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An Efficient SFBC Pilot-Aided Channel Estimation Method for MIMO-OFDM Systems over Mobile Frequency Selective Fading Channels

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Abstract—**An iterative pilot-aided channel estimation technique for space frequency block coded (SFBC) multiple-input multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) systems is proposed. Traditionally, when channel estimation techniques are utilized, the SFBC information signals are decoded one block at a time. In the proposed algorithm, multiple blocks of SFBC information signals are decoded simultaneously. The proposed channel estimation method can thus significantly reduce the amount of time required to decode information signals compared to similar channel estimation methods proposed in the literature. The proposed method is based on the maximum likelihood (ML) approach which offers linearity and simplicity of implementation. An expression for the pairwise error probability (PEP) is derived based on the estimated channel. The derived PEP is then used to determine the optimal power allocation for the pilot sequence. The performance of the proposed algorithm is demonstrated in high frequency selective channels, for different number of pilot symbols, using different modulation schemes. The algorithm is also tested under different levels of Doppler shift and for different number of transmit and receive antennas. The results show that the proposed scheme minimises the error margin between slow and high speed receivers compared to similar channel estimation methods in the literature.**

I. INTRODUCTION

Motivated by the increasing demand for high spectral efficiency and high transmission speed due to applications such as audio, video and internet services [1, 2], wireless communications has migrated from narrowband to broadband communications. In practice, broadband transmissions are susceptible to impairments experienced in time varying or frequency selective channels. To address this challenge, multiple-input multiple-output (MIMO) systems have been combined with orthogonal frequency division multiplexing (OFDM). This concept, referred to as MIMO-OFDM, has been successfully used to counteract the effects of multipath fading. MIMO-OFDM has already been adopted for present and future broadband communication standards such as LTE and WiMax. Furthermore, the performance of MIMO-OFDM systems can be enhanced by the use of coding techniques such as: space time block coding (STBC), space frequency block coding (SFBC), and space time frequency block coding (STFBC). For all of these coding schemes, it is worth mentioning that orthogonality of the coding matrices is a key factor to enhance system performance.

The combination of MIMO and STBC has gained wide popularity since Alamouti introduced the well known integration of MIMO and STBC in [3] which consists of data coded through space and time to improve transmission reliability. MIMO and STBC have been adopted for the 802.11n standard [4] due to the potential of such combination to achieve higher data rate and to provide more reliable performance compared to traditional single antenna communications [5, 6]. OFDM on the other hand is a multiplexing technique used to reduce the effect of frequency selective channels on transmitted data. In OFDM, the data stream that is to be transmitted is divided into multiple parallel streams and transmitted through the wideband channel which is therefore divided into a number of parallel narrowband sub-channels and each with lower rate data stream. OFDM has been widely used because of its simplicity of implementation especially in the digital domain when discrete Fourier transform

(DFT) is utilized. Moreover, OFDM is bandwidth efficient since the parallel subcarriers are orthogonal to each other and as a result overlap each other without causing interference. With the use of cyclic prefix, OFDM has also been proven to be a robust modulation technique in multipath fading environments [7, 8].

Practical STBC systems utilizing channel estimation algorithms are posed with huge challenges when operating in frequency selective multipath channels. One viable solution to counteract this is the use of OFDM in combination with STBC, referred to as STBC-OFDM. STBC-OFDM allows systems to combat long channel impulse response by transmitting parallel symbols over many orthogonal subcarriers, thus yielding unique reduced complexity and enhanced physical layer capabilities [9]. In the frequency domain, SFBC systems can also be combined with OFDM to yield SFBC-OFDM [10]. STBC-OFDM and SFBC-OFDM have shown similar performance in slow fading environments under the assumption that the channel parameters are known at the receiver. However, in environments where the propagation experiences high Doppler shift, SFBC-OFDM has shown better performance compared to STBC-OFDM [11, 12]. Furthermore, given a total transmission bandwidth, the efficiency of SFBC-OFDM can be increased with the use of a higher number of subcarriers. This is because it is unlikely that a large number of neighbouring subcarriers will experience identical fade even in frequency selective channels. Another advantage of SFBC-OFDM systems is that only half of the decoder memory is required compared to STBC-OFDM where the decoding is performed within only one OFDM symbol [12].

Channel estimation is crucial to the performance of MIMO-OFDM systems and has therefore attracted much attention since the pioneering work was introduced in [13]. More work was done later in [14, 15] for STBC-OFDM and recently for SFBC-OFDM [12, 16]. Amongst the proposed methods, DFT based channel estimation using maximum likelihood (ML) decoding in OFDM systems that utilize preamble sequence [17], and minimum mean square error (MMSE) channel estimators [18, 19] have become popular. Both techniques have been demonstrated to be competent at the cost of high complexity involved in calculating the

pseudo inversion of the matrix. Hence, lower complexity channel estimation methods were proposed in [20, 21], the downside of these methods however is that they introduce performance degradation when the Doppler shift reaches values higher than 40Hz.

In this paper a robust iterative channel estimation method is proposed for SFBC-OFDM systems operating in highly mobile scenarios. The main contribution of the paper is that the channel estimation technique presented in this work is a SFBC-OFDM method based on ML decoding. In [19], a family of iterative receivers are evaluated and compared in term of performance and computational complexity where it is shown that generally low complexity methods are providing the best tradeoff. In contrast with other methods proposed in the literature, our method benefits from reduced computational complexity as it does not require any matrix inversion at the receiver. Similar to [22], the proposed channel estimation method achieves a low complexity of $O(n^2)$ and is evaluated under the WiMax standard but offers a different approach using orthogonal codes rather than weighting factors. The method also shows less performance degradation compared to the design of [20, 21] in high mobility scenarios. As a result of the orthogonal property of SFBC, it has been possible to derive exact and simple analytical expressions to estimate the unknown channel parameters. Another contribution of the paper is the investigation of the effect of channel estimation error on the coding gain. Based on PEP analysis an upper bound on the training design is derived followed by a power allocation analysis. In addition, the proposed method is suitable for any number of transmit or receive antennas as well as any type of modulation scheme for pilot and data subcarriers. Finally, with the proposed method, each OFDM symbol is divided into groups, and each group is decoded simultaneously according to the number of pilot subcarriers used. Once the number of groups is defined, each one of them is assigned to a number of pilot subcarriers. The scheme is designed such that the number of pilot subcarriers employed equals the number of frequency slots required to transmit one SFBC training block. The use of groups reduces the number of computations linearly with respect to the number of pilot subcarriers used and each group can be assigned to a user to initiate a low complexity

multi-user MIMO-OFDM system. In the literature, a method based on Genetic Algorithm where the number of user can be higher than the number of receive antennas was proposed in [23] . However, contrary to [23], our proposed method would exploit this interesting property but keep its simplicity and low computational complexity. In addition, the proposed method could be adapted to Space Time Frequency Block Coding to exploit time and frequency domain and therefore enhance the performance such as that proposed in [18]. Compared with [18],since the proposed method does not require any matrix inversion, the computational complexity would be low. Our method also has the benefits of robustness and higher accuracy.

The rest of the paper is organized as follows. Section II describes the system and channel model considered in this paper. Section III presents the proposed iterative channel estimation method and some discussion on the frame structure. PEP and power allocation analysis are given in Section IV and Section V respectively. In Section VI, computational complexity of the proposed method is discussed and compared with other methods proposed in the literature. Simulation results and discussions are provided in Section VII. Finally, Section VIII concludes the paper.

Notation: A bold-face upper case letter denotes a matrix, while a bold-face lower case letter denotes a vector; $(\cdot)^*, (\cdot)^T, (\cdot)^H, (\cdot)^\dagger$ denote conjugate, transpose, Hermitian and pseudo-inversion operations respectively, $tr\{\cdot\}$ and $arg\{\cdot\}$ are the trace and argument function respectively, $E\{\cdot\}$ represents the expectation of a random variable; $\|X\|_F$ denotes the Frobenius norm of the matrix X, $|x|$ denotes the absolute value of x, I_N is an $N \times N$ identity matrix, finally $j = \sqrt{-1}$

II. SYSTEM MODEL

A. Transmitter

The design is based on a SFBC-OFDM wireless communication system with N_t transmit antennas and N_r receive antennas. At time t, a binary data block X(t) of q bits is scrambled and mapped using a set of predefined constellation diagrams (BPSK, QPSK, 16-QAM, 64-

QAM, 256-QAM) resulting in a symbol stream $\boldsymbol{s} = [s_0, s_1, \dots, s_{K_s-1}]^T$, with K_s being the total number of data subcarriers. The subcarriers are sub-divided into B blocks of subcarriers given by $B = K_s/N_{FS}$, where N_{FS} denotes the number of frequency slots per block required to generate the SFBC matrix. The modulated pilot sequence is given by $\boldsymbol{p} = [p_0, p_1, \dots, p_{K_p-1}]^T$, where K_p represents the total number of pilot subcarriers which must be a multiple of N$_{FS}$. The pilot subcarriers are sub-divided into B_p blocks of subcarriers given by $B_p = K_p/N_{FS}$. The pilot sequences are scattered in the data signals at regular intervals resulting in a data stream $\bar{\boldsymbol{s}} = [\bar{s}_0, \bar{s}_1, \dots, \bar{s}_{K-1}]^T$ where $K = K_s + K_p$. The data is then sent to the SFBC encoder which is based on Alamouti's encoding method [3]. For each antenna, an N-point inverse fast Fourier transform (IFFT) is applied to convert the data signals from frequency domain to time domain. Finally, cyclic prefix is added to each OFDM symbol and the data is transmitted simultaneously from different antennas.

B. Channel

It is assumed that OFDM symbols are transmitted over a Rayleigh multipath channel from transmit antenna i to receive antenna j. The Rayleigh fading multipath channel can be described as:

$$h_{i,j}(\tau,t) = \sum_{l=0}^{L_{i,j}} \varsigma_{i,j,l}(t)\delta(\tau - \tau_{i,j,l}) \tag{1}$$

where L is the number of paths, $\tau_{i,j,l}$ is the delay of the l_{th} path, $\varsigma_{i,j,l}(t)$ is the corresponding gain, $\delta(t)$ is the Dirac function. All paths $\varsigma_{i,j,l}(t)$ are assumed to be independent of each other. The frequency response of the channel on the k_{th} subcarrier between the i_{th} transmit antenna and the j_{th} receive antenna can be expressed as:

$$H_{i,j}(k) = \sum_{l=0}^{L_{i,j}} \alpha_{i,j,l}(k)e^{-\frac{j2\pi k\tau_{i,j,l}}{N_{FFT}}} \tag{2}$$

where N$_{FFT}$ is the FFT size and $\alpha_{i,j,l}(k)$ represents the amplitude of the path l of the channel between the i_{th} transmit antenna and the j_{th} receive antenna at the k-th subcarrier .

C. Receiver

The reverse of the procedure at the transmitter is adopted at the receiver. Data is received and down converted, cyclic prefix is removed and FFT operation is performed. Alamouti's encoding scheme offers a simple decoding algorithm when channel parameters are known at the receiver. By introducing guard interval (GI) into each OFDM symbol, intersymbol interference (ISI) caused by multipath propagation can be sufficiently prevented, assuming that the guard interval is greater than the maximum delay spread of channel. In this work, it is assumed that the time, phase and frequency synchronization has been achieved using the repeated preamble sequence. However, transmission in a mobile communication environment is impaired by Doppler spread. The orthogonality among different subcarriers is affected due to Doppler spread. Therefore, interchannel interference (ICI) may still exist due to frequency offset estimation errors or unexpected Doppler shifts. Since our proposed algorithm is based on SFBC-OFDM, the SFBC codeword components are subjected to the same channel and therefore provide higher immunity against fading channels with severe temporal and frequency diversity. In this research, in order to compensate the effect of ICI, SFBC-OFDM has been considered with isolated and equi-spaced pilots. Separating the sub-carriers holding the codeword components within the coherence bandwidth has been shown in [24, 25] as an efficient technique to reduce ICI effect on time invariant channel estimation technique such as the one proposed in this paper.

Therefore, the transmitted sequence across N_t transmit antennas passes through a frequency selective channel with additive white Gaussian noise so that the received signal between the i_{th} transmit antenna and the j_{th} receive antenna, once the OFDM demodulation is applied, can be expressed as:

$$r_j(k) = \frac{\sigma_T}{\sqrt{N_t}} \sum_{l=0}^{L_{i,j}} s_i(k - \Delta_l) H_{i,j}(k) + n(k) \tag{3}$$

where $r(k)$ represents the received signal at the k_{th} subcarrier at the t_{th} time interval, σ_T represents the total transmitted energy across all the N_t transmit antennas in each time slot,

n_{th} represents the white Gaussian noise with variance $\sigma_n^2/2$ per dimension and Δ_l represents the number of delay to sample interval at the l_{th} path. $s_i(m)$ represents the transmitted signal at k_{th} subcarrier for the l_{th} path interval of the time domain signal vector $s_i(m) = \mathcal{L}_N S_l$ where $\mathcal{L}_N$ is the N x N IDFT matrix with entries $\mathcal{L}(m,n) = \frac{1}{N} e^{\frac{j2\pi mn}{N_{FFT}}}$. $S_i = [S_i[0] \dots S_i[N-1]]^T$ is the length N_{FFT} input data vector with $S_i[k]$ being the k_{th} symbol sent by the i_{th} transmit antenna. The receive signal for the k_{th} subcarrier can therefore be expressed as:

$$R_i[k] = \sum_{i=1}^{N_t} X_i[0]H_i[k]S[k] + \sum_{i=1}^{N_t} X_i[n-k]H_i[n]S[n] + N[k] \tag{4}$$

where n is the index of the subcarriers used for data transmission only and X_i is an interference coefficient given by:

$$X_i[m] = \sum_{n=1}^{N-1} e^{\frac{j2\pi nm}{N}} = \frac{\sin \pi m}{N \sin \frac{\pi}{N} m} e^{j\pi(1-\frac{1}{N})m} \tag{5}$$

Once the transmitted data has been decoded, the output of the combiner is fed to the ML detector which computes the optimum ML decision metric J_m over the set $\bar{s}$ and decides in favour of the symbol group that minimizes the metric J_m [13].

$$J_m = \sum_{j=1}^{N_r} \sum_{t=1}^{K-1} \left\| \mathbf{r}_{j,t} - \frac{\sigma_T}{\sqrt{N_t}} \sum_{i=1}^{M_t} \mathbf{H}_{i,j,t} \mathbf{s}_{i,t} \right\|^2 \tag{6}$$

ML has been preferred to other detectors like MMSE because of its linearity and simplicity and due to the fact that better performance can be achieved for simpler implementation.

III. Two Stage Channel Estimation Method

In this section, the proposed pilot design and channel estimation algorithm are described in detail.

A. *Basic principles of pilot design and channel estimation for SFBC-OFDM systems*

The channel estimation method is based on a two-step procedure which is described in Fig. 1.
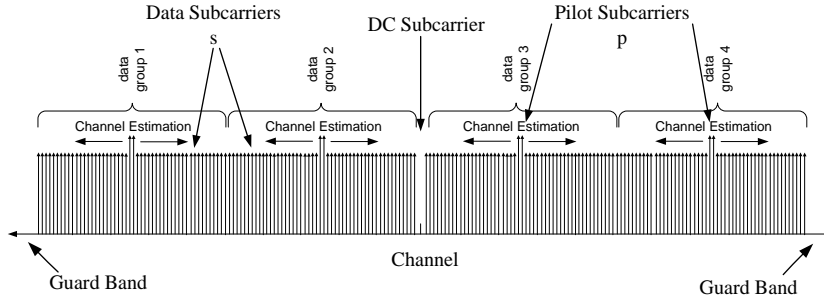
Fig. 1: Diagram of the Channel Estimation Method for Two Transmit Antennas

The first step occurs at the transmitter side where, according to the application used, a specific number of subcarriers K_s and K_p are assigned for data and pilot transmission respectively. As discussed earlier, each OFDM symbol is comprised of B blocks of data subcarriers and B_p blocks of pilot subcarriers. In order to implement the channel estimation method, each OFDM symbol is divided into G groups, where each g_{th} group is made up of one block of pilot symbols and B/B_p blocks of data symbols. Thus within each group one block of pilot symbols is positioned such that an equal number of data blocks can be found on each side. In other words, the pilot symbols are centralized within the B/B_p blocks of data symbols. The length of each group varies according to the number of pilot subcarriers per block. From Fig. 1, for 2 transmit antennas with 160 data subcarriers and 8 pilot subcarriers, this implies that, $N_t = 2$, $K_s = 160$ and $K_p = 8$ respectively. Using the Alamouti scheme of [3], the number of transmit antennas N_t is equivalent to the number of frequency slots N_{FS}, thus $N_{FS} = 2$. The OFDM symbol is divided into $B_p = K_p/N_{FS}$ groups, which means that 4 groups will be created. The number of blocks of data subcarriers per group would be equal to B/B_p which in this case is equal to 20. Therefore the block of pilot subcarriers is set between data subcarrier 20 and 21 for each group. By doing so, an equivalent number of 20 subcarriers can be found on each side of the pilot subcarriers which will be decoded simultaneously at the receiver and therefore reduce the computation complexity of the system.

The second step occurs at the receiver side, the pilot sequence known at the receiver is used to estimate the channel frequency response of the corresponding subcarriers. Then, under the

assumption that the channel parameters for two adjacent SFBC coded subcarriers are similar, the channel estimated by the pilot subcarriers is used to recover the adjacent SFBC-coded data subcarriers. Using the earlier example, channel parameters for pilot subcarriers between data subcarriers 20 and 21 of each group can be estimated and then used to simultaneously decode adjacent SFBC coded data subcarriers 19, 20 and 21, 22. Once adjacent data subcarriers have been recovered, channel parameters for the corresponding data subcarrier can be estimated and used to decode the next set of adjacent data subcarriers 17, 18 and 23, 24. The estimated data becomes the new pilot sequence, which will first be used to estimate the channel parameters, and then used to recover the next set of data symbols. With the assumption that the channels remain constant for two adjacent SFBC coded blocks, the next pair of symbols can be jointly estimated in the lower side of the pilot subcarriers as well as in the upper side. Finally, with the use of groups, the entire OFDM symbol is decoded faster than the traditional SFBC-OFDM decoder. This new decoding method improves the computation and memory efficiency of the system.

B. Least Square Channel Estimation

A sequence of L_p pilot code vectors which form the orthogonal SFBC coded matrix $\mathbf{P} = \left[\mathbf{p_0}, \mathbf{p_1}, \dots, \mathbf{p_{L_p-1}}\right], \mathbf{p}_{l_p} = [p_0,\ p_1, \dots, p_{K_p-1}]^T$ is assumed known at the receiver and used to perform the channel estimation at the pilot subcarriers. The received pilot matrix $\mathbf{R_p} = \left[\mathbf{rp_0}, \mathbf{rp_1}, \dots, \mathbf{rp_{L_p-1}}\right], \mathbf{rp}_{l_p} = [rp_0,\ rp_1, \dots, rp_{K_p-1}]^T$ can be expressed as:

$$\mathbf{R_p} = \frac{\sigma_p}{\sqrt{N_t}} \mathbf{P H_p} + \mathbf{N_p} \tag{7}$$

where σ_p^2 represents the total transmitted pilot energy across all the N_t transmit antennas in each time slot, $\mathbf{H_p} = \left[\mathbf{hp_0}, \mathbf{hp_1}, \dots, \mathbf{hp_{L_p-1}}\right], \mathbf{hp}_{l_p} = [hp_0,\ hp_1, \dots, hp_{L_p-1}]^T$ and $\mathbf{N_p} = \left[\mathbf{np_0}, \mathbf{np_1}, \dots, \mathbf{np_{L_p-1}}\right]$ represents the white Gaussian noise.

In the next phase, data is transmitted simultaneously from the N_t transmit antennas and the received matrix $\mathbf{R_s}$ can be expressed as:

$$\mathbf{R_s} = \frac{\sigma_s}{\sqrt{N_t}}\mathbf{SH} + \mathbf{N_s} \tag{8}$$

where similar to (7), σ_S^2 represents the total transmitted data energy across all the N_t transmit antennas in each time slot, $\mathbf{S} = [\mathbf{s_0}, \mathbf{s_1}, ..., \mathbf{s_{L_s-1}}]$, $\mathbf{s}_l = [s_0, s_1, ..., s_{K_s-1}]^T$ with L being the number of data vectors and K_S denotes the number of data subcarriers, $\mathbf{H} = [\mathbf{h_0}, \mathbf{h_1}, ..., \mathbf{h_{L_s-1}}]$, $\mathbf{h}_l = [h_0, h_1, ..., h_{L_s-1}]^T$ and $\mathbf{N} = [\mathbf{n_0}, \mathbf{n_1}, ..., \mathbf{n_{L_s-1}}]$ represent the channel and white Gaussian noise respectively. As in [26], the least square (LS) estimate of the channel $\mathbf{H_p}$ in (5) is given as:

$$\mathbf{H_p} = \mathbf{R_p}\frac{\sqrt{N_t}}{\sigma_p}\mathbf{P}^\dagger \tag{9}$$

where $\mathbf{P}^\dagger = \mathbf{P}^H(\mathbf{PP}^H)^{-1}$. Due to the orthogonality property of the matrix $\mathbf{P}$, $\mathbf{P}^\dagger$ can be simplified such that $\mathbf{P}^\dagger = \mathbf{P}^H$. Combining (9) and (7), the following is obtained:

$$\mathbf{H_p} = \mathbf{H_p} + \frac{\sqrt{N_t}}{\sigma_p}\mathbf{P}^\dagger\mathbf{N_p} \tag{10}$$

with $E\{\mathbf{N_p}\mathbf{N_p}^H\} = \sigma_n^2 N_r$ and $\tilde{\mathbf{H}}_\mathbf{p} = \mathbf{H_p} - \mathbf{H_p}$, the estimated variance of $\tilde{\mathbf{H}}_\mathbf{p}$ is:

$$
\begin{aligned}
\sigma_h^2 &= E\left\{\left\|\tilde{\mathbf{H}}_\mathbf{p}\right\|_F^2\right\} \\
&= \frac{\sigma_n^2}{\sigma_p^2}N_t N_r tr\left\{(\mathbf{PP}^H)^{-1}\right\}
\end{aligned}
\tag{11}
$$

Due to the orthogonality property of the LS estimator, $\mathbf{H_p}$ and $\tilde{\mathbf{H}}_\mathbf{p}$ are uncorrelated. Moreover, substituting $\tilde{\mathbf{H}}_\mathbf{p}$ into (7), the following can be obtained:

$$r_{p_{i,t}} = \frac{\sigma_p}{\sqrt{N_t}}\sum_{j=1}^{N_t}p_{j,t}h_{p_{i,j,t}} + \underbrace{\frac{\sigma_p}{\sqrt{N_t}}\sum_{j=1}^{N_t}p_{j,t}h_{p_{i,j,t}} + n_{p_{i,t}}}_{z_p} \tag{12}$$

where $\tilde{h}_{p_{i,j,t}}$ and $h_{p_{i,j,t}}$ are elements of the matrix $\tilde{\mathbf{H}}_{\mathbf{p}}$ and $\mathbf{H}_{\mathbf{p}}$ respectively and the variance of z_p can be expressed as:

$$\sigma_z^2 = E\left\{\left|z_p\right|^2\right\} = \frac{\sigma_p^2}{N_t}\sum_{j=1}^{N_t}\left|p_{j,t}\right|^2\sigma_h^2 + \sigma_n^2 \tag{13}$$

IV. PERFORMANCE ANALYSIS

In this section, the performance of the ML detection technique is analysed by looking at the upper Chernoff bound on error probability. In the analysis, LS channel estimation is taken into account. Assuming the data sequence $\mathbf{s}$ is transmitted and after decoding, the receiver selects the code vector $\tilde{\mathbf{S}}$ where $\tilde{\mathbf{S}} = [\tilde{\mathbf{s}}_0,\ \tilde{\mathbf{s}}_1, \dots, \tilde{\mathbf{s}}_{K_s-1}]^T$ and $\tilde{\mathbf{s}}_l = [\tilde{s}_0,\ \tilde{s}_1, \dots, \tilde{s}_{K_s-1}]^T$. Using the minimum distance rule in (6), the following can be deduced:

$$\mathbf{S} = \arg\left\{\min_{\mathbf{X}=[\mathbf{x}_1,\mathbf{x}_2,\dots,\mathbf{x}_{K_s}]}\sum_{l=1}^{K_s}\left\|\mathbf{r}_l - \frac{\sigma_s}{\sqrt{N_t}}\mathbf{H}_{\mathbf{p}}\mathbf{x}_l\right\|^2\right\} \tag{14}$$

Substituting (10) into (14), we obtain:

$$\mathbf{S} = \arg\left\{\min_{\mathbf{X}=[\mathbf{x}_1,\mathbf{x}_2,\dots,\mathbf{x}_{K_s}]}\sum_{l=1}^{K_s}\left\|\mathbf{r}_l - \frac{\sigma_s}{\sqrt{N_t}}\mathbf{H}_{\mathbf{p}}\mathbf{x}_l - \frac{\sigma_s}{\sigma_p}\mathbf{N}_{\mathbf{p}}\mathbf{P}^\dagger\mathbf{x}_l\right\|^2\right\} \tag{15}$$

The received data signal $\mathbf{r}_l$ in (13) can be expressed using (3) and (6) as $\mathbf{r}_l = \frac{\sigma_s}{\sqrt{N_t}}\mathbf{s}_l\mathbf{H} + \mathbf{n}_l$.

Substituting the expression of $\mathbf{r}_l$ into (13) when the channel estimation is perfect, that is $\mathbf{H}_{\mathbf{p}} = \mathbf{H}_{\mathbf{p}} = \mathbf{H}$ leads to:

$$\mathbf{S} = \arg\left\{\min_{\mathbf{X}=[\mathbf{x}_1,\mathbf{x}_2,\dots,\mathbf{x}_{K_s}]}\sum_{l=1}^{K_s}\left\|\frac{\sigma_s}{\sqrt{N_t}}\mathbf{H}(\mathbf{s}_l - \mathbf{x}_l) + \mathbf{n}_l - \frac{\sigma_s}{\sigma_p}\mathbf{N}_{\mathbf{p}}\mathbf{P}^\dagger\mathbf{x}_l\right\|^2\right\} \tag{16}$$

where the noise vector $\mathbf{n}_l$ denotes the Gaussian noise.

The PEP can therefore be expressed as:

$$P(\mathbf{S} \to \mathbf{S}) = P_r\left\{\sum_{l=1}^{K_s}\left\|\frac{\sigma_s}{\sqrt{N_t}}\mathbf{H}(\mathbf{s}_l - \tilde{\mathbf{s}}_l) + \mathbf{n}_l - \frac{\sigma_s}{\sigma_p}\mathbf{N}_{\mathbf{p}}\mathbf{P}^\dagger\tilde{\mathbf{s}}_l\right\|^2\right\} \tag{17}$$

From the Gaussian distribution of $\mathbf{n}_l$, it can be shown that the conditional PEP, conditioned on the channel matrix $\mathbf{H}$, is given by:

$$P(\mathbf{S} \to \mathbf{S}|\mathbf{H}) = Q\left(\sqrt{\frac{\frac{\sigma_s^2}{N_t}d^2(\mathbf{S}-\mathbf{S})}{2\sigma_z^2}}\right) \qquad (18)$$

where $Q(\bullet)$ denotes the complementary error function also called the "Q" function and is

limited by $Q(x) \leq \dfrac{1}{2}e^{-\frac{x^2}{2}}$ with $x \geq 0$ and $d^2(\mathbf{S}-\mathbf{S})$ is the Euclidean distance between the two

space-frequency codewords $\mathbf{S}$ and $\mathbf{S}$ which is given by:

$$d^2(\mathbf{S}-\mathbf{S}) = \sum_{t=1}^{L}\sum_{i=1}^{N_r}\left|\sum_{j=1}^{N_t}h_{i,j,t}(s_{i,t}-\tilde{s}_{i,t})\right|^2 \qquad (19)$$

From (18) and the definition of the Q function, we have the following Chernoff bound:

$$P(\mathbf{S} \to \mathbf{S}|\mathbf{H}) \leq \frac{1}{2}\exp\left(-\frac{\frac{\sigma_s^2}{N_t}d^2(\mathbf{S}-\mathbf{S})}{4\sigma_z^2}\right) \qquad (20)$$

Substituting (19) into (20), the Chernoff bound can be further written as:

$$P(\mathbf{S} \to \mathbf{S}|\mathbf{H}) \leq \frac{1}{2}\exp\left(-\frac{\frac{\sigma_s^2}{N_t}\sum_{t=1}^{L}\sum_{i=1}^{N_r}\left|\sum_{j=1}^{N_t}h_{i,j,t}(\mathbf{s}_{i,t}-\tilde{\mathbf{s}}_{i,t})\right|^2}{4\sigma_z^2}\right)$$

$$\leq \frac{1}{2}\exp\left(-\frac{\frac{\sigma_s^2}{N_t}\sum_{j=1}^{N_r}h_{i,j}Dh_{i,j}^H}{4\sigma_z^2}\right) = \frac{1}{2}\exp\left(-\frac{\sigma_s^2 h_j Dh_j^H}{4N_t\sigma_z^2}\right) \qquad (21)$$

where h_i is the i_{th} column of $\mathbf{H}$ and $D = (\mathbf{s}_{i,t}-\tilde{\mathbf{s}}_{i,t})(\mathbf{s}_{i,t}-\tilde{\mathbf{s}}_{i,t})^H$.

Finally, the upper bound of the PEP at high SNR becomes:

$$P(\mathbf{S} \to \mathbf{S}) \le \prod_{t \in \rho(\mathbf{s},\tilde{\mathbf{s}})} \left(\frac{1}{1 + \left|\mathbf{s}_t - \tilde{\mathbf{s}}_t\right|^2 \dfrac{\sigma_s^2 \sigma_{\hat{h}}^2}{4N_t \sigma_z^2}} \right)^{N_r}$$

$$\le \quad (d_p^2)^{-N_r} \left(\underbrace{\frac{\sigma_s^2 \sigma_{\hat{h}}^2}{4N_t \sigma_z^2}}_{f(\square)} \right)^{-\delta_h N_r} \tag{22}$$

where $\rho(\mathbf{s},\tilde{\mathbf{s}})$ denotes the set of time instances $t=1, 2, ..., L_s$, such that $\left|\mathbf{s}_t - \tilde{\mathbf{s}}_t\right|^2 \ne 0$, δ_h defines

the number of space frequency symbols in which the two codewords $\mathbf{S}$ and $\mathbf{S}$ differs and d_p^2

is the product of the squared Euclidean distance between the two space-frequency symbol

sequences $d_p^2 = \prod_{t \in \rho(\mathbf{s},\tilde{\mathbf{s}})} \left|\mathbf{s}_t - \tilde{\mathbf{s}}_t\right|^2$.

V.   OPTIMUM POWER ALLOCATION FOR LEAST SQUARE CHANNEL ESTIMATION

In this section, the power allocation problem is addressed for pilot and data subcarriers.

Different scenarios have been considered. In the first scenario pilot subcarriers are assigned

higher transmit power than data subcarriers. In the second scenario data and pilot subcarriers

have equal transmit power. To this end, the following assumptions have been made;

$$\sigma_{\hat{h}}^2 = \sigma_h^2 - \frac{\sigma_n^2}{\sigma_p^2} N_t N_r, \quad \sigma_z^2 = E\left\{\left|z_p\right|^2\right\} = \frac{\sigma_p^2}{N_t} \sum_{j=1}^{N_t} \left|p_{j,t}\right|^2 \sigma_h^2 + \sigma_n^2, \quad \sigma_p^2 = \frac{\alpha L}{K_p}, \quad \sigma_s^2 = (1-\alpha)\frac{L}{K_s} \quad \text{and}$$

$SNR = \dfrac{s_h^2}{s_n^2}$; where α represents the normalised block power allocated to the pilot part, L

represents the number of OFDM symbols, K_p and K_s represent the length of the pilot and data

blocks respectively. Therefore, the function of α using (20) can be deduced:

$$f(\alpha) = \frac{\sigma_s^2 \sigma_{\hat{h}}^2}{4N_t \sigma_z^2} = \frac{\alpha SNRL\left(1 - \alpha + \dfrac{N_t N_r}{SNRL}\right) - N_t N_r}{4N_t \alpha (K_s - N_t N_r) + 4N_t^2 N_r \nu} \tag{23}$$

where $\nu = \sum_{j=1}^{N_t} \left|p_{j,t}\right|^2$.

Differentiating (23) and equating the result to zero, the optimum solution for the pilot power factor can be derived as follows:

$$\alpha = \begin{cases} \dfrac{1}{2}\left(1+\dfrac{K_s}{SNRL}\right) & ; K_s = N_t N_r \\[3mm] \dfrac{8N_t^2 N_r vSNRL - 8N_t N_r \sqrt{N_t SNRL\left(N_t vSNRL\left(v^2-1+K_s\right)+K_s\left(K_s-N_t\right)\right)}}{8N_t SNRL\left(N_t N_r - K_s\right)} & ; K_s \neq N_t N_r \end{cases}$$

(24)

Total mean square error (MSE) given as $tr\{\sigma_{\hat{h}}^2\}$ where $\sigma_{\hat{h}}^2$ is given in (11) must satisfy the following inequality:

$$Total\ MSE = \frac{s_n^2}{s_p^2} N_t N_r \leq L \tag{25}$$

From (22), two scenarios can be further investigated. The symmetric case is first considered where $K_s = N_t N_r$ then the asymmetric case is considered where $K_s \neq N_t N_r$.

A. The case of $K_s = N_t N_r$

Substituting (24) into (25), the SNR range can be found and the minimum optimum pilot power factor α when MSE=L is given as:

$$SNR \geq \frac{K_s}{2\sigma_n^2 N_t N_r - L} \quad and \quad \alpha_{min} = \frac{\sigma_h^2 N_t N_r}{2\sigma_h^2 N_t N_r - K_s} \tag{26}$$

The range of the optimum power when the SNR satisfies the inequality in (26) can be defined as

$$\frac{\sigma_h^2 N_t N_r}{2\sigma_h^2 N_t N_r - K_s} \leq \alpha < \frac{1}{2}\left(1+\frac{L_s}{SNRL}\right) \tag{27}$$

If SNR does not satisfy the inequality in (26), the probability of detection error will increase due to the unreliable total MSE of the channel estimation. In such cases, according to the range of α given in (27), the minimum value of α will be used because $f(\alpha)$ is a monotonically increasing function of α. To summarise, the optimum pilot power can be written as:

$$\alpha = \begin{cases} \dfrac{\sigma_h^2 \mathrm{N}_t N_r}{2\sigma_h^2 \mathrm{N}_t N_r - K_s} & ; SNR \geq \dfrac{\sigma_h^2 \mathrm{N}_t N_r}{2\sigma_h^2 \mathrm{N}_t N_r - K_s} \\[4mm] \dfrac{1}{2}\left(1 + \dfrac{L_s}{SNRL}\right) & ; Otherwise \end{cases} \tag{28}$$

B. The case of $K_s \neq N_t N_r$

Similar to the case of $\mathrm{K}_s = \mathrm{N}_t \mathrm{N}_r$, substituting (24) into (25), the SNR range can be found and the minimum optimum pilot power factor α is given as:

$$\frac{\sqrt{N_t SNRL\left(N_t \nu SNRL\left(\nu^2 - 1 + K_s\right) + K_s\left(K_s - N_t\right)\right)}}{SNR} \geq \frac{1}{\sigma_n^2 \mathrm{N}_t\left(N_t N_r - K_s\right) - \mathrm{N}_t L \nu}$$

and $\qquad\qquad \alpha_{\min} \leq \dfrac{N_t^2 \nu \sigma_h^2 N_r}{1 - \sigma_h^2 \mathrm{N}_t\left(\mathrm{N}_t N_r - K_s\right)}$ $\qquad\qquad$ (29)

The range of the optimum power when the SNR satisfies the inequality in (29) can be defined by:

$$\alpha_{\min} \leq \alpha \leq 1 - \frac{\mathrm{N}_t N_r \nu}{L} \tag{30}$$

To summarise, the optimum pilot power can be defined as follows:

$$\alpha = \begin{cases} \alpha_{\min} & ; when\ inequality \\ & (27)\ is\ not\ satisfied \\[4mm] \dfrac{8N_t^2 N_r \nu SNRL - 8N_t N_r \sqrt{N_t SNRL\left(N_t \nu SNRL\left(\nu^2 - 1 + K_s\right) + K_s\left(K_s - N_t\right)\right)}}{8N_t SNRL\left(N_t N_r - K_s\right)} & ; Otherwise \end{cases} \tag{31}$$

VI. COMPUTATIONAL COMPLEXITY

The computational complexity is one of the important factors of the estimator performance. In this section the computational complexity of the proposed scheme is evaluated and capital O notation represents the computational complexity and n represents the matrix size. With the help of [35], the computational complexity of the proposed channel estimation method is $O(N_p^2 N_t + N_p N_t + \min(N_p, N_t)^3)$. Based on the formula of the computational complexity, it is clear that increasing the number of pilot symbols would lead to an increase

of complexity such that for example, increasing the number of pilot from eight to sixteen, the complexity would increase by 200%.

Table I shows a comparison of the computational complexity of the proposed algorithm with algorithm proposed in [27]. The number of complex multiplication in the method proposed in this paper is of the same order to the Least-Square method proposed in [27]. However, the order of the complexity of the linear minimum mean square error (LMMSE) method proposed in [27] is higher than our proposed method.

Table I: Computational Complexity of the proposed algorithm

Algorithm	Number of complex Multiplication	Order
LS [27]	$O(N_p(N_p + N_t N_p + 1))$	$O(n^2)$
LMMSE [27]	$O(N_p(N_p + 1) + N_p^4(N_t + 1))$	$O(n^4)$
Proposed Method	$O(N_p^2 N_t + N_p + \min(N_p, N_t)^3)$	$O(n^2)$

VII. SIMULATION RESULTS

In this section, simulation results are presented for the proposed channel estimation method. Simulation results and examples are also given for the power analysis discussed in Section V. The performance of the proposed iterative channel estimation technique has been evaluated according to the specifications described in the WiMax standard for fixed and mobile wireless communications. Simulation results are presented for two and four transmit antennas and for different number of receive antennas. The proposed method was tested under different levels of Doppler shift and for different number of pilot subcarriers.

The simulation scenario adopted for the analysis of SFBC-OFDM is as follows: a 16-QAM modulation format is employed and the number of transmit and receive antennas utilized are given as N_t=2, 4 and N_r=1, 2, 3, 4. In addition, σ_h^2 and ν have been set to 1 and $K_s = N_t$. The

system has a 3.5 MHz channel bandwidth and a carrier frequency of 2.5GHz. Specific simulation parameters are presented in Table II.

Allocation of the subcarriers of the OFDM frame is made according to the IEEE802.16e (WiMax) standard [28]; indices of -128~-101 and 101~127 are reserved for guard interval, 0 is for the DC subcarrier, -100~-1 and 1~100 are defined as the chosen subcarriers in which -88, -63, -38, -13, 13, 38, 63 and 88 are pilot subcarriers and the remaining are specified as data subcarriers. However, in order to achieve more accurate channel estimation in fast fading environments, the pilot symbols are relocated to adjacent subcarriers. This is particularly useful for SFBC schemes where adjacent subcarriers experience similar channel gain. Thus the pilot subcarriers have been redefined for this work as -76, -75, -26, -25, 25, 26, 75 and 76 for two transmit antennas. For a higher number of transmit antennas, adjacent subcarriers would also be defined in order to have an equal number of data subcarriers on each side of the pilot subcarriers (for example, subcarrier -4, -3, -2, -1, 1, 2, 3 and 4 would be used for four transmit antennas and therefore 96 data subcarriers would be found on each side of the pilot subcarriers). Such values have been defined in order to have pilot subcarriers regrouped to achieve more accurate channel estimation as the channel gains of two adjacent subcarriers can be considered to be approximately equal.

Table II: Simulation Parameters

FFT Size (Nfft)	256
Number of Active Subcarriers (N_{used})	200 (192 for data, 8 for pilot)
Number of Guard Subcarriers	28 low, 27 high
Channel Bandwidth	3.5MHz
Sampling Rate (Fs)	2.28MHz (n=57/50)
Distance between Adjacent Subcarrier (Δf)	8.9kHz
Useful Symbol Duration (T_b)	0.112ms
Guard Time (T_g)	28.07µs
Total Symbol Duration (T_s)	140 µs
Modulation	16 QAM

Doppler Shift	50, 100Hz
SUI	1
Transmit Antenna	2, 4
Receive Antenna	1, 2, 3, 4

$f(\alpha)$ given in (23) as a function of α is shown in Fig.2 for different values of SNR for 2 transmit antennas and one receive antenna. It can be observed that the value of $f(\alpha)$ increases with the values of SNR which causes the value of PEP expressed in (22) to reduce. From Fig.2, the maximum value of $f(\alpha)$ can be evaluated at $\alpha=0.45$ for different SNR values of 30dB, 20dB, 15dB and 10dB that matches the numerical value obtained from (24). In addition, it can be seen that the performance of the proposed power allocation method outperforms the one in [29].

Fig. 2: $f(\alpha)$ (c.f.(21)) vs. α at different SNR for MIMO-OFDM systems

In Fig.3, bit error rate (BER) vs SNR plot is shown for the Alamouti matrix G2 in [30] where the performance of the proposed channel estimation method is compared with the LMMSE channel estimation method of [31]. From Fig.3, it is observed that the BER performance of the proposed channel estimation method is close to that of [31]. However, the proposed channel estimation method offers lower complexity and better computation. Looking at (10) and (11) in [31], it can be observed that the derivation given in this paper for channel

estimation is simpler than the one given in [31]. The results of the proposed method still shows better performance compared to the one given in [32] where the performance degradation observed for 2 transmit antennas ranges from 3dB to more than 10dB as estimation error increases. In SFBC schemes, detection is achieved within one OFDM symbol, therefore, systems are less subject to Doppler interference than STBC-OFDM systems. By looking at [33], it can be seen that STBC-OFDM systems reach their BER limits faster than SFBC-OFDM systems.

From Fig. 3, Fig. 4 and Fig. 5, it can be seen that as the number of transmit or receive antennas increase, the BER performance also improves. Matrices G2 and G4 of [30] have been used to investigate the performance of the proposed channel estimation method. Performance has also been investigated for higher number of pilot subcarriers (Np) as well as higher number of data subcarriers. It has been observed that a higher number of pilot subcarriers per OFDM symbol achieve better performance at the cost of bandwidth inefficiency. Indeed, if more pilot subcarriers are used, the number of data subcarriers is reduced. Simulation for a higher number of data subcarriers have also been conducted and have shown a degradation which increases with the number of subcarriers. The proposed method offers a trade-off between accurate channel estimation and efficient bandwidth usage as more pilots would allow the algorithm to perform more accurate channel estimation at the cost of less transmitted data.

With the proposed iterative channel estimation technique, the grouping of symbols improves the computational efficiency of the system. When the number of pilot symbols is increased, the number of groups is also increased. This means that the decoding time is also reduced as two sets of SFBC-OFDM symbols are decoded concurrently in each group. Moreover, with the use of SFBC-OFDM, decoding can take place within one OFDM symbol which when compared to STBC-OFDM, saves half of the memory used. Indeed, in order to decode the data in STBC-OFDM, the system needs to save two OFDM symbols in contrast to SFBC-OFDM where the system needs to save only one OFDM symbol. Thus, it can be concluded

that the proposed SFBC-OFDM channel estimation method is a good candidate for high mobility environments.
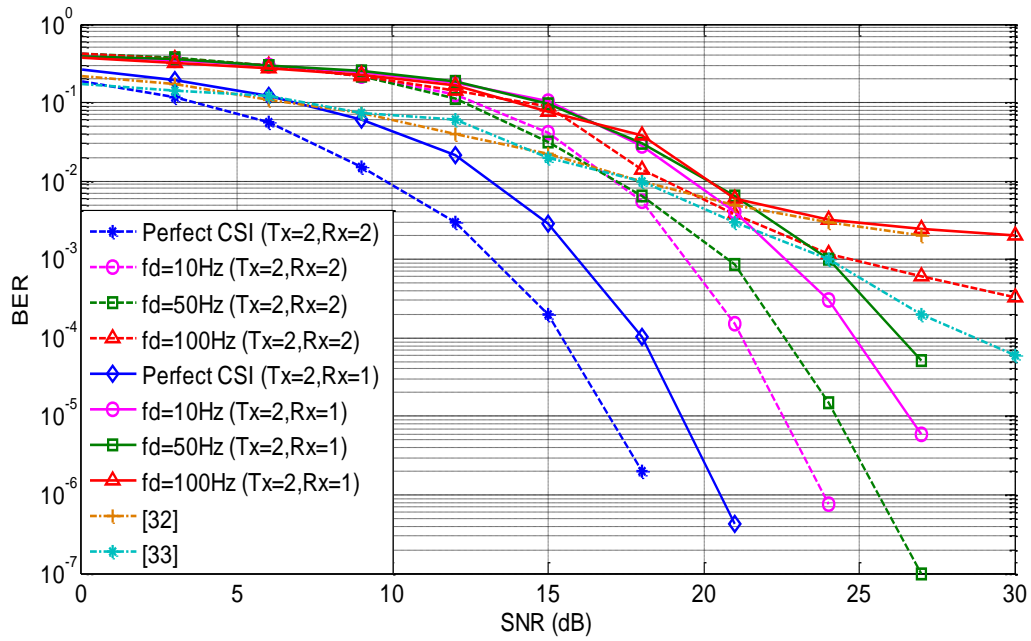

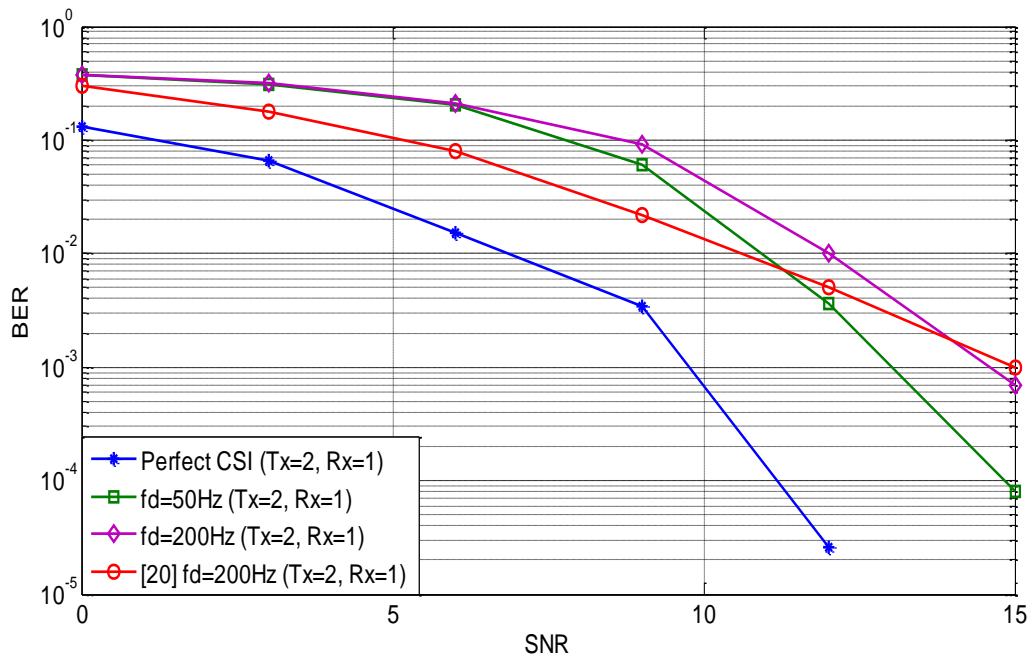Fig. 3: Performance results for 2 transmit antennas with 16-QAM

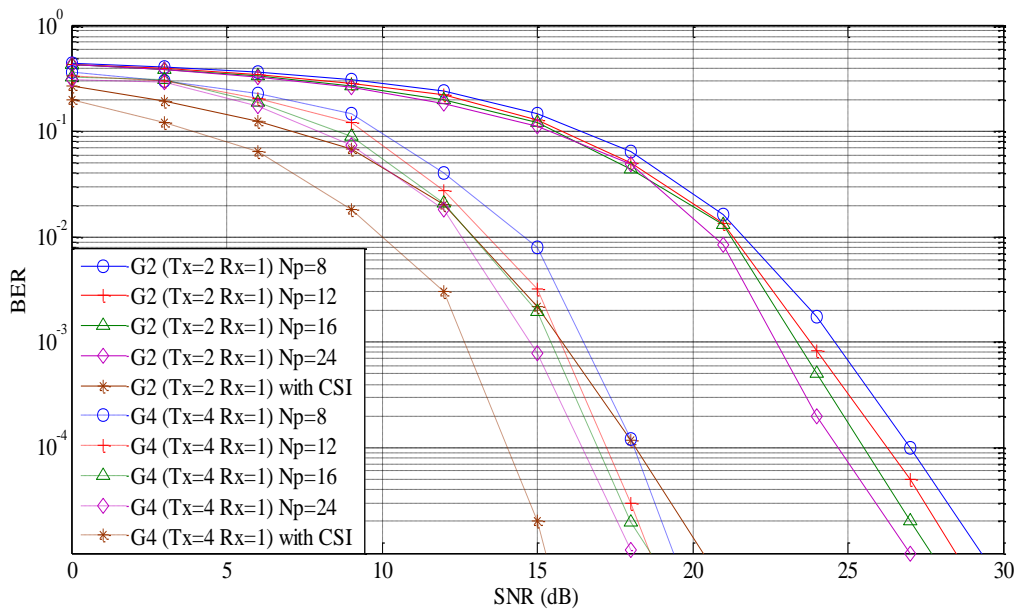Fig. 4: Performance results for 2 transmit antennas with QPSK

Fig. 5: Effect of number of pilot subcarriers on the performance of the channel estimation technique for 16-QAM

Fig. 6 shows the performance of the proposed method for different Doppler shifts for a SNR value of 20dB. It can be observed that as the speed increases the performance of the channel estimation method decreases due to the high frequency selectivity of the channel. Since the channel estimation method relies on an iterative process, the BER performance is significantly affected by Doppler shift. At high levels of Doppler shift, the performance degrades compared to when low levels of Doppler shift is experienced.
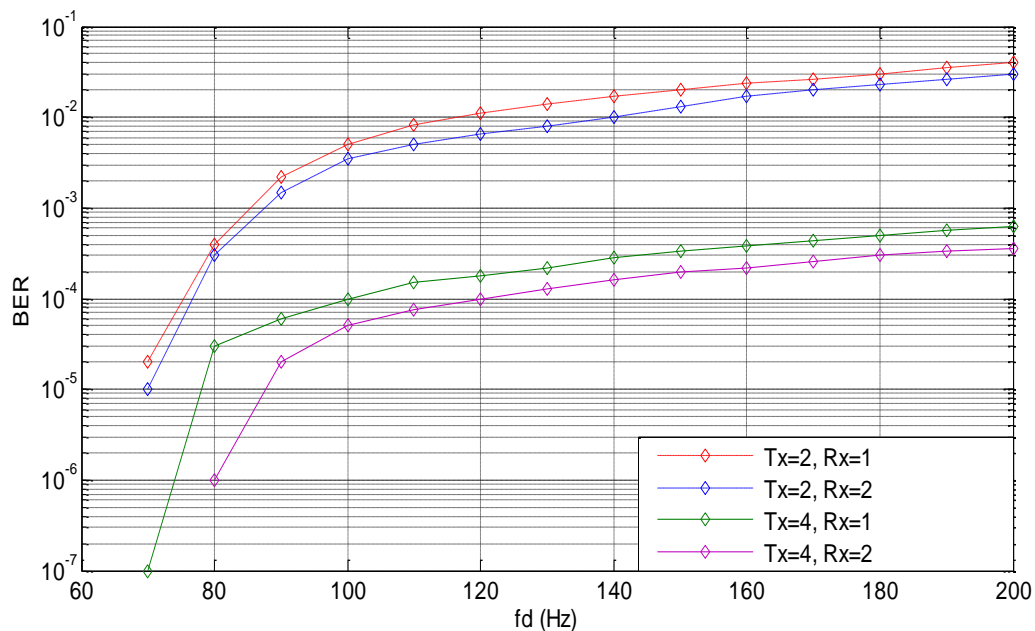

Fig. 6: Performance of the proposed channel estimation technique with variable Doppler shift

Finally Fig. 7 shows the Normalized MSE (NMSE) performance of the proposed method. The NMSE is defined by:

$$NMSE(channel) = \frac{\sum\limits_{i=1}^{K_s} \left\| h_{i,j,k} - \tilde{h}_{i,j,k} \right\|^2}{\sum\limits_{i=1}^{K_s} \left\| h_{i,j,k} \right\|^2} \tag{32}$$

It can be observed from Fig. 7 that higher number of antennas provide better channel estimation performance due to a higher number of replicas at the receiver and therefore channel estimation is more accurate. In addition, compared with [21] and [34], it can be seen that the proposed channel estimation technique performs better for two and four transmit antennas respectively. The performance of the proposed channel estimation method outperforms the method in [21] and [34] by values from 3dB to more than 10dB. Moreover the method proposed in [21] and [34] reaches saturation limit faster.
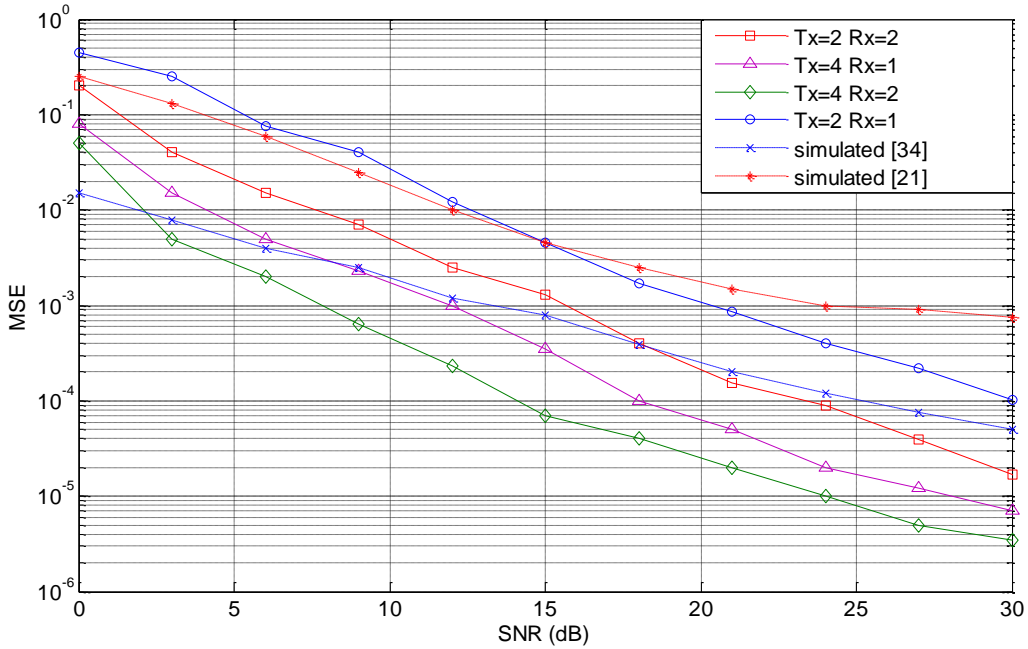

Fig. 7: NMSE performance of the channel estimation method

VIII. CONCLUSION

A new iterative channel estimation method has been proposed for SFBC MIMO-OFDM systems. Compared to existing channel estimation methods in the literature, the proposed method offers the advantage of computational efficiency as multiple blocks of SFBC information signals are decoded simultaneously. In addition, the information signals utilized

in this work are designed to have an orthogonal structure, thus matrix inversion is not required at the receiver. This significantly reduces the amount of time required to decode information signals. Simulation results show that the proposed method outperforms some other existing low-complexity techniques in the literature. Simulations results also show that the length of training sequence employed for channel estimation offers a trade-off between accurate channel estimation and efficient bandwidth usage. In other words, more pilots would enable the algorithm to perform more accurate channel estimation at the cost of less transmitted data.

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