Collusive Decisions and Punishment under Demand and Cost Uncertainty

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Abstract

This paper provides a wider approach to competitive behaviour in sectors affected by a slump in demand, based on the contribution of the original paper by Green and Porter (1984). In addition to the cooperative solution where firms may enter a reversionary episode without breaking the collusion, the model of this study takes into consideration two forms of punishment as a result of future uncertainty; a total breakdown in collusion if there is no trust between its participants; or the formation of a new one by charging a lower common price as a disciplinary act. It is shown that under demand and cost uncertainty firms may have the incentives to choose a different short-run solution compared to the solution under certainty.

Keywords: Bertrand Competition; Collusion; Punishment; Demand uncertainty

***JEL* Classification**: D21; D43; L13

1. Introduction

A significant matter for both industrial organization and game theory economists is the collusive agreements formed between firms in order to achieve a desirable outcome. Such collusions are formed whenever market participants consider this action necessary in order to maximize their profits. The major interest is focused on the nature and the degree of implicit collusion that can be sustained through strategic interactions and production decisions according to past and present information about every firm's actions. The reasoning of forming collusions lies on the forces of competition that provoke a spirited debate about the intentions of firms signing such contracts. A number of collusions are formed in order to reduce the degree of competition in sectors that face a great slump in demand. Other contracts are formed as well even under demand peaks in order for firms to extract the maximum amount of consumer surplus. This may be achieved by increasing their profits and exploiting the maximum amount of consumption especially from goods characterized by a high degree of inelasticity.

George Stigler (1964) provided a dynamic interpretation of oligopoly theory based on Edward Chamberlin's theory (1933) of equilibrium solution were firms have to cooperate in order to maximize their joint profits. Nevertheless, the main attribute of this paper is the fact that Stigler took into consideration the case that collusion cannot be sustained for too long due to its instability as Nash equilibrium. This solution gives birth to incentives for defection under certain circumstances. Therefore, in order for such actions to be avoided there must be an endogenously determined "self-policing" way that will sustain the signed contracts between the participating firms.

Under product homogeneity and an industry structure immune to entry, Stigler focused his interest on the "secret price cutting". This action is motivated by specific fluctuations in markets, such as an unexpected increase in demand for a given price. It provides sufficient incentives to participants in breaking the contract with collusion and charge a lower price to attract a greater portion of consumers and thus, attain more profits. In order for such behaviour to be avoided, Stigler considered the collusion as a "Leviathan" (following the notion given by Thomas Hobbes) whose work is to protect the interacting members from exogenous shocks and impose penalties to anyone who would try to deviate from their contract. This whole reasoning supports that strategic pricing decisions are dependent on detailed market conditions specification, like the fact that the number of sellers is very small while the number of heterogeneous buyers is quite large. Also, an unsustainable equilibrium solution can be attained under some restrictions imposed by the collusion. The firms can avoid infinite reversionary episode, such as Cournot competition, which will result in less discounted profits in the long run compared to the ones that can be attained under the act of cooperation.

According to this reasoning, two formulations of the cartel problem treat non-cooperative collusion in a rigorous way. Osborne (1976) provided a reaction function equilibrium in which firms respond to changes in output by other firms in order to maintain their initial share of industry output. Knowing that competitors will choose the same optimal strategy, each firm will realize that it does not pay to deviate from the collusive output level, due to short-term or even long-term losses. Friedman (1971), on the other hand, outlined a strategy in which firms respond to suspected cheating. Such actions infer from a fall in the market price below the pre-agreed level. If future profit flows are discounted sufficiently slowly, then a firm would reduce the discounted value of its returns by failing to collude.

Instead of focusing on the characteristics that define monopoly power as mentioned above, Green and Porter (1984) indicated a direct expansion of Stigler's original paper by reintroducing the assumption of imperfect information. They presented a model where price cutting is a rational choice for firms under specific circumstances without defecting from their contracting agreement. They argued that under demand uncertainty, optimal incentive equilibrium may involve an episodic recourse to a short-term unprofitable solution (i.e. price war), but it cannot be clearly defined if the explaining outcome is the same with the one under certainty.

They showed that collusive conduct might be characterized by repeated episodes that may result in price and profit falls, a fact which is triggered by a decrease in the observed price for their goods. This outcome leaves no place for the view of an industry in which firms are acting on abortive attempts to form collusion. Therefore, if any collusion is formed under demand uncertainty, then no firm will ever defect due to the lack of information that cannot allow a cost-benefit analysis of expected future returns. Nevertheless, when low prices are observed, this signals an increase in demand for a certain product, by rendering the participation in reversionary episodes a rational choice for every firm. The final observations from these results show that price instability will be intense between normal and reversionary periods due to the stable pattern of prices when firms have decided to collude.

A generalized version of this paper has been presented by Abreu, Pearce and Stacchetti (1986) that introduced an *optimal pure strategy symmetric equilibrium* of a class of games that expand the Green-Porter model. They found that a constrained efficient solution is described by two "acceptance regions" in the signal space and two actions. This means that in the efficient equilibrium, players will choose to produce the output of collusion as long as the value of the signal falls in this set. Otherwise, they will switch to their reversionary strategy and keep playing that strategy as long as the signal falls in the other set. A larger set of strategies and less severe punishment will generally result in a loss of efficiency due to firms' constrained ability to discriminate between cooperation and defection. Thus, after one period of the best equilibrium, players will be instructed either to choose the worst or to restart the best equilibrium strategy.

The objective of the present paper is to provide a general interpretation of how collusions work based on the model presented by Green and Porter (1984). The main intention is to present a point of view, under the fact that threat conditions are regarded as credible based on the market power that every firm possess; the higher that power, the more endurable a firm will be by triggering competition as a form of punishment for the ones who have defected from the contracts of collusion. These restrictions can transform this solution into a sustainable one by changing the payoffs of every player and providing them an efficient outcome compared to the one acquired by the non-cooperative solution.

The main argument has two parts; the first one provides a description of the collusion structure that is about to be studied in terms of industry conduct. The second shows that even if collusive conduct results in reversionary episodes as a rational choice in which price and profit levels sharply decrease, firms may prefer to bestow a form of punishment upon collusion participants. This action occurs based on a high degree of future uncertainty that may render them unavailable to undertake such reversionary actions due to cost elements, even during the period of competition. This means that their costs might unexpectedly rise by leading to a forced price increase as a result of a marginal cost increase. As a result, it will render the products of such firms unattractive, given that their competitors continue to charge a competitive price level.

The paper is organized as follows: section 2 presents the main assumptions of the model; section 3 provides the formulation of the model; section 4 presents the solution of the model; and section 5 offers the concluding remarks.

2. Collusion Decisions under Demand and Cost Uncertainty

The model that will be studied is based on the fact that demand and cost fluctuations are not directly observed by other firms which may lead to unstable industry performance. The main structure reflects a market sector in which demand is deteriorating due to a slump in aggregate consumption. When firms notice that their profits are rapidly decreasing, they will choose to undertake an act of collusion in order to both secure their short-term profits and maximize their long-term expected returns as well, over the time horizon.

The model consists of a super game defined by firms' actions according to their incentives and the signals they receive from the market environment. They choose to compete under Bertrand behaviour by identifying a "trigger quantity" which may motivate firms to enter a reversionary episode. The time horizon includes $k=0,1,2,\dots K$ time periods and $t=0,1,2,\dots T$ time sub-periods. Sub time periods denote whether collusion is in a normal or a reversionary state, while time periods denote the decisions of collusion. Such decisions may refer to an outcome similar to Green and Porter's or an outcome of punishment based on future uncertainty. Specifically, Green and Porter argued that it is optimal for all firms to enter a reversionary episode which is triggered by an observed price reduction (Cournot behaviour) as long as "the marginal return to a firm from increasing its production in normal periods is offset exactly by the marginal increase in risk of suffering a loss in returns by triggering a reversionary episode" (1984: p.93). In addition, since product homogeneity along with an accurate realization of competitors' cost functions hold, then there will be no need for punishment.

The main intention of this chapter is to overcome the assumption of fully observing competitors' cost functions and by adding the element of uncertainty and speculation, to render the option of punishment credible. The industry that this model might appropriately describe is characterized by four features.

- First, the industry is assumed to be stable over time by rendering any expectation made to be rational based on the available information that firms are called to use. This assumption is necessary in order for this model to result in temporary stability.

- Second, the decision variable is the relative price set by firms which leads to Bertrand competition[1].
- Third, there is private information about cost decisions which sets uncertainty as a crucial factor of forming or deviating from collusion[2]. Therefore, an accurate idea can be formed regarding only the production costs of their competitors.
- Fourth, the set of information that firms use to monitor whether the collusion is in a collusive (normal) or reversionary state has to be imperfectly correlated with their conduct. This means that no direct compliance is allowed because reversion would never occur.

3. The model

As mentioned above, a game of K periods and T sub-periods is assumed that incorporates the decisions and strategic interactions of the participating firms. The game starts at $k=0$, $t=0$ when it is assumed that the participants form collusion and charge a price level which maximizes their joint profits. Consider an oligopoly of N firms which produce a differentiated product in a stationary and time separable environment, like the one described by Friedman (1971). It is assumed that if $i=1,2,\ldots,N$ indicates the number of firms, then $\pi_i\colon R^2{}_+ \rightarrow R$ is the return function of firm i and $\pi_i=\pi_i(p_i,q_i)$, where p_i is the set of price decisions and q_i is the output produced corresponding to quantity demanded for a certain price level (expressed in logarithms). If β is the discount factor and firms are assumed to be risk neutral, then they are called to maximize their long-run value function $\mathrm{E}[\sum_{k=0}^{\infty}\beta^k \sum_{t=0}^{\infty}\beta^t\pi_i(p_i,q_i)]$.

The observed demand function is given by

$$Q_{it}{}^d = A_i\left(\frac{P_{it}}{P_t}\right)^{-b_i}\left(\frac{M_t}{I_{it}{}^{\lambda}P_t}\right)^{\mu_i}\left(\frac{Z_{it}}{P_t}\right)^{\zeta_i^1} \tag{1}$$

where:

- A_i, is a constant that captures any shock in demand
- $P_{it}\colon R_+ \rightarrow R_+$, is the relative price charged by firms

[1] It is assumed that quality improvement during this game remains the same due to restrictions in investment, but differentiation in products exists, because of the set of actions undertaken over the periods prior to the slump in demand.

[2] Despite the fact that the Nash equilibrium assumption presupposes that firms have an accurate idea of their competitors' cost functions, private knowledge renders very difficult for variables such as quality investment or liabilities to be observed.

- P_t: $R_+ \rightarrow R_+$ is the industry's aggregate price level
- $(\frac{M_t}{I_{it}{}^{\lambda} P_t})$ is the wealth effect or the realization of liquidity from the public
- $(\frac{Z_{it}}{P_t})$ is the expected/undertaken investment in product quality

In this point, as denoted by Green and Porter (1984), it is assumed that firms choose their strategies from an infinite sequence $s_i=(s_{i0}, s_{i1}, s_{i2}, \ldots)$ where s_{i0} is a determinate initial price level p_{i0}, and s_{it+1}: $R_+^{t+1} \rightarrow R_+$ determines i's price level at time $t+1$ as a function of past output produced by $s_{it+1}(q_0, \ldots, q_t)=p_{it+1}$. Also, it is assumed that a price decision taken at time t is dependent on past pricing decisions formed by both j competitors and firm i, thus confirming the assumption of rational choices, where $p_{it}=p_{it}(p_{i0}, p_{i1}, \ldots, p_{it-1}, p_{j0}, p_{j1}, \ldots, p_{jt-1})$ and p_{jt} indicates the pricing decisions of competitors.

A strategy profile $(s_1, \ldots, s_n)$ determines recursively a stochastic process of output, which in turn induces a probability distribution on the space of infinite sequences of such variable. Expectations with respect to this distribution will be denoted by $E_{s_1,\ldots,s_n}$. This means that a Nash Equilibrium is a strategy profile $(s_1^*, \ldots, s_n^*)$ that satisfies

$$E_{s_1,\ldots,s_n}[\sum_{t=0}^{\infty}\beta^t \pi_i(s_{it}(q_0, \ldots, q_{t-1}), q_t)] \leq E_{s_1{}^*,\ldots,s_n{}^*}[\sum_{t=0}^{\infty}\beta^t \pi_i(s_{it}{}^*(q_0, \ldots, q_{t-1}), q_t)] \Leftrightarrow$$

$$E_{s_1,\ldots,s_n}[\sum_{k=0}^{n}\beta^k \sum_{t=0}^{\infty}\beta^t \pi_i{}^k(s_{it}(q_0, \ldots, q_{t-1}), q_t) \leq$$

$$E_{s_1{}^*,\ldots,s_n{}^*}[\sum_{k=0}^{\infty}\beta^k \sum_{t=0}^{\infty}\beta^t \pi_i{}^k(s_{it}{}^*(q_0, \ldots, q_{t-1}), q_t)] \quad (2)$$

for all firms i and feasible strategies s_{it}, where π_i^k indicates the profit level at time k.

On this basis, firms start their production at $k=0$, $t=0$ under a commonly accepted price when the slump in demand persists. The reasoning behind this process is based on the degree of influence each firm possesses. The higher that degree is, the higher the amount of output produced by that firm will be. As long as quantity demanded remains under a threshold $\hat{q}^k$ (the value of $\hat{q}$ at time k) which is commonly accepted by all participants as the "*trigger quantity*" that will result in Bertrand competition, collusion is sustained and firms keep on charging a common price level. If for some reason, this threshold is overcome due to improvement in demand conditions, like an expansionary policy that bolsters aggregate income or demand, then at least one firm will reduce its price to the competitive level, by leaving no other option to the rest but to follow such action.

In this model, the element of uncertainty does not provide an outcome based on mutual trust. In fact, three cases emerge after the increase of the observed quantity demanded above the trigger threshold; the first is the one where the trust of collusion is not broken and thus, firms return to charging the initial price level; the second declares a crumble in the relationship of the collusion members that leads to a new collusion under which the price level charged is lower than the initial one; the last reflects a complete lost in trust which leads to an infinite Bertrand competition for $k=1,2,\ldots, K$ where the strongest firm(s) will survive.

Initially, assume that $p^k=\{p_1^k,\ldots,p_N^k\}$ is a profile of monopolistic pricing choices for each firm and $p^{Bk}=\{p_1^{Bk},\ldots,p_N^{Bk}\}$ is a Bertrand pricing profile. For simplicity, the case for $k=0$ will be considered. An output level $\hat{q}^m$ is chosen along with a length of time t to be normal if (i) $t=0$ or (ii) $t-1$ was normal and $\hat{q}^k \geq q_{t-1}^{dk}$ or (iii) $t-T$ was normal and $q_{t-T}^{dk} > \hat{q}^k$, where $q_t^{dk}= q_t^{dk}(p_t^k)$ indicates the observed demand function for time k. For any other case, define t to be reversionary. Each firm faces a pricing strategy set

$$p_t = \begin{cases} p_i^m, & \text{if } t \text{ is normal under no punishment in effect} \\ p_i^1, & \text{if } t \text{ is normal under punishment in effect} \\ p_i^B, & \text{if } t \text{ is reversionary} \end{cases}$$

It is optimal for firms to charge a fixed common price $\bar{p}^k$[3] in normal periods and p^{Bk} in reversionary periods. The analysis starts from the first collusion. The joint expected profits that firms have to maximize for $k=0$ are given by

$$\pi_t{}^m = \sum_{i=1}^N \gamma_i = \bar{p_t}^m q_t{}^{dm}(\bar{p_t}^m, z_t) - \sum_{i=1}^N c_{it}(q_{it}{}^{pm}, l_{it}) \tag{3}$$

where

$$\bar{p}_t{}^m = p_t = \sum_{i=1}^N \frac{\psi_i}{\sum_{i=1}^N \psi_i} p_{it}{}^m \tag{4}$$ [4]

$$q_{it}{}^{pm} = \frac{\psi_i}{\sum_{i=1}^N \psi_i} q_t{}^m \tag{5}$$

[3] The common price charged by collusion at time $k=0$ is denoted by $\overline{p}^m$.
[4] See Rotemberg (1982a).

$$q_t{}^m = q_t{}^{dm}(\bar{p}_t{}^m, z_t) = \sum_{i=1}^{N} q_{it}{}^{pm} \tag{6}$$

The variable q_{it}^{p} corresponds to the quantity produced by firm i; q_t^{dm}= $q_t^{dm}(\bar{p}_t{}^m, z_t)$ refers to the observed demand function of the collusion at *k=0, t=0;* q_t^m is the observed quantity demanded for price $\bar{p}^m$; z_t denotes a vector of determinants of collusion's demand curve; l_{it} refers to a vector of cost determinants for each individual firm *i*, and ψ_i reflects the influence that firm *i* possesses in the operating sector. Therefore, for the last factor holds that

$$\sum_{i=1}^{N} \frac{\psi_i}{\sum_{i=1}^{N} \psi_i} = 1 \tag{7}$$

where $\theta_i = \frac{\psi_i}{\sum_{i=1}^{N} \psi_i}$ is the weighted average of individual production in collusion[5].

The expected profits of individual firm *i* participating in collusion are given by

$$\gamma_{it}(\bar{p}^m) = \bar{p}^m \frac{\psi_i}{\sum_{i=1}^{N} \psi_i} q_t{}^{dm}(\bar{p}^m, z_t) - c_{it}\left(\frac{\psi_i}{\sum_{i=1}^{N} \psi_i} q_t{}^m, l_{it}\right) \tag{8}$$

However, if production is increased beyond the threshold point by an individual firm (due to an increase in observed demand), then that firm will start charging a competitive Bertrand price $p_i{}^B$ by forcing the remaining *N-1* firms to follow such strategy as well. Under Bertrand competition, the expected profits for each firm are given by

$$\delta_{it}(p_{it}{}^B) = p_{it}{}^B q_{it}{}^{Bd}(p_{it}{}^B, z_{it}) - c_{it}(q_{it}{}^{Bp}, l_{it}) \tag{9}$$

where q_{it}^{Bp} corresponds to the quantity produced by firm *i* when charging p_i^B.

4. Definition of Value Functions

Let $V_i^m(\bar{p}^m)$ be the expected discounted present value of firm *i* if $p_i{}^m = \bar{p}^m$ in normal periods. Let also *Pr(.)* denote probability with respect to the distribution of θ_i which follows the same properties as ψ_i, dependent on demand shocks. Also, *Prb(.)* denotes the probability with discrete density that defines the volume of output produced in every sub-period *t*. If it is also assumed that $\gamma_i(p_i^m)>\delta_i(p_i^B)$, the value function for each firm satisfies the following

[5] This factor can also be viewed as the degree of market power of firm *i*.

equation

$$V_i(p_i{}^m = \bar{p}^m) = \gamma_i(\bar{p}^m) + \beta \Pr(q^{dm} \leq \hat{q}^m) V_i{}^m(\bar{p}^m)$$

$$+\Pr(q^{dm} > \hat{q}^m)(Prb(q^p = q^{Bp}))^{T-1} Prb(q^p = \theta_i q^m)[\sum_{t=1}^{T-1} \beta^t \delta_i(p_i{}^B) + \beta^T V_i{}^m]$$

$$+\Pr(q^{dm} > \hat{q}^m)(Prb(q^p = q^{Bp}))^{T-1} Prb(q^p = \theta_i{}^1 q^1)[\sum_{t=1}^{T-1} \beta^t \delta_i(p_i{}^B) + \beta^T V_i{}^1]$$

$$+\Pr(q^{dm} > \hat{q}^m)(Prb(q^p = q^{Bp}))^{T-1} Prb(q^p = q^{Bp})[\sum_{t=1}^{T} \beta^t \delta_i(p_i{}^B)] \quad (10)$$

The first term of the right hand side reflects the returns that every firm *i* expect to receive if the agreement for charging a fixed price level $\bar{p}^m$ persists, as long as the quantity demanded threshold is not overcome. The remaining three terms capture the implications of deviating from the pre-agreed price level due to an increase in observed demand. Specifically, the second term reflects the assumption presented by Green and Porter where Cournot (Bertrand in this case) competition persists for *T-1* sub-periods and in time *T* collusion reverts back in charging the initial monopolistic price level. The third term provides the first form of punishment; after competing in Bertrand terms for *T-1* sub-periods, most of the firms believe that such behaviour will be repeated. In order to punish such actions by minimizing intertemporal expected occurring losses, they agree in forming another collusion under which they charge a price level $\overline{p}^1 < \bar{p}^m$. This action materializes because even if at least one firm starts charging $p_i{}^B$, the participants will not be able to identify that firm because all of them will adopt the same strategy almost instantaneously.

This assumption may not accurately correspond to reality, but it is of great help to this analysis for emerging its dynamic elements. If firms could observe the one who would be deviating every single time, then they could adopt various strategies. They could either bestow penalties on this firm, or if the deviating firm had higher market power than the rest, all of them would be forced to charge competitive prices, where in the long-run only the strongest firm would survive. This effect is captured by the last term of this equation. It indicates a complete breakdown in collusion agreements and gives the signal for an all-out competition among participants, thus rendering any agreement about future collusion impossible.

Another difference from the original paper concerns the probability that determines the volume of output produced. It will be set as $r_{k+1} = (Prb(q^p = q_i{}^{Bk}))^{T-1}$the probability which shows how long Bertrand competition will last. In the original paper, it is assumed that

$r_1\left(Prb(q^p = q_i{}^{pk})\right) = 1$ and thus, the duration of charging a competitive price is determined only by $\Pr(q^{dm} > \hat{q}^m)$. In the present case, the duration of such competition is determined by $r_1(\Pr(q^{dm} > \hat{q}^m))$ and according to firm decisions of how they will respond in time *T*, their strategy is given by $Prb(q^p = \theta_i q^m)$ if they choose to return to the initial collusion; $Prb(q^p = \theta_i{}^1 q^1)$ if they choose to form another collusion; and $Prb(q^p = q_i{}^{Bp})$ if they choose not to cooperate. For this reason holds that

$$r_1[Prb(q^p = \theta_i q^m) + Prb\left(q^p = \theta_i{}^1 q^1\right) + Prb(q^p = q_i{}^{Bp})] = 1 \tag{11}$$

In this point, by taking logarithms of (1) at *k=0, t=0,* it follows that

$$q_{it}{}^d = a_i + \mu_i(m_t - \mathrm{I}_{it}{}^{\lambda} - \overline{p}^m) + \zeta_i^1(z_{it} - \overline{p}^m) \tag{12}$$

In this equation, it is seen that the real price effect of this sector is not taken into consideration since every firm charges the same nominal price level and acts like a monopolist whose products do not have any substitutes[6]. This means that elasticity b_i will be fixed responding to the agreed price level and won't impose any changes in demand for the output of collusion. Intuitively, this outcome is consistent with the assumptions of this model because (12) indicates that a change in demand will take place only if there is a change in μ_i or ζ_i^1 that can occur due to fluctuations in the liquidity capacity of the public or the quality of investment. Either way, a change in observed demand does not result from a change in the elasticity of demand with respect to nominal price. Since it has been assumed that production always corresponds to the level of observed demand, it holds that

$$q_{it}{}^d = q_{it}{}^p = \theta_i q_t{}^m \Leftrightarrow q_t{}^m = q_t{}^{dm}(\bar{p}^m, z_t) = \frac{a_i + \mu_i\left(m_t - I_{it}{}^{\lambda} - \bar{p}^m\right) + \zeta_i{}^1(z_{it} - \bar{p}^m)}{\theta_i} \tag{13}$$

Based on the assumptions made for *Pr(.)*, it holds that

$$Pr(q^{dm} \leq \hat{q}^m) = \Pr\left(\frac{a_i + \mu_i\left(m_t - I_{it}{}^{\lambda} - \bar{p}^m\right) + \zeta_i{}^1(z_{it} - \bar{p}^m)}{\hat{q}^m} \leq \theta_i\right) = 1 - F\left(\frac{q_{it}{}^d}{\hat{q}^m}\right) \tag{14}$$

The last element of this analysis corresponds to the incentives of punishment. In order for such action to be credible, all firms must abandon a Pareto optimal condition and choose a different one, less preferable than the initial. This means that the expected profits and the

[6] It holds because $\sum_{i=1}^{N} \theta_i p_{it} = p_t$ (see equation 4).

expected value from an action of punishment must be less than the initial expected returns from collusion. For this reason, since it has already been assumed that $\gamma_i(p_i^m)>\delta_i(p_i^B)$, it must also hold that $V_i^m(\overline{p}^m)-V_i^1(\overline{p}^1)\geq 0$. If (10), (11) and (14) are substituted in this inequality it holds that

$$V_i^1(\overline{p}^1)\leq \frac{\gamma_i(\overline{p}^m)-\delta_i(p_i^B)}{1-\beta+(\beta-\beta^{\mathrm{T}})F(\frac{q_i^d}{q^m})+r_1F(\frac{q_i^d}{q^m})\beta^{\mathrm{T}}\Pr(q_i^B)}+\frac{\delta_i(p_i^B)}{1-\beta}$$

$$-[\frac{r_1F(\frac{q_i^d}{q^m})\beta^{\mathrm{T}+1}\Pr(q_i^B)}{(1-\beta)\{1-\beta+(\beta-\beta^{\mathrm{T}})F(\frac{q_i^d}{q^m})+r_1F(\frac{q_i^d}{q^m})\beta^{\mathrm{T}}\Pr(q_i^B)\}}]\delta_i(p_i^B) \qquad (15)$$

The first and the second term of the right hand side is the same as in the model of Green and Porter. They indicate the single-period gain in returns to colluding plus the expected discounted value of firm *i* in Bertrand environment. This was the sum of the value of firm *i* when there was no punishment. In this model, expression (15) reflects the fact that the value function includes an extra element; the expected gain in entering an infinite Bertrand competition for more than one periods or for the rest of the game. If the right hand side is greater than the form of punishment under which firms create a new collusion with lower price (i.e. $V_i^1(\bar{p}^1)$), then firms will have the incentives not to choose this new form of punishment.

Specifically, (15) provides the main outcome of this paper. The act of punishment indicates the risk that firms may be willing to take in order to discipline their collusion. If most of them are determined to sustain such collusion in the long-run to secure and form a strong arsenal against future uncertainty, then they may also be willing to force such punishment upon the colluding firms. This action can minimize any unnecessary losses and keep the firms on operating by both ensuring their survival and effort to recover their losses after the emerged slump in demand. In addition, the act of punishment is a way of exploiting the weakest firms by revealing their cost elements through their inability to keep on operating under a lower price level. This way, the remaining members of the collusion and especially the ones on the margin of operating under the new price level, will be forced to abide by the contracted rules.

Therefore, if firms intend to impose a form of punishment, they will have to maximize the gap between the two forms of collusion at *k=0* and *k=1* and thus, by substituting (2) in (15) it is obtained

$$V_i^m(\overline{p}^m) - V_i^1(\overline{p}^1) \leq V_i^m(p_i^m) - V_i^1(p_i^1) \qquad (16)$$

The first-order partial derivative for (16) is

$\frac{\vartheta[V_i{}^m(p_i{}^m) - V_i{}^1(p_i{}^1)]}{\vartheta p_i{}^m}$ =0 for every firm *i*.

So, it holds that

$$0= [(1-\beta)+(\beta-\beta^{\mathrm{T}})F(\frac{q_i^{\ d}}{q^m})+r_1F(\frac{q_i^{\ d}}{q^m})\beta^{\mathrm{T}}\Pr(q_i^{\ B})]\gamma_i^{'}(p_i^{\ m})$$

$$-[(\beta-\beta^{\mathrm{T}})\xi+r_1\beta^{\mathrm{T}}\Pr(q_i^{\ B})\xi](\gamma_i(p_i^{\ m})-\delta_i(p_i^{\ B}))$$

$$-[(r_1\beta^{\mathrm{T}+1}\Pr(q_i^{\ B})\xi)(1-\beta+(\beta-\beta^{\mathrm{T}})F(\frac{q_i^{\ d}}{q^m})+r_1F(\frac{q_i^{\ d}}{q^m})\beta^{\mathrm{T}}\Pr(q_i^{\ B})]\frac{\delta_i(p_i^{\ B})}{1-\beta}$$

$$+[(r_1F(\frac{q_i^{\ d}}{q^m})\beta^{\mathrm{T}+1}\Pr(q_i^{\ B}))((\beta-\beta^{\mathrm{T}})\xi+r_1\beta^{\mathrm{T}}\Pr(q_i^{\ B})\xi)]\frac{\delta_i(p_i^{\ B})}{1-\beta}$$

$$-\frac{\vartheta V_i{}^1(p_i{}^1)}{\vartheta p_i{}^m} \qquad (17)$$

where $\xi=\frac{1}{q^m}F^{'}(\frac{q_i^{\ d}}{q^m})[\mu_i(\frac{\partial m_t}{\partial p_i^{\ m}}-1)+\zeta_i^1(\frac{\partial z_{it}}{\partial p_i^{\ m}}-1)]$ and $\gamma_i{}'(p_i{}^m)=\frac{\vartheta\gamma_i}{\vartheta p_i{}^m}$

Equation (17) states that the marginal return to a firm from reducing its price in normal periods ($\gamma_i{}'(p_i{}^m)$) must be equal to the sum of (i) the marginal increase in risk of suffering a loss in returns ($\gamma_i(p_i{}^m) - \delta_i(p_i{}^B)$), (ii) the discounted expected profits from maintaining an infinite Bertrand competition ($\frac{\delta_i(p_i{}^B)}{1-\beta}$)and (iii) the marginal value from entering a new collusion by triggering a reversionary episode[7]. Without (ii) and (iii), this equation reflects the incentives of charging a lower price level when observed demand is

[7] If this equation holds, then the participants will be indifferent in choosing between the alternative forms of punishment.

increased beyond the trigger quantity and subsequently, return to the initial collusion. In the present model, it reflects the incentives of participants to punish any defection from the initial collusion. The only term that has further to be defined is the expected marginal value of a new collusion under a change in p_i^m. If a similar function like V_i^m is assumed, then the decision variable that would affect V_i^1, would be the price set $\bar{p}^1$ under which firms are called to set a new fixed price $\bar{p}^1 < \bar{p}^m$.

Equation (17) reflects the set of strategies that firms have in their disposal in order to exploit the benefits of collusion and maximize their intertemporal gains. The form of punishment in forming new collusions will not be adopted, only when individual profits from collusion *k* are equal or slightly less than the ones under Bertrand competition (for *k=0*, the initial monopoly *m* holds). This means that firms will stop adopting the first form of punishment as long as $p_i^{\,k} = p_i^{\,Bk}$ and

$$\gamma_i^{\,k}\left(p_i^{\,k}\right) \leq \delta_i^{\,k}\left(p_i^{\,Bk}\right) \Leftrightarrow p_i^{\,k}\theta_i^{\,k}q^k - c_i(q_i^{\,p}) \leq p_i^{\,Bk}q_i^{\,Bk} - c_i(q_i^{\,Bp})$$

$$\Leftrightarrow p_i^{\,k} \leq \frac{c_i(q_i^{\,Bp}) - c_i(q_i^{\,p})}{q_i^{\,Bk} - \theta_i^{\,k}q^k} \tag{18}$$

The right hand side of (18) indicates the difference between the risk in average cost that firm *i* will undertake under Bertrand competition and the benefit in average cost that firm *i* faces if the choice of producing $\theta^k q^k$ is maintained without defecting from collusion. As long as the price choice falls below that difference, then it pays no more to use as a method of punishment (or sustain) the formation of a new collusion by charging a lower common price because $p_i^{\,k} \leq p_i^{\,B} = mc_i$, thus signaling negative profits.

5. Concluding Remarks

According to such results, there are two final observations about the formal model of collusion under demand and cost uncertainty. First, the higher the operating cost of individual firm *i*, the lower the incentives of deviating from collusion will be. However, given the fact that collusion cannot observe the sequence of firms that cause a reversionary episode, then if a form of punishment is chosen, the weakest firms will be the first to face the consequences. On the other hand, if some of the firms with a high ψ_i value are expected to deviate, then Bertrand competition will be chosen. This happens because the degree of distrust among firms overcomes the degree of profit loss due to uncertainty. In equilibrium, the frequency of a reversionary episode to occur is given by $F(\frac{{q_i}^d}{\hat{q}^m})$.

Second, firms know that a higher observed demand level does not reflect simultaneous low pricing strategies by competitors. Consequently, it is rational for them to participate in reversionary episodes as long as there is belief that no punishment will occur[8]. A reversionary episode is just a temporary switch to Nash equilibrium in non-contingent strategies. It would not pay any firm to deviate unilaterally from its Nash strategy in this temporary situation as was presented by the original paper. This behaviour is expressed by equation (17) and as long as it is satisfied, firms will be able to choose a form of punishment as the optimal reaction. This strategy may be adopted even if entering a reversionary episode was the optimal choice, as it would be suggested if the terms of punishment $\frac{\delta_i}{1-\beta}$ and $\frac{\partial {V_i}^1({p_i}^1)}{\partial {p_i}^m}$ were excluded.

The structure of this model has tested a more general case, as well as provided a general outcome compared to the one of Green and Porter by trying to provide a degree of convergence between theory and reality. Some of the assumptions may still have quite a significant gap from reality, but the main point was to formulate a model of rational strategic choices consistent with Nash equilibrium where punishment is taken into account. In marked contrast, such actions play an essential role in maintaining an ongoing scheme of collusive incentives.

The traditional views would predict the transience of collusion in a market marked by these episodes of price instability, and a breakdown of collusion at the beginning of competition by eliminating such effect. However, this model suggests that industries under certain structural characteristics will exhibit demand and industry fluctuations as a feature of a stable, time-stationary pattern of output if the operating firms are colluding[9].

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[8] This outcome corresponds to the one proposed by Green and Porter (1984).

[9] See Appendix B1 for the stochastic process of output which arises in the equilibrium of the model.

Appendix

A similar approach adopted by Green and Porter (1984) is accounted under which the observed output process $\{Y_t\}_{t \in N}$ is determined by three processes; $\{Q_t^m\}_{t \in N}$ that reflects the output process when t is normal (if the industry sets p_i^m monopoly price for $k=0$), $\{Q_t^B\}_{t \in N}$ the output process which would ensue $\forall k$ if t is reversionary (if the industry is under Bertrand competition by charging $p_i^B=mc_i$) and $\{Q_t^k\}_{t \in N}$ the output process that occurs when $k>0$ and t is normal (if the industry sets a new price set $p_i^k<p_i^{k-1}<p_i^m$) when the formation of new collusion manifests. As in original model, it is assumed that the time period ends at $k=1$ which shows that up to two new collusions can be formed. Whether the observed output level is obtained by one of the three sets, it is determined by a process $\{W_t\}_{t \in N}$ that specifies the condition the industry is under (normal, reversionary or normal after punishing a reversionary episode). Also, $\{Y_t\}_{t \in N}$ is the only component of the joint process $\{(W_t, Q_t^m, Q_t^B, Q_t^1, Y_t)\}_{t \in N}$ which is observed.

In this point, define a *switching process* to be determined by a probability space (Ω, β, m), a state space S, a subset $N \subseteq S$, and five sequences of random variables $\{W\}= \{W_t: \Omega \rightarrow S\}_{t \in N}$, $\{Y\}= \{Y_t: \Omega \rightarrow R\}_{t \in N}$, $\{Q^m\}= \{Q_t^m: \Omega \rightarrow R\}_{t \in N}$, $\{Q^B\}= \{Q_t^B: \Omega \rightarrow R\}_{t \in N}$ and $\{Q^1\}= \{Q_t^1: \Omega \rightarrow R\}_{t \in N}$ that satisfy the following conditions

$(Q^m) \cup (Q^B) \cup (Q^1)$ is a set of independent random variables (I)

(Q^m) is identically distributed with *c.d.f. G*, (II)

(Q^B) is identically distributed with *c.d.f. H*, (III)

(Q^1) is identically distributed with *c.d.f., J*, (IV)

(W) is a Markov process with stationary transition probabilities[10] (V)

For $k=0$ and $\forall t$, $S_t \in N \Rightarrow Y_t= Q_t^m$ (VI)

For $k=1$ and $\forall t$, $S_t \in N \Rightarrow Y_t= Q_t^1$ (VII)

[10] A Markov process is described by memorylesness which is why the current decisions of pricing strategies have embodied the interactions of previous strategies. In this way, past observations are not needed and thus, the Markov properties can be used in order to test the stochastic process of output.

For $\forall k$ and $\forall t$, $S_t \notin N \Rightarrow Y_t = Q_t^B$ (VIII)

The special case of a switching process usually studied occurs when $S=\{0,1\}$ and $N=\{0\}$, where $\{W\}$ is a Bernoulli process which is independent of (I). In the case of a collusive output process, G, J and H denote the *c.d.f.*'s normal (under no punishment and punishment actions) and reversionary output distribution when $S=\{0,1,\ldots,T-1\}$ and $N=\{0\}$. The Markov process $\{W\}$ is defined recursively by starting with an arbitrary initial $W_0: \Omega \rightarrow S$, and then imposing

If $W_T=0$ and $Q_T^m \geq \hat{q}^m$, then $W_{T+1}=0$ (IX)

If $W_T=0$ and $Q_T^1 \geq \hat{q}^1$, then $W_{T+1}=1/2$ (X)

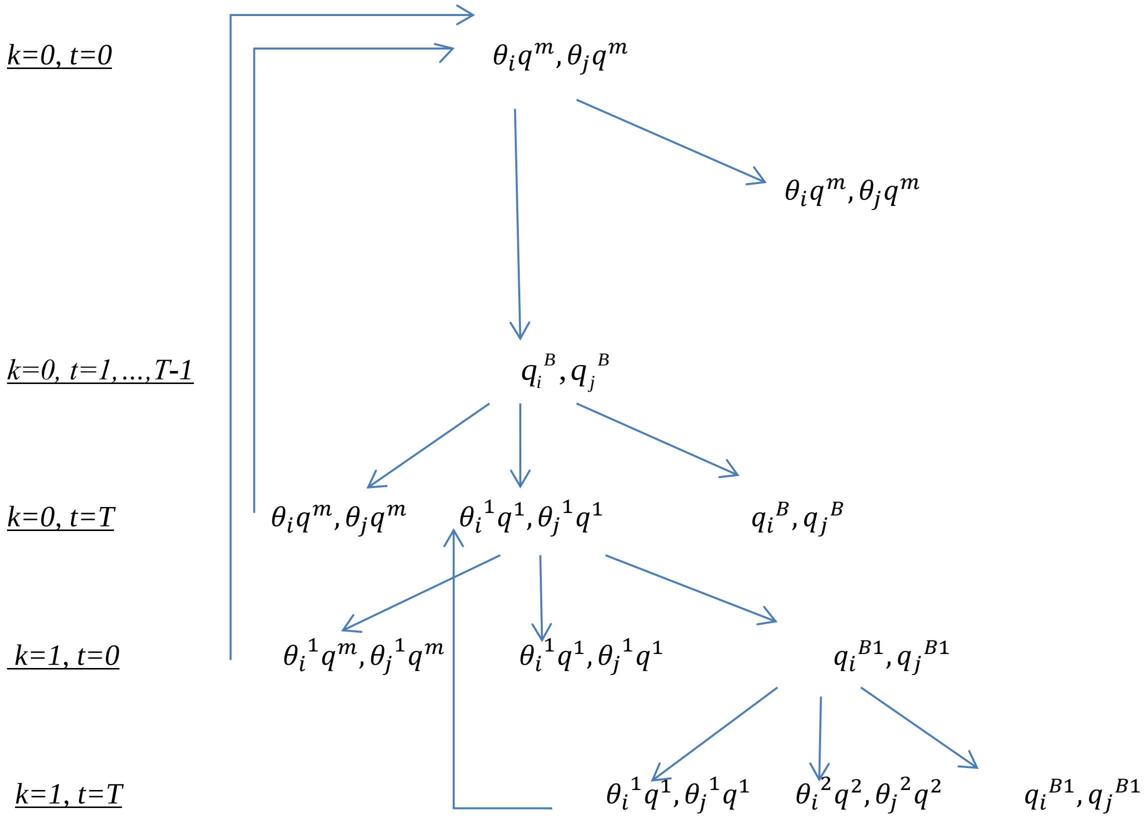
If $W_T=0$ and $Q_T^m \leq \hat{q}^m$ or $Q_T^1 \leq \hat{q}^1$, then $W_{T+1}=1$ (XI)

If $W_T=v$, $1 \leq v < T-1$, then $W_{T+1}=v+1$ (XII)

If $W_T=T-1$, then $W_{T+1}=0$ or $W_{T+1}=1/2$ (XIII)

The process $\{W\}$ defined by *(IX)-(XIII)* is a Markov process with stationary transition probabilities because $\{Q^m\}$ is *i.i.d*, based on *(I)* and *(II)*. The transition graph of $\{W\}$ is shown in Figure 11 as a sequential game, in which each arrow reflects a transition probability. The aim is to show that W_0 can be chosen in such a way that $\{Y\}$ will be a stationary ergodic process. Conditions *(VI)-(VIII)* show that if Y_t is a function of (Q_t^m, Q_t^1, Q_t^B) it will be sufficient to prove that the joint process $\{Q^m, Q^1, Q^B\}$ is ergodic. As argued in the Appendix of the original paper (Green and Porter, 1984), this process is ergodic if it is a stationary Markov process having a unique invariant distribution, such that if W_1 is defined by *(IX)-(XIII)*, then $\{Y_0, Q_0^m, Q_0^1, Q_0^B\}$ and $\{Y_1, Q_1^m, Q_1^1, Q_1^B\}$ have identical distributions according to Breiman (Theorem 7.18, 1968).

Figure 11: *Strategies that firms can choose when a temporary shock in demand occurs.*



According to this figure it is seen that the dominant strategy under certainty would be the one where firm *i* maximizes its long-run value function by maximizing its $\theta_i^{\,k} q^m$. This occurs when $\theta_i^{\,k}$ tends to unity by reflecting that monopolistic power has been acquired by the remaining firm(s), thus preventing any threat of competition. For this very reason, Bertrand competition will be the optimal choice for firm *i* only if

$$\sum_{k=0}^{K} \beta^k \sum_{t=0}^{T} \beta^t \, \delta_i^{\,k}(p_i^{\,Bk}) \geq \sum_{k=0}^{K} \beta^k \sum_{t=0}^{T} \beta^t \, \delta_j^{\,k}(p_j^{\,Bk}) \tag{B1.1}$$

This expression shows that if the expected value of entering a Bertrand competition is greater than the expected value of any other competitor, then firm *i* will have the incentives to enter an infinite reversionary episode and cause a breakdown in collusion in order to acquire monopolistic power.

On the other hand, when uncertainty is introduced as presented in this model, then Bertrand competition will not be the optimal solution as long as two conditions are met: there is no overconfidence about individual cost functions being much lower than the remaining firms'; and there is no total collapse in trust among the participating firms. For this reason, as in the model of Green and Porter, the optimal solution would be the sustainability of collusive actions and if punishment is necessary, then firms will have the incentives to form a new agreement. The resulting collusion will be sustained only in the short-run and return to the initial (optimal) agreement ($p=\bar{p}^m$) afterwards, if trust is restored among the remaining participants. This means that charging a common price $\bar{p}^1$ from a price set p_i^1 will be a short-run solution since in normal periods holds that

$$\gamma_i{}^k(p_i{}^m) \geq \gamma_i(p_i{}^m) \geq \gamma_i{}^1(p_i{}^1) \tag{B1.2}$$

This shows that the lower the number of the remaining firms in the operating sector, the greater the incentives to return to the initial charging price $\bar{p}^m$ will be in order for profits to be maximized under the constraint of uncertainty. As a result, based on inequality (15), $\forall k$ it will also hold that

$$V_i{}^k(p_i{}^m) \geq V_i(p_i{}^m) \geq V_i{}^1(p_i{}^1) \Leftrightarrow$$

$$\sum_{k=0}^{K} \beta^k \sum_{t=0}^{T} \beta^t \gamma_i{}^k(p_i{}^m) \geq \sum_{k=0}^{K} \beta^k \sum_{t=0}^{T} \beta^t \gamma_i(p_i{}^m) \geq \sum_{k=0}^{K} \beta^k \sum_{t=0}^{T} \beta^t \gamma_i{}^1(p_i{}^1) \tag{B1.3}$$

Consequently, the long-run equilibrium choices under uncertainty can either result in firms sustaining a collusive act by charging $\bar{p}^k$ in the short-run and $\bar{p}^m$ in the long-run or by charging $p_j{}^{Bk}$when there are no incentives in forming a new collusion by at least one firm (through firm i's lack of trust or expectations for eliminating its competitors).

References

Abreu, D., Pearce, D., & Stacchetti, E. (1986). “Optimal cartel equilibria with imperfect monitoring”. *Journal of Economic Theory, 39*(1), 251-269.

Breiman, L., (1968) "Probability Reading", Addison-Wesley.

Chamberlin, E. (1933), “The Theory of Monopolistic Competition.” Cambridge: Harvard University Press.

Friedman, J. W., (1971) "A Non-cooperative Equilibrium for Supergames", *Review of Economic Studies*, 28, 1-12.

Green, E. J., & Porter, R. H. (1984). “Noncooperative collusion under imperfect price information”. *Econometrica: Journal of the Econometric Society*, 87-100.

Osborne, D. K., (1976) "Cartel Problems", *American Economic Review*, 66, 835-844.

Rotemberg, J.J., (1982a) “Monopolistic price adjustment and aggregate output.” *Review of Economic Studies* 49, pp. 517-531.

Stigler, G. J., (1964) "A Theory of Oligopoly," *Journal of Political Economy*, 72, pp. 44-61.