

# A Simple Test of the External Shock Model for the Prompt Emission in Gamma-Ray Bursts

Enrico Ramirez-Ruiz<sup>1,2,3</sup>, Jonathan Granot<sup>4</sup>

## ABSTRACT

It is demonstrated here that if the prompt GRB emission is produced by the simplest version of the external shock model, a specific relation should prevail between the observed duration, isotropic equivalent energy, and photon peak energy. In essence, this relation arises because both the burst duration and the typical energy of the emitted synchrotron photons depend on the same combination of the, usually poorly constrained, external density at the deceleration radius,  $n_{\text{dec}}$ , and initial bulk Lorentz factor,  $\Gamma_0$ . This has the fortunate consequence of making the relation independent of both  $\Gamma_0$  and  $n_{\text{dec}}$ . Unless the efficiency of electron acceleration is very low, synchrotron gamma-rays from the external shock would fail to meet the current observational constraints for the vast majority of GRBs, including those with a smooth, single peak temporal profile. This argues either against an external shock origin for the prompt emission in GRBs or for changes in our understanding of the microphysical and radiation processes occurring within the shocked region.

*Subject headings:* gamma-rays: bursts – hydrodynamics – ISM: jets and outflows

## 1. Introduction

The simplest version of the standard fireball model for gamma-ray bursts (GRBs) involves a spherical explosion taking place in a uniform or a stratified surrounding medium. When an explosion deposits a large amount of energy into material with a much smaller amount of rest energy within a compact volume, an ultra-relativistic pair fireball is formed

---

<sup>1</sup>Institute for Advanced Study, Einstein Drive, Princeton, NJ 08540, USA

<sup>2</sup>Chandra Fellow

<sup>3</sup>Department of Astronomy and Astrophysics, University of California, Santa Cruz, CA 95064, USA

<sup>4</sup>KIPAC, Stanford University, P.O. Box 20450, MS 29, Stanford, CA 94309, USA

(Cavallo & Rees 1978; Paczyński 1986; Goodman 1986). The large pressure of the explosion causes the fireball to expand, and the thermal energy of the explosion is transformed into bulk kinetic energy due to strong adiabatic cooling of the particles in the comoving frame. Because of the Thomson coupling between the particles and photons, most of the original explosion energy is eventually carried by the baryons that were originally mixed into the explosion (Shemi & Piran 1990). This bulk kinetic energy cannot be efficiently radiated as gamma rays unless it is converted back to internal energy (i.e. the velocities of the protons must be re-randomized). This requires shocks, and in order to tap a reasonable fraction of the total kinetic energy, the shocks must be (at least mildly) relativistic.

Impact on an external medium would randomize about half of the initial energy merely by reducing the expansion Lorentz factor by a factor of  $\sim 2$  (Rees & Mészáros 1992). Alternatively, internal shocks may form within the outflow: for instance, if the Lorentz factor of the outflow varied by a factor  $> 2$ , then the shocks that developed when fast material overtakes slower material would be internally (at least mildly) relativistic. There is a general consensus that the longer complex bursts must involve internal shocks, while simple smooth profiles could arise from an external shock interaction (Sari & Piran 1997a,b; Ramirez-Ruiz & Fenimore 2000; Nakar & Piran 2002; McMahon et al. 2004; Ramirez-Ruiz & Merloni 2001). The latter would in effect be the beginning of the afterglow.

An external shock moving into a medium with a smooth density profile would naturally result in a burst with a simple time-profile. Angular variations within the outflow might still cause variability in the light curve, but variations on very small angular scales ( $\theta < \Gamma_0^{-1}$ , where  $\Gamma_0$  is the initial Lorentz factor) are required in order to produce the large variability of the prompt GRB emission (Fenimore et al. 1999; Dermer & Mitman 1999). A blobby external medium could produce significant variability only if the covering factor of blobs is low, implying modest efficiency. Furthermore, the resulting variability in the light curve would be small if produced close to or after the deceleration radius, or if the portion of the ejecta that collides with a blob is decelerated significantly (Nakar & Granot 2006).

The purpose of this *Letter* is to demonstrate that if the prompt emission is produced by the simplest version of the external shock model, this implies a specific relation between the observed duration, isotropic equivalent energy (or luminosity), and photon peak energy, which is apparently incompatible with observations. This relation is derived in § 2 and compared to observations in § 3. The implications are discussed in § 4 along with possible caveats.

## 2. External Shock Model

In the simplest version of the external shock model, the outflow is approximated by a uniform thin shell. A forward shock is driven into the external medium by the outflowing ejecta, while the latter is decelerated by a reverse shock (and/or by  $pdV$  work across the contact discontinuity that separates it from the shocked external medium). The dynamics of a spherical shock wave eventually approaches a self-similar evolution (Blandford & McKee 1976) which depends only on the explosion energy  $E$  and on the external mass density  $\rho_{\text{ext}} = n_{\text{ext}}m_p$  (the Lorentz factor depends only on their ratio,  $E/\rho_{\text{ext}}$ ). If the initial GRB outflow is collimated, an additional parameter – the jet initial half-opening angle,  $\theta_0$ , is required in order to specify the flow. However, for  $\Gamma_0\theta_0 \gg 1$  (as appears to be the case from afterglow modeling; Panaitescu & Kumar 2002) the dynamics at early times – before the jet break time (as long as  $\Gamma > \theta_0^{-1}$ ) do not significantly deviate from the spherical case, where the true kinetic energy  $E$  is replaced by its isotropic equivalent value  $E_{\text{iso}}$ . Therefore, it is still valid to adopt the spherical dynamics for the prompt emission from the external shock in this case as well.

Most of the energy is transferred to the shocked external medium at the deceleration radius,  $R_{\text{dec}}$ , where the inertia of the swept-up external matter starts to produce an appreciable slowing down of the ejecta. For a given shock dynamics, the luminosity and spectrum of the emitted radiation are determined by the fractions  $\epsilon_B$  and  $\epsilon_e$  of internal energy in the shocked fluid that are carried, respectively, by the magnetic field and by relativistic electrons, as well as by the shape of the electron distribution function.

As seen in the rest frame of the downstream fluid, most of the mass and of the kinetic energy of the incoming upstream fluid is in protons (or other ions), unless the external medium is highly enriched in  $e^\pm$  pairs. Therefore, a simple isotropization of the velocities of the upstream particles at the shock transition would give the electrons only a very small fraction of the total internal energy ( $\sim m_e/m_p$ ). This would imply a very small radiative efficiency, since the radiation is emitted primarily by electrons. For a radiatively efficient system, physical processes must therefore transfer a large fraction of the swept-up energy to the electron component. The energy of the particles can be further boosted by diffusive shock acceleration as they scatter repeatedly across the shock interface, acquiring a power law distribution  $dN_e/d\gamma_e \propto \gamma_e^{-p}$  at  $\gamma_e > \gamma_m$ .

The strength of the magnetic field is another major uncertainty. Most of the required magnetic field must typically be generated in-situ, presumably through plasma instabilities or turbulent motions, but its strength has yet to be derived from first principles. The standard prescription is to assume that the magnetic field energy density  $U'_B = (B')^2/8\pi$  is a fixed fraction  $\epsilon_B$  of the downstream proper internal energy density,  $B' = (32\pi\epsilon_B n_{\text{ext}} m_p c^2 \Gamma^2)^{1/2}$ ,

where primed quantities are measured in the comoving frame.

The typical (minimal) electron energy is given by

$$\gamma_m = \frac{m_p}{m_e} \left( \frac{p-2}{p-1} \right) \frac{\epsilon_e}{\xi_e} \Gamma, \quad (1)$$

where  $\Gamma$  is the Lorentz factor of the fluid behind the forward shock, and  $\xi_e$  is the number of relativistic electrons (or positrons) per proton, which for a proton-electron plasma is equal to the fraction of the electrons that are accelerated to relativistic energies.<sup>1</sup>

The peak synchrotron photon energy is given by

$$E_p \approx \Gamma \frac{heB'\gamma_m^2}{2\pi m_e c} = \frac{42 \text{ keV}}{(1+z)} g^2 \epsilon_B^{1/2} \epsilon_e^2 \xi_e^{-2} n_0^{1/2} \Gamma_2^4, \quad (2)$$

where  $\Gamma_2 = \Gamma(R_{\text{dec}})/100$ ,  $n_0$  is  $n_{\text{dec}} = n_{\text{ext}}(R_{\text{dec}})$  in units of  $\text{cm}^{-3}$ , and  $g = 6(p-2)/(p-1)$  (where  $g = 1$  for  $p = 2.2$ ). For  $\rho_{\text{ext}} = n_{\text{ext}} m_p = Ar^{-k}$  (with  $k < 3$ ) we have

$$R_{\text{dec}} = \left[ \frac{(3-k)E_{\text{iso}}}{4\pi A c^2 \Gamma_0^2} \right]^{1/(3-k)} = \left[ \frac{(3-k)E_{\text{iso}}}{4\pi n_{\text{dec}} m_p c^2 \Gamma_0^2} \right]^{1/3}, \quad (3)$$

$$T_{\text{dec}} = (1+z) \frac{R_{\text{dec}}}{a c \Gamma_0^2}, \quad (4)$$

where<sup>2</sup>  $\Gamma_0 = \Gamma(R_{\text{dec}}) = \Gamma_{\text{dec}}$ ,  $a = 2a_2$  with  $a_2 \approx 1$ , and

$$n_{\text{dec}} \equiv n_{\text{ext}}(R_{\text{dec}}) = \frac{A}{m_p} R_{\text{dec}}^{-k}. \quad (5)$$

In the external shock model the duration of the GRB is  $T_{\text{GRB}} \sim T_{\text{dec}}$ , and therefore

$$\frac{\epsilon_B^{1/2} \epsilon_e^2}{\xi_e^2} \approx \frac{4.3 a_2^{3/2} \sqrt{1+z}}{g^2 \sqrt{3-k}} \left( \frac{E_p}{100 \text{ keV}} \right) \left( \frac{T_{\text{GRB}}}{20 \text{ s}} \right)^{3/2} \sqrt{\frac{10^{50} \text{ erg}}{E_{\text{iso}}}}. \quad (6)$$

Since  $\epsilon_B, \epsilon_e \lesssim 1/3$ , we can write

$$\Psi \approx \frac{67 a_2^{3/2} \sqrt{1+z}}{g^2 \sqrt{3-k}} \left( \frac{E_p}{100 \text{ keV}} \right) \left( \frac{T_{\text{GRB}}}{20 \text{ s}} \right)^{3/2} \sqrt{\frac{10^{50} \text{ erg}}{E_{\text{iso}}}} \lesssim \xi_e^{-2}. \quad (7)$$

---

<sup>1</sup>It is assumed here that all the relativistic electrons take part in the power law distribution of energies; the definitions of  $\epsilon_e$  and  $\xi_e$  would not include possible additional components, such as a thermal component.

<sup>2</sup>Note that the initial Lorentz factor of the outflow can be higher than  $\Gamma_0$  (if the reverse shock is relativistic).

This relatively simple relation between different observable quantities arises since  $T_{\text{dec}} \propto R_{\text{dec}}/\Gamma_{\text{dec}}^2 \propto E_{\text{iso}}^{1/3}(n_{\text{dec}}\Gamma_{\text{dec}}^8)^{-1/3}$  while  $E_p \propto \Gamma B' \gamma_m^2 \propto \epsilon_B^{1/2}(\epsilon_e/\xi_e)^2(n_{\text{dec}}\Gamma_{\text{dec}}^8)^{1/2}$ , so that both  $E_p$  and  $T_{\text{GRB}} \sim T_{\text{dec}}$  depend on  $n_{\text{dec}}$  and  $\Gamma_{\text{dec}}$  only through the combination  $n_{\text{dec}}\Gamma_{\text{dec}}^8$ . Therefore, the dependence on both  $n_{\text{dec}}$  and  $\Gamma_{\text{dec}}$  (which are hard to determine from observations) can be eliminated by taking the combination  $E_p T_{\text{dec}}^{3/2} \propto \epsilon_B^{1/2}(\epsilon_e/\xi_e)^2 E_{\text{iso}}^{1/2}$ .

The strength of Eq. 7 is that it depends mainly on quantities that can either be directly measured, like the peak photon energy ( $E_p$ ) and the duration of the GRB ( $T_{\text{GRB}}$ ), or that can be reasonably constrained by observations. Here  $E_{\text{iso}}$  is the isotropic equivalent kinetic energy of the outflow, which for a reasonable radiative efficiency,  $\epsilon_\gamma \sim 0.5$ , is of the order of the isotropic equivalent energy output in gamma-rays,  $E_{\gamma,\text{iso}}$ , that is measured directly.

### 3. Comparison to Observations

In order to compare the limit imposed by Eq. 7 with observations, we use the following observational properties derived by Ghirlanda et al. (2004) and Kaneko et al. (2006):  $T_{90}$ ,  $E_p$  and  $E_{\gamma,\text{iso}}$ . In Fig. 1 we show the distribution of  $\Psi$  as a function of  $T_{90}$ ,  $E_p$  and  $E_{\gamma,\text{iso}}$ . Filled circles are *typical* long bursts from the sample compiled by Ghirlanda et al. (2004), while the empty circles are the four long GRBs found so far to be spectroscopically associated with type Ic supernovae (Kaneko et al. 2006). Of the latter, three have a smooth, single peak temporal profile (while GRB 030329 has two peaks). Only two bursts have  $\Psi < 1$  while most bursts (and in particular those associated spectroscopically with supernovae) have  $\Psi \gg 1$ . Fig. 2 shows the maximal value of  $\xi_e$  that is allowed according to Eq. 7,  $\xi_{e,\text{max}} = \Psi^{-1/2}$ .

There are some necessary limitations to our approach. The choice of  $\epsilon_e = \epsilon_B = 1/3$  that has been used in the definition of  $\Psi$  in Eq. 7 is conservative. More typical values that are inferred from afterglow modeling ( $\epsilon_e \sim 0.1$  and  $\epsilon_B \sim 0.01$ ) would result in the values of  $\xi_e$  being smaller by a factor of  $8.0(\epsilon_e/0.1)^{-1}(\epsilon_B/0.01)^{-1/4}$  when compared to  $\xi_{e,\text{max}}$ . It is also important to note that  $E_{\gamma,\text{iso}}$  is used as an estimate for the isotropic equivalent kinetic energy  $E_{\text{iso}}$ . This would increase the value of  $\xi_{e,\text{max}}$  by a factor of  $[(1 - \epsilon_\gamma)/\epsilon_\gamma]^{1/4}$ , where  $\epsilon_\gamma$  is the  $\gamma$ -ray efficiency:  $E_{\text{iso}}/E_{\gamma,\text{iso}} = (1 - \epsilon_\gamma)/\epsilon_\gamma$ . However, even for  $\gamma$ -ray efficiencies as low as  $\epsilon_\gamma \sim 10^{-2}$ ,  $\xi_{e,\text{max}}$  would only increase by a factor of  $\sim 3$ .

### 4. Discussion

It has been shown that the simplest version of the external shock model implies a relation between different observed quantities of the GRB (Eq. 7), which can conveniently be

expressed in the form  $\xi_e \lesssim \xi_{e,\max} = \Psi^{-1/2}$ , where  $\xi_e$  is the number of accelerated electrons per proton. Naively, for the standard assumption that  $\xi_e \approx 1$ , one would expect  $\Psi \sim 10^{-2} - 10^{-1}$  for typical values of  $\epsilon_e \sim 0.1$  and  $10^{-3} \lesssim \epsilon_B \lesssim 0.1$ . It is conceivable, however, that only a small fraction of the electrons participate in the acceleration process (i.e.  $\xi_e \ll 1$ ).<sup>3</sup>

A comparison with observations shows, however, that  $\Psi \gg 1$  (and  $\xi_{e,\max} \ll 1$ ) for the vast majority of GRBs (Figs. 1 and 2). This implies that GRBs could arise from synchrotron emission in the external shock only if the efficiency of electron acceleration in relativistic collisionless shocks is very low ( $\xi_e \ll 1$ ). An external shock origin might still be possible if the radiation process responsible for the gamma rays is other than synchrotron radiation (e.g., Wang et al. 2006). Alternatively, the prompt gamma-ray emission might arise from completely different mechanism, such as internal shocks (e.g. Ramirez-Ruiz & Lloyd-Ronning 2002).

It should be noted that afterglow observations already provide interesting constraints on the efficiency of electron acceleration (Eichler & Waxman 2005). Current observations imply that the characteristic energy of accelerated electrons is comparable to the proton post-shock temperature. They also imply that the efficiency  $\xi_e$  is similar for highly relativistic and sub-relativistic shocks and plausibly suggest that  $\xi_e \sim 1$ . However, even values of  $\xi_e$  as low as  $\sim m_e/m_p$  cannot be ruled out, since currently testable afterglow predictions remain unchanged for  $(E_{\text{iso}}, n_{\text{ext}}) \rightarrow (E_{\text{iso}}, n_{\text{ext}})/\xi_e$  and  $(\epsilon_e, \epsilon_B) \rightarrow \xi_e(\epsilon_e, \epsilon_B)$  for any  $\xi_e$  in the range  $m_e/m_p \leq \xi_e \leq 1$  (Eichler & Waxman 2005).

Estimates of the energy in the afterglow shock from late time radio observations when the flow is only mildly relativistic and starts to approach spherical symmetry (often called “radio calorimetry”; Frail, Waxman & Kulkarni 2000; Berger, Kulkarni & Frail 2004; Oren et al. 2004; Frail et al. 2005; Granot et al. 2005) typically yield  $E_k \sim 10^{51.5}$  erg assuming  $\xi_e = 1$ . However, as noted by Eichler & Waxman (2005), afterglow observations actually constrain  $\xi_e E_k$  rather than  $E_k$ . The true kinetic energies at late times are thus given by  $E_k \sim 10^{52.5}(\xi_e/0.1)^{-1}$  erg. The initial energy content of the outflow could be even larger due to early radiative losses (i.e., during the prompt GRB and early afterglow stages). It is difficult to accurately account for the magnitude of such losses, as they depend on poorly known questions about postshock energy exchange between protons and electrons. Nevertheless, a lower limit on the radiated energy is given by  $E_\gamma = E_{\gamma,\text{iso}}(1 - \cos \theta_0) \approx E_{\gamma,\text{iso}}\theta_0^2/2$  (additional energy may be radiated outside the observed photon energy range, or during the early

---

<sup>3</sup>It should be pointed out that in principle even  $\xi_e > 1$  is possible, especially near  $R_{\text{dec}}$ , due to pair enrichment of the ambient medium from pair production by gamma-ray photons that are scattered on the external medium (Ramirez-Ruiz et al. 2002; Mészáros et al. 2001; Kumar & Panaitescu 2004; Beloborodov 2002; Thompson & Madau 2000).

afterglow). Other possible channels of energy loss are the escape of accelerated non-thermal protons from the blast wave (high energy cosmic rays) or the production of high energy neutrinos via pion decay. Any added losses would inevitably lead to a further increase in the energy requirements. Therefore, very small values of  $\xi_e$  would imply very large energy contents.

Another test of the simple external shock model is provided by a comparison of the correlation found by Firmani et al. (2006) between the isotropic equivalent luminosity, the burst duration, and the peak energy, with that predicted by Eq. 6. In the cosmological frame of the GRB this correlation reads  $L_{\text{iso}} \propto E_p^{1.62 \pm 0.08} T_{0.45}^{-0.49 \pm 0.07}$ , where  $T_{0.45}$  is defined by Firmani et al. (2006) to be the time during which 45% of the counts above background are measured (which is expected to scale linearly with  $T_{\text{GRB}} = T_{90}$ ). This is in disagreement with Eq. 6, which, for a reasonably small scatter in  $\epsilon_B^{1/2}(\epsilon_e/\xi_e)^2$ , gives  $L_{\text{iso}} \propto E_p^2 T_{\text{GRB}}^2$ .

It is natural to hope that the values of  $\epsilon_B$ ,  $\epsilon_e$ ,  $p$  and  $\xi_e$  are universal, since they are determined by the microphysics of the collisionless shock. However, the wide distribution of  $\Psi$  values seen in Fig. 2 suggests otherwise. That is, in the simplest version of the external shock model, a large scatter in  $\epsilon_B^{1/2}(\epsilon_e/\xi_e)^2$  is required. The presence of a significant number of non shock-accelerated electrons in the external shock ( $\xi_e \ll 1$ ) appears to be more prominent for the sub-sample of bursts found to be spectroscopically associated with a supernova (Fig. 2), most of which have a smooth temporal profile (Kaneko et al. 2006). The low values of  $\xi_e$  do not, however, increase the total energy requirements to unreasonable values for these events as they have rather low values of  $E_{\text{iso}}$ . Under this interpretation, a wide range of shock microphysical parameters may be the rule, rather than the exception.

In conclusion, observations of the prompt emission in GRBs with known redshifts, which are becoming far more accessible in the *Swift* era, can provide an important diagnostic of the external shock model. Current observational constraints do not allow for efficient electron acceleration in the external shock, if its synchrotron emission produces the observed prompt gamma-ray emission. Although there is no a priori reason to suspect that  $\xi_e$  should be large,  $\xi_e \ll 1$  would dramatically increase the total kinetic energy budget.

We are especially grateful to Stan Woosley and Eli Waxman for discussions. This work is supported by IAS and NASA through a Chandra Postdoctoral Fellowship award PF3-40028 (ERR) and by the Department of Energy under contract DE-AC03-76SF00515 (JG).

## REFERENCES

- Beloborodov, A. M. 2002, *ApJ*, 565, 80
- Berger, E., Kulkarni, S. R., & Frail, D. A. 2004, *ApJ*, 612, 966
- Blandford R. D., & McKee C. F., 1976, *Phys. Fluids*, 19, 1130
- Cavallo, G., & Rees, M. J., 1978, *MNRAS*, 183, 359
- Dermer, C. D., & Mitman, K. E. 1999, *ApJ*, 513, L5
- Fenimore, E. E., Cooper, C., Ramirez-Ruiz, E., Sumner, M. C., Yoshida, A., & Namiki, M. 1999, *ApJ*, 512, 683
- Firmani, C., Ghisellini, G., Avila-Reese, V., & Ghirlanda, G. 2006, *MNRAS*, 370, 185
- Frail, D. A., Waxman, E., & Kulkarni, S. R. 2000, *ApJ*, 537, 191
- Frail, D. A., et al. 2005, *ApJ*, 619, 994
- Eichler, D., & Waxman, E. 2005, *ApJ*, 627, 861
- Ghirlanda, G., Ghisellini, G., & Lazzati, D. 2004, *ApJ*, 616, 331
- Goodman, J., 1986, *ApJ*, 1986, 308, L47
- Granot, J., Ramirez-Ruiz, E., & Loeb, A. 2005, *ApJ*, 618, 413
- Kaneko, Y., et al. 2006, *ApJ* in press (astro-ph/0607110)
- Kumar, P., & Panaitescu, A. 2004, *MNRAS*, 354, 252
- McMahon, E., Kumar, P., & Panaitescu, A. 2004, *MNRAS*, 354, 91
- Mészáros, P., Ramirez-Ruiz, E., & Rees, M. J. 2001, *ApJ*, 554, 660
- Nakar, E., & Piran, T. 2002, *ApJ*, 572, L139
- Nakar, E., & Granot, J. 2006, submitted to *MNRAS* (astro-ph/0606011)
- Oren, Y., Nakar, E., & Piran, T. 2004, *MNRAS*, 353, L35
- Paczynski, B., 1986, *ApJ*, 308, L43
- Panaitescu, A., & Kumar, P. 2002, *ApJ*, 571, 779

- Ramirez-Ruiz, E., & Fenimore, E. E. 2000, *ApJ*, 539, 712
- Ramirez-Ruiz, E., & Merloni, A. 2001, *MNRAS*, 320, L25
- Ramirez-Ruiz, E., MacFadyen, A. I., & Lazzati, D. 2002, *MNRAS*, 331, 197
- Ramirez-Ruiz, E., & Lloyd-Ronning, N. M. 2002, *New Astronomy*, 7, 197
- Rees, M. J., & Mészáros, P. 1992, *MNRAS*, 258, 41
- Sari, R., & Piran, T. 1997a, *ApJ*, 485, 270
- Sari, R., & Piran, T. 1997, *MNRAS*, 287, 110
- Shemi, A., & Piran, T., 1990, *ApJ*, 365, L55
- Thompson, C., & Madau, P. 2000, *ApJ*, 538, 105
- Wang, X.-Y., Li, Z., Waxman, E., & Mészáros, P. 2006, submitted to *ApJ* (astro-ph/0608033)

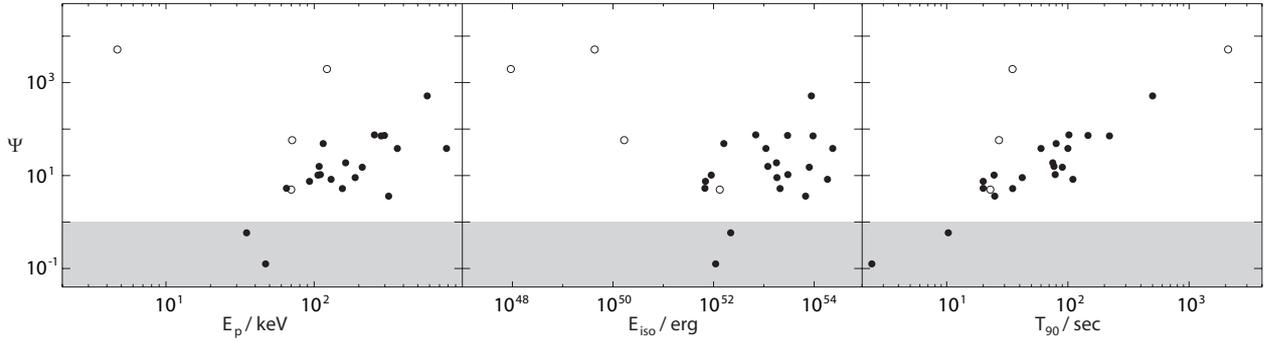


Fig. 1.—  $\Psi$  as a function of  $T_{90}$ ,  $E_p$  and  $E_{\gamma,\text{iso}}$  for GRBs with established redshifts (black symbols) from Ghirlanda et al. (2004) and for all 4 GRBs with spectroscopically determined SNe (open symbols) from Kaneko et al. (2006).

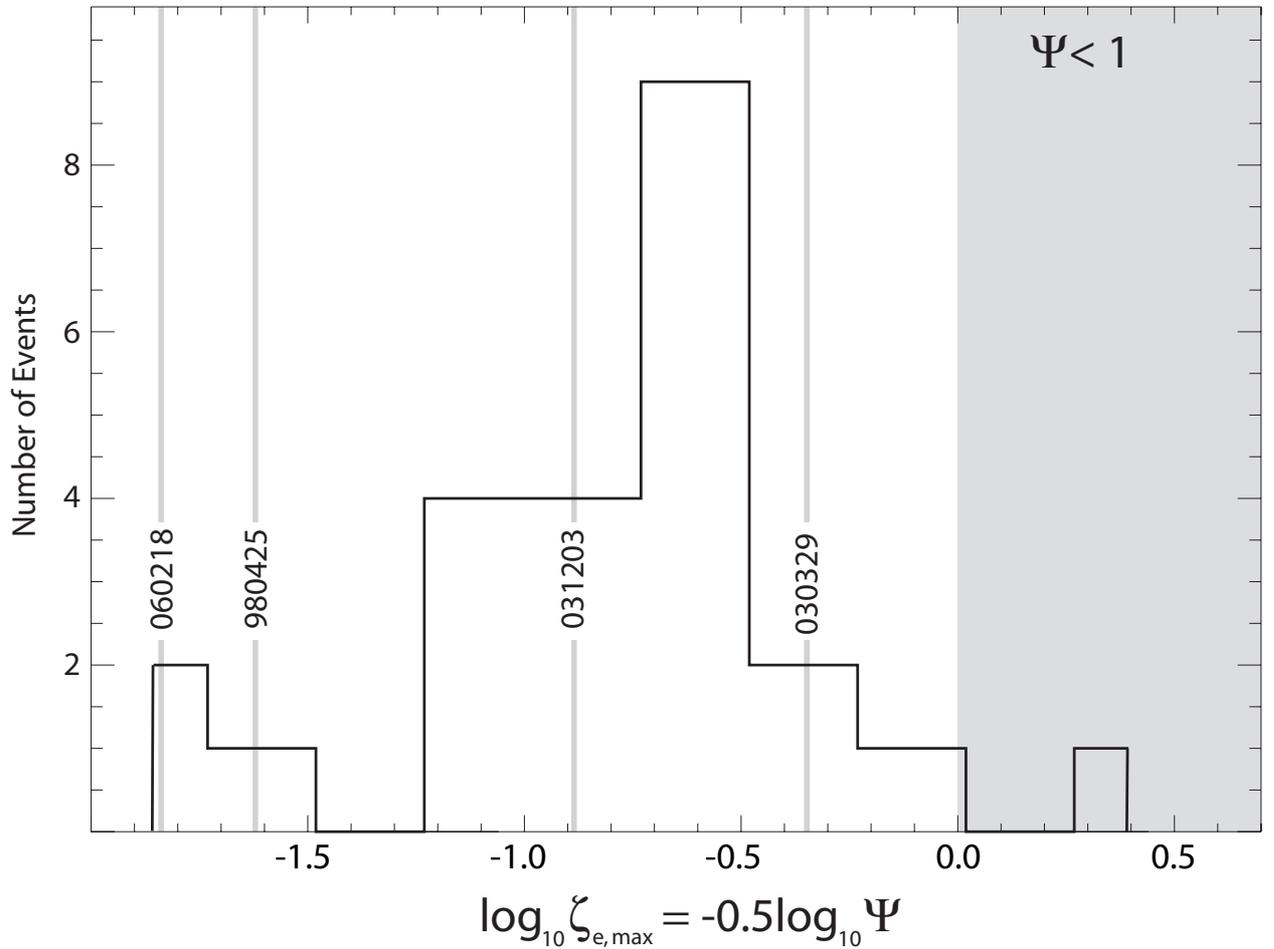


Fig. 2.— Histogram of the inferred values of  $\xi_{e,\text{max}} = 1/\sqrt{\Psi}$  for the GRBs in Fig. 1.