channel.

Assuming that both beam-splitters are balanced and equal we choose the modes such that the action of the first beam splitter is described by [2–6,10]

$$a^\dagger = (\mathit{1}^\dagger + \mathit{2}^\dagger)/\sqrt{2}, \quad b^\dagger = (\mathit{1}^\dagger - \mathit{2}^\dagger)/\sqrt{2}\,. \tag{1}$$

The photons following channel $\mathit{1}$ are delayed by a tunable phase ϕ and are subsequently mixed with the $\mathit{2}$-channel to form the detector modes $A^\dagger$ and $B^\dagger$:

$$\mathit{1}^\dagger = e^{i\phi}\,(A^\dagger + B^\dagger)/\sqrt{2}, \quad \mathit{2}^\dagger = (A^\dagger - B^\dagger)/\sqrt{2}\,, \tag{2}$$

see Fig. 1. For the case of a single photon entering the interferometer through mode $a^\dagger$ we thus receive the final state

$$|\psi\rangle_{out} = \frac{(1+e^{i\phi})|1\rangle_A|0\rangle_B + (-1+e^{i\phi})|0\rangle_A|1\rangle_B}{2} \tag{3}$$

and the well known classical photo-detector response probabilities or intensities

$$I_A = P_A(\phi) = \langle A^\dagger A\rangle = \frac{1}{2}\,(1+\cos\phi) \tag{4}$$

$$\text{and} \quad I_B = P_B(\phi) = \langle B^\dagger B\rangle = \frac{1}{2}\,(1-\cos\phi)\,. \tag{5}$$

Note that for a single photon entering through mode $b^\dagger$ the role of I_A and I_B or the signs are exchanged. A simple calculation furthermore confirms that this state is intensity balanced, that is, inside the interferometer intensities are equal

$$I_{\mathit{1}} = \langle \mathit{1}^\dagger\,\mathit{1}\rangle = \frac{1}{2} = I_{\mathit{2}} = \langle \mathit{2}^\dagger\,\mathit{2}\rangle\,. \tag{6}$$

Let us summarize and generalize this result. The input modes expressed in terms of the output modes are

$$a^\dagger = \frac{(1+e^{i\phi})\;A^\dagger + (-1+e^{i\phi})\;B^\dagger}{2}\,, \tag{7}$$

as can easily be read off eq. (3). The result for $b^\dagger$ is

$$b^\dagger = \frac{(-1+e^{i\phi})\;A^\dagger + (1+e^{i\phi})\;B^\dagger}{2}\,. \tag{8}$$

For a conventional Young's double slit setup the chances of a single photon to pass slit $\mathit{1}$ are equal to it passing slit $\mathit{2}$, for every passing photon $|1\rangle_{into} \mapsto (|1\rangle_{\mathit{1}} + |1\rangle_{\mathit{2}})/\sqrt{2} = a^\dagger|0\rangle$ [2–4]. Every Fock-state component results in a symmetric binomial distribution of photons, since they randomly pass one hole or the other [10]. In order to find the general expression for these binomial states, let us recast this result in the operator language employed in the first equation of (1). We find that an arbitrary pure input state for a conventional Young's setup has the form

$$|\psi\rangle_{in} = \sum_\nu c_\nu \frac{a^{\dagger\nu}}{\sqrt{\nu!}}\,|0\rangle\,. \tag{9}$$

Using eq. (7) it immediately follows that

$$I_A = \langle A^\dagger A\rangle = \frac{N}{2}\,(1+\cos\phi)\,, \tag{10}$$

where N is the average photon number of $|\psi\rangle_{in}$ in equation (9). Comparing this expression with (4) we see that the textbook case of a conventional Young's double-slit indeed always shows classical behaviour.

III. GENERAL TWO-MODE INTERFEROMETERS

Let us now study the general case of any pure states occupying modes $a^\dagger$ and $b^\dagger$ simultaneously. In order to get a better understanding for what we are in search of, let us however briefly remind ourselves of some basics of the theory of optical coherence. The absolute value of the relative first-order coherence function $g^{(1)} = G^{(1)}_{12}/\sqrt{G^{(1)}_{11}G^{(1)}_{22}}$ is connected with the visibility V of the interference pattern by [2–4,6]

$$V = |g^{(1)}|\frac{2\sqrt{I_{\mathit{1}}I_{\mathit{2}}}}{I_{\mathit{1}}+I_{\mathit{2}}}\,. \tag{11}$$

In our case of equal intensities this yields the result that visibility equals first order coherence [4,6]. With eq. (10) we have, in other words, shown that in a Young's double slit setup 'all' light states [8] appear to be first-order coherent, meaning $|g^{(1)}| = 1$ everywhere [4]. Since this is obviously the largest possible degree of first-order coherence the signatures of a modification due to quantum statistics can only lead to its *suppression*.

In order to study the general case of any pure states occupying modes $a^\dagger$ and $b^\dagger$ simultaneously we will, for simplicity, expand this state in terms of the modes $\mathit{1}$ and $\mathit{2}$ *inside* the interferometer.

$$|\psi\rangle_{inside} = \sum_{\mu\nu} c_{\mu,\nu}\frac{\mathit{1}^{\dagger\mu}\,\mathit{2}^{\dagger\nu}}{\sqrt{\mu!\nu!}}\,|0\rangle = \sum_{\mu\nu} c_{\mu,\nu}\,|\mu,\nu\rangle\,. \tag{12}$$

With eqs. (2) we find $A^\dagger A = \frac{1}{2}(1^\dagger 1 + 2^\dagger 2 + e^{-i\phi} 1^\dagger 2 + e^{i\phi} 1 2^\dagger)$ and the corresponding expectation value for state (12) is

$$\begin{aligned}\langle A^\dagger A\rangle &= \sum_{\mu\nu} |c_{\mu,\nu}|^2 \, \frac{\mu+\nu}{2} \\ &+ \tfrac{1}{2} \sum_{\mu\nu} \Big(e^{-i\phi} \, c^*_{\mu,\nu} c_{\mu-1,\nu+1} \sqrt{\mu(\nu+1)} \\ &\qquad + e^{i\phi} \, c^*_{\mu,\nu} c_{\mu+1,\nu-1} \sqrt{(\mu+1)\nu} \quad \Big) \, . \end{aligned} \tag{13}$$

Of the state's density matrix only the diagonal terms contribute to the background intensity and only the first off-diagonal terms determine the interference pattern. To completely erase the interference pattern we want to get rid of the first off-diagonal terms. For single photon states that is obviously impossible if one maintains the restriction of balanced illumination of both interferometric paths. Indeed assuming $|c_{10}| = 1/\sqrt{2}$ immediately recovers – up to a phase – our classical result (4).

Adding a second photon changes this picture considerably. Let us first consider the conventional Young's setup again, a two-photon Fock-state incident in mode $a^\dagger$ impinging on the first beam-splitter $\mathcal{B}$ leads to the 'inside' state

$$|\psi\rangle_{inside} = \mathcal{B}\frac{a^{\dagger\,2}}{\sqrt{2}}|0\rangle = \frac{|2,0\rangle + \sqrt{2}\,|1,1\rangle + |0,2\rangle}{2} \, . \tag{14}$$

The amplitudes precisely match the corresponding weight factors in the general intensity expression (13) rendering this state first-order coherent. Removal of the $|1,1\rangle$-term will completely suppress this first-order coherence. The two-photon state furthest deviating from the conventional Young's mono-mode input is obviously the balanced bimodal input state

$$|\psi\rangle_{inside} = B a^\dagger b^\dagger |0\rangle = \frac{|2,0\rangle - |0,2\rangle}{\sqrt{2}} \tag{15}$$

which, indeed, does not show any first order interference. This state is intensity balanced inside the interferometer. It therefore only remains to be shown that this state's non-classical behaviour cannot be attributed to some kind of randomness in phase. Guided by the discussion of the general intensity expression (13) we can expect that a second-order coherence measurement [7] (two-photon coincidence detection with probability P_{AA}) should give us an interference pattern of higher order since it connects diagonal density matrix elements with second off-diagonal elements. The corresponding expectation value for the detection of two photons in channel A has indeed perfect visibility

$$P_{AA}(\phi) = \frac{1}{2}\langle A^\dagger A^\dagger A A\rangle = \frac{1}{4}(1-\cos 2\phi) \, , \tag{16}$$

here the prefactor $\frac{1}{4}$ normalizes the sum of all detection probabilities per shot to unity. We note that the effective de Broglie wavelength of this interference effect is halved with respect to the single photon detection case [11,12]. For our present considerations the most striking feature is the perfect visibility of this interference pattern which holds for P_{AA}, P_{BB} and $P_{AB} \equiv P_{BA}$ as well. These are the probabilities to detect two photons in coincidence in the respective channels indicated by the subscripts [11]. It is not difficult to show that a second-order pattern with perfect visibility cannot coexist with a first-order pattern.

With these perfect second-order visibilities state (15) has to be free of random phase fluctuations. We are led to consider a state that has a stable relative phase ϕ_{12} across both arms of the interferometer and illuminates both channels of the interferometer with equal intensity. When we seek a classical description of this state inside the interferometer before it passes the phase shifter we have to choose a combination of coherent states $|\alpha\rangle$ of the form

$$|\psi\rangle_{inside} = |\alpha\rangle_1 |e^{i\phi_{12}}\alpha\rangle_2 \, . \tag{17}$$

Classically we would expect this state to be first-order coherent, quantum mechanically, however, *the first-order coherence is suppressed.*

It is clear from our discussion of the general intensity expression (13) that, by moving to the second off-diagonal of the density matrix, we have found the simplest possible case of complete suppression of first-order coherence through quantum statistics.

IV. TRANSITION FROM CLASSICAL TO QUANTUM CASE

In order to study the transition from classical to quantum behaviour let us look at states that linearly interpolate ($\eta \in [0,1]$) between the above extremes (14) ($\eta = 1$) and (15) ($\eta = 0$)

$$|\psi\rangle_{in} = \left(\sqrt{\eta}\, \frac{a^{\dagger 2}}{\sqrt{2}} + \sqrt{1-\eta}\, a^\dagger b^\dagger \right) |0\rangle \, . \tag{18}$$

These states' first-order visibilities are $V_Q = \eta$, as can be seen from

$$I_A = 1 + \eta\cos\phi \;\text{ and }\; I_B = 2 - I_A \, . \tag{19}$$

The respective channel intensities inside the interferometer are no longer balanced

$$I_1 = 1 - \sqrt{2\eta(1-\eta)} \;\text{ and }\; I_2 = 2 - I_1 \, ; \tag{20}$$

we will therefore have to compare classical and quantum behaviour using a channel intensity-independent measure, namely the ratio of first order coherence functions $|g^{(1)}_Q|/|g^{(1)}_C|$, where the subscripts stand for 'quantum' and 'classical' case. Because of the stable relative phase ϕ_{12} across both arms of the interferometer and our assumed perfect mode match [8] we expect $|g_C^{(1)}|=1$ [4,6]. From eq. (11) we can hence infer that the expected classical visibility only depends on the channel intensities, $V_C = 2\sqrt{I_1 I_2}/(I_1+I_2)$, which are given above. Dividing the visibilities yields $V_Q/V_C = |g_Q^{(1)}|/|g_C^{(1)}| = |g_Q^{(1)}|$. The modulus of the quantum-statistically suppressed first-order coherence function

$$|g_Q^{(1)}(\eta)| = \frac{\eta}{\sqrt{(1-\sqrt{2\eta\,(1-\eta)})\,(1+\sqrt{2\eta\,(1-\eta)})}}\ , \quad (21)$$

falls far below the classically expected value of '1' when we approach the bimodal input state (15) at $\eta = 0$.

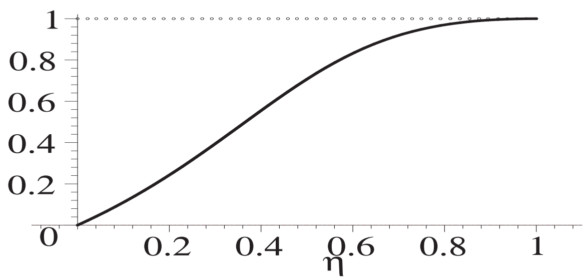

FIG. 2. Plot of the modulus of the quantal first-order coherence function $|g_Q^{(1)}(\eta)|$ of state (21) compared with the classical value $|g_C^{(1)}(\eta)|=1$, dotted line.

In the quantum-classical comparison leading to FIG. 2 we assumed the same channel intensities I_1 and I_2 and stability of the cross-phase ϕ_{12}. One can compare the classical to the quantum case employing a different set of assumptions. We could for example keep the input intensities I_a and I_b matched and also vary the relative phase between modes $a^\dagger$ and $b^\dagger$ by, say, multiplying the latter with a phase factor: $e^{i\beta}b^\dagger$. Input state $|\psi_{in}\rangle$ of eq. (18) now depends on η and β and so does its classical counterpart

$$|\psi\rangle_{inside} = \left|\sqrt{1+\eta}\right\rangle_1 \left|e^{i\beta}\sqrt{1-\eta}\right\rangle_2 . \quad (22)$$

Here the Glauber-coherent state amplitudes [1] match the input intensities $I_a = 1+\eta$ and $I_b = 2 - I_a$ of state (18). The interference signals seen by detector A for the phase tunable version of (18) are

$$I_{A,Q}(\eta,\beta) = 1 + \eta\cos\phi - \sqrt{2\eta\,(1-\eta)}\sin\beta\sin\phi \quad (23)$$

and

$$I_{A,C}(\eta,\beta) = 1 + \eta\cos\phi - \sqrt{(1+\eta)(1-\eta)}\sin\beta\sin\phi\ . \quad (24)$$

for its classical counterpart (22). The resulting visibilities are the pythagorean combinations of the trigonometric coefficients

$$V_Q(\eta,\beta) = \sqrt{\eta^2 + 2\eta(1-\eta)\sin^2\beta} \quad (25)$$

$$\text{and} \quad V_C(\eta,\beta) = \sqrt{\eta^2 + (1+\eta)(1-\eta)\sin^2\beta}\ . \quad (26)$$

The quantum-statistical suppression $V_Q(\eta,\beta) \le V_C(\eta,\beta)$ is again strongest for $\eta = 0$.

The two classical models, just presented, are incompatible with each other since the second model's input intensities I_a and I_b do not yield the first model's channel intensities I_1 and I_2 and vice versa. Consequently this discussion is to some extent arbitrary but it still serves to show that a physically reasonable classical model cannot explain the quantum-statistical suppression of first-order coherence.

V. PREPARING THE STATE

For experimental confirmation of the results presented here it is important to realize how easy it is to prepare the interpolation state, let us therefore rewrite eq. (18) with η as a mixing angle

$$|\psi\rangle_{in} = \left(\cos\eta\,\frac{a^{\dagger 2}}{\sqrt{2}} + \sin\eta\; a^\dagger b^\dagger\right)|0\rangle\ . \quad (27)$$

This form can be synthesized using spontaneous parametric down-conversion [2,3,6] and passive linear devices. Let us think of a configuration where signal $1^\dagger$ and idler $2^\dagger$ photons of a single pair are mode-matched except for orthogonal polarization, say horizontal 'H' and vertical 'V'. The initial state is thus in the state $1_H^\dagger 2_V^\dagger|0\rangle$. Next the 2-mode's polarization is turned such that it overlaps 1 by an amount $\cos\eta$, subsequently the two spatial modes 1 and 2 are mixed with a polarization-insensitive beam splitter according to transformation (2). This yields the combination

$$\begin{aligned}|\psi\rangle &= \frac{A_H^\dagger + B_H^\dagger}{\sqrt{2}} \cdot \frac{\cos\eta(A_H^\dagger - B_H^\dagger) + \sin\eta(A_V^\dagger - B_V^\dagger)}{\sqrt{2}}|0\rangle \\ &= \left[\frac{a^\dagger}{\sqrt{2}}\left(\cos\eta\frac{a^\dagger}{\sqrt{2}} + \sin\eta b^\dagger\right) - \frac{c^\dagger}{\sqrt{2}}\left(\cos\eta\frac{c^\dagger}{\sqrt{2}} - \sin\eta b^\dagger\right)\right]|0\rangle\ ,\end{aligned}$$

where the identifications $A_H \mapsto a$, $(A_V - B_V)/\sqrt{2} \mapsto b$ and $B_H \mapsto c$ have been made. The presence of the $c^\dagger$ photon allows us to identify the unwanted $c^\dagger b^\dagger$-component which otherwise would pollute the contribution from the desired first two terms which constitute state (27).

VI. CONCLUSION

To conclude, we have given a general proof for the equivalence of the first-order coherence function for the classical and the quantum case measured with a conventional Young's double-slit setup. We have shown that this equivalence does not hold for arbitrary two-mode interferometers and have derived the simplest possible case for maximal quantum-statistical suppression of first-order coherence. This case was analyzed by comparing it to two classical scenarios. Finally have we shown how to synthesize the quantum states used in this paper in order to allow for experimental confirmation of the predicted quantum-statistical suppression of first-order coherence.

ACKNOWLEDGMENTS

I wish to thank Janne Ruostekoski for discussions.

---

[1] By 'classical' I mean 'behaving like a Glauber-coherent state'.

[2] D. F. Walls and G. J. Milburn, *Quantum Optics*, (Springer, Berlin, 1995).

[3] M. O. Scully and M. S. Zubairy, *Quantum Optics*, (Cambridge Univ. Press, Cambridge, 1997).

[4] R. Loudon, *The Quantum Theory of Light*, (Clarendon Press, Oxford, 1984).

[5] U. Leonhardt, Measuring the Quantum State of Light, (Cambridge Univ. Press, Cambridge, 1997).

[6] L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics*, (Cambridge Univ. Press, Cambridge, 1995).

[7] I use the terminology of references [2–4] and refer to $g^{(1)}$ as a *first* order function since it is of first order in the measured intensities, this is at variance with the terminology in other parts of the literature [6] where it is refered to as a function of *second* order, in terms of amplitudes.

[8] I will only consider pure states populating single spatio-temporal modes [9] (quasi-monochromatic, one polarization) of sufficient transversal coherence length as to coherently illuminate both slits. In the case of paths overlapping at the beam splitters I always assume perfect matching of these modes. The generalization to mixtures of such pure states is straightforward [4].

[9] U. M. Titulaer and R. J. Glauber, Phys. Rev. **145**, 1041 (1966).

[10] R. A. Campos, B. E. A. Saleh, and M. C. Teich, Phys. Rev. A **40**, 1371 (1989).

[11] O. Steuernagel, *De Broglie Wavelength Reduction for a Multi-photon Wave Packet*, submitted.

[12] E. J. S. Fonseca, C. H. Monken, and S. Padua, Phys. Rev. Lett. **82**, 2868 (1999).