Theoretical determination of the radiation force for a spherical particle illuminated by a focused laser beam

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Abstract

Trapping forces on dielectric spheres in single beam laser tweezers are computed. A focused beam description based on an exact solution of Maxwell's equations is compared to the 5th order Gaussian beam approximation due to Barton and Alexander. Forces on water droplets suspended in air and on polystyrene spheres suspended in water, exerted by beams focused to varying degree, are calculated. It is demonstrated that the 5th order approximation is accurate for almost paraxial beams (numerical aperture NA<0.25), as compared to the exact treatment. However, for strongly focused beams the 5th order approximation breaks down. Thus it is established that accurate beam description is vital for modeling optical traps, since in order to hold a particle effectively in a single beam trap a strongly focused beam is required.

1 Introduction

Starting with the experiments of Ashkin in 1970 [1] it became possible to use laser beams to trap a variety of particles, including living cells [2], organelles within cells [3] and even larger objects like the giant amoeba [4]. There are various types of such "laser tweezers" in use, including single and the multi beam ones. It is the aim of this work to present a theoretical model of single beam laser tweezers which uses a computational method developed by Barton *et al*. [5]. Since the model presented in [5] is based on the electromagnetic (EM) field derived from the 5th order Gaussian beam approximation [6], the accuracy of the results obtained using this model is uncertain. Hence in order to test the accuracy of the results, we replace the 5th order approximation by EM fields that are exact solutions to Maxwell's equations [7]. Very recently another model has been presented by Mazolli *et al*. [8], which is based on the Debye-type integral representation of the laser beam as a superposition of plane EM waves. The main difference between Mazolli *et al*. [8] and Barton *et al*. [5] as well as the model presented here is that Mazolli *et al*. [8] take truncations of the beam by the focusing lens into account. This is not the case in the model of Barton *et al*. [5] and the model presented here.

2 Theory and results

Lorenz [9] and Mie [10] formulated a theory with which the EM fields inside and outside a sphere can be calculated, when a plane incident wave is scattered by the sphere. Barton *et al*. [5] have used this theory, generalised to an arbitrary incident field, to compute radiation forces and torques exerted on a spherical particle. Ulanowski and Jones [11] have written a set of computer programs to model the trapping forces as derived by Barton *et al*. [5] for real relative refractive indices n, which is adequate for calculating the trapping forces for nonabsorbing particles. Here the programs are used with Barton's 5th order beam approximation, and the exact solutions to Maxwell's equations presented by Dorizzi [7], based on the order 01 from the family of scalar, non-paraxial beam solutions derived by Ulanowski and Ludlow [12]. The latter is referred to as the "exact" treatment. Both treatments are used to calculate the trapping forces, exerted by beams focused to varying extent, on water droplets suspended in air (Fig. 1 and 2), and on polystyrene spheres suspended in water (Fig. 3).

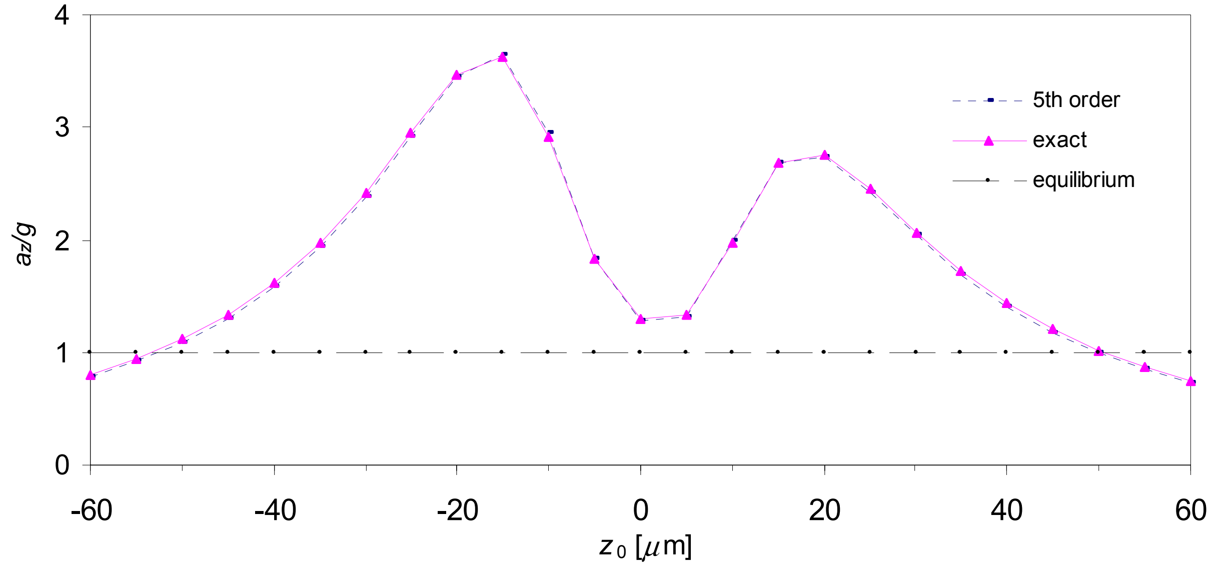

Fig. 1. Comparison of the restoring acceleration based on the 5th order Gaussian beam approximation (doted curve) and the "exact" treatment (solid curve), along the propagation axis for optical levitation of a water droplet in air, using a focused, linearly polarised laser beam, diameter of water droplet d=4.96 μm, n=1.334, wavelength λ=0.5145 μm, beam waist radius w_0=1μm, NA≈0.246 and beam power P=3.5 mW.

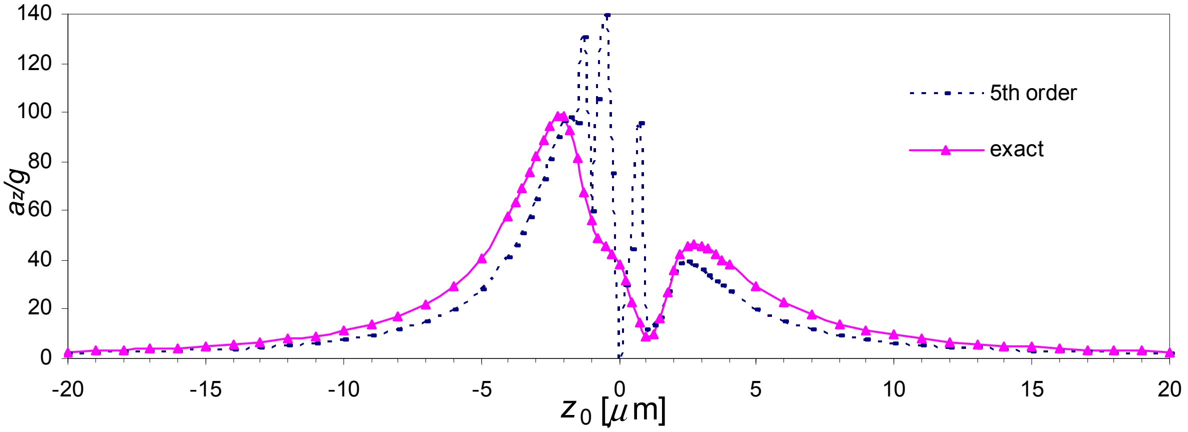

Fig. 2. Comparison of the restoring acceleration based on the 5th order approximation (doted curve) and the "exact" treatment (solid curve), along the propagation axis for optical levitation of a water droplet in air, using a focused, linearly polarised laser beam, d= 2 μm, λ_0 = 0.5145 μm, w_0=0.231μm, and P=3.5 mW.

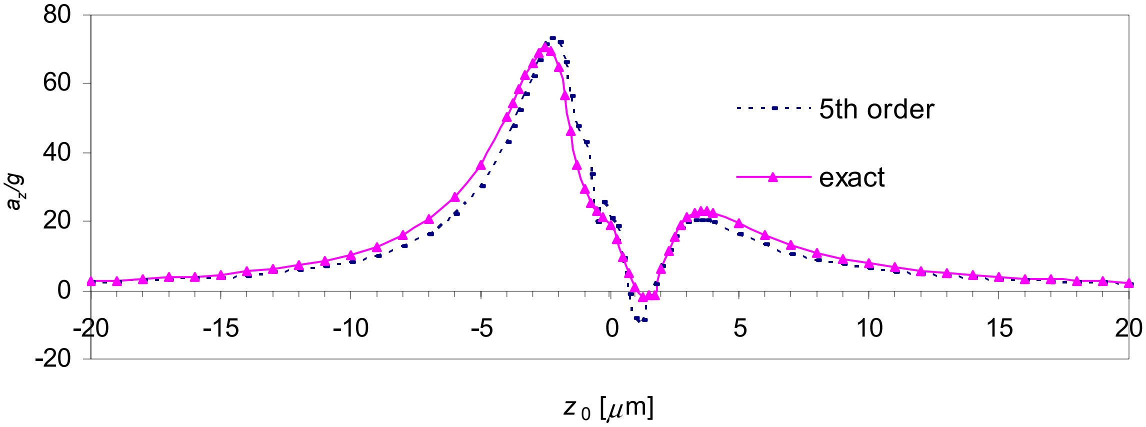

Fig. 3. Comparison of the restoring acceleration based on the 5th order Gaussian beam approximation (doted curve) and the “exact” treatment (solid curve), along the propagation axis for optical trapping of a polystyrene sphere in water, using a focused, linearly polarised laser beam, d = 2.0 µm, λ_0 = 0.5145 µm, w_0=0.231µm, P= 3.5 mW.

3 Discussion

Optical trapping forces have been calculated using the method presented by Barton *et al.* [5], which is valid for arbitrary EM fields. Fig. 1 shows that the trapping forces calculated using the 5th order approximation are accurate for a nearly paraxial Gaussian beam (NA=0.246), when compared to the exact treatment. The RMS error in this case is 1.4%. However, for strongly focused beams the 5th order approximation becomes unreliable: we observe in Fig. 2 a breakdown of the approximation when trapping a water droplet in air. From Fig. 3 it can be seen that when trapping a polystyrene sphere in water the 5th order approximation does not break down completely, but there is an RMS error of 48%. It has thus been established that it is important to have a focused laser beam description that is accurate under non-paraxial conditions, since in order to trap a particle effectively the laser beam needs to be strongly focused.

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