RESOURCE EFFICIENCY PROFILING

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ABSTRACT

Consider a set of decision-making units (DMUs, e.g. branches ,departments, firms) which employ a variety of resources to produce multiple outputs. The units being compared may be in the public sector e.g. health or education, so that the outputs and inputs need not be measured in monetary terms. We present two methods for evaluating the relative efficiency with which each individual resource input is utilised at each organizational unit. Unlike DEA (data envelopment analysis) a large data set is not required and the discriminatory power is shown to be higher.

KEYWORDS:

Efficiency, data envelopment analysis, performance measurement, production, productivity.

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INTRODUCTION

Our starting point is the definition of efficiency used in science and engineering, namely, output/input . There the efficiency value naturally lies between zero and unity (100%), but in an organisational context we need to impose constraints to ensure all the scores remain in this range. Once an efficiency score , $\boldsymbol{E}$, is obtained it is possible to calculate target values for inefficient DMUs . This can be done in two ways :--

(i) keep output fixed , in which case the target input is $\boldsymbol{E}$ * (current input)

(ii) keep the input fixed so that the target output is (current output) / $\boldsymbol{E}$.

Once we have multiple inputs and/or outputs we encounter the question of how to aggregate these when there is no pricing system available and the units of measurement differ.

In recent years there has been a great deal of interest in a technique (DEA) which claims to measure the overall technical efficiency of a DMU relative to others which carry out the same type of activities; the efficiency is calculated as the sum of weighted outputs divided by the sum of weighted inputs . DEA applies linear programming to find for *each* DMU its set of weights which will give it the maximum score subject to the conditions that none of the scores exceed unity. Users of DEA sometimes calculate target values as above for individual inputs and outputs, however these are derived from the overall score and so this may not be an appropriate way of proceeding.

Whilst there is a wealth of literature on the estimation of overall technical efficiency, there is very little indeed that deals with a separate efficiency measure for each input when there are multiple outputs; Kumbhakar[4] assumed a particular type of output function (namely Cobb-Douglas: the output is equated to the product of the inputs raised to powers) and also assumed that technical efficiency followed a half-normal distribution and was time-invariant. His stochastic frontier model was applied to U.S. railroad data spanning 25 years and the results indicated that the most inefficient railroad used 40%-50% more labour and 6%-9% more fuel than those on the efficient frontier. Kopp [3] suggested another approach to resource efficiency measurement and he too required the assumption of a known production function relating output to the inputs. Both these papers deal with the case of a single output or require that everything is reducible to this form.

In this paper two efficiency profiling methods are presented which enable the relative (technical) efficiency of individual inputs to be evaluated for each decision-making unit. There are a number of advantages that such an approach has over the efficiency score provided by the currently popular DEA models:

(1) It enables the source and extent of inefficiencies in the individual DMU to be more precisely determined: a particular DMU may be efficient in its utilisation of one resource (e.g. sales staff) and inefficient in its utilisation of another (e.g. clerical staff). Such directive information may assist local managers in improving the efficiency of their DMU.

(2) It rules out the possibility of effectively ignoring some inputs at some DMUs by attaching zero or near-zero weights to them. (It would clearly not be sensible to use efficiency scores for input target-setting when no account has been taken of that input in the efficiency assessment.) For the methods of this paper these problems will be avoided by providing a score for each input at every DMU.

(3) Only those outputs to which a given resource acts as an input will be considered in the assessment of that input. As well as being intuitively sensible this rules out the unjustifiable appearance of efficiency by, for instance, placing all the weight on a single input and a single output which are not causally related.

(4) The number of dimensions and 'free parameters' for the new LP model will be fewer than in the equivalent DEA model and so one would expect greater discrimination between the DMUs and a lower proportion appearing to be 100% efficient.

METHOD USING INDIVIDUAL WEIGHTS

Suppose that resource $\boldsymbol{I_i}$ acts as an input to s outputs $\boldsymbol{O_r}$ (r =1,...,s); this may be a subset of the outputs. Note that a different resource may act as an input to a different set of outputs, possibly fewer or more. The relative efficiency (E_{ik}) with which resource i is being utilised by

DMU k is evaluated using the following L.P.(linear program) in which the u-variables are being solved for, and the *O's* and *I's* are the observed output and input values.

$$\text{Maximize } E_{ik} = \frac{\sum_{r=1}^{s} u_{irk} O_{rk}}{I_{ik}} \qquad (1)$$

$$\text{subject to } \frac{\sum_{r=1}^{s} u_{irk} O_{rj}}{I_{ij}} \leq 1 \quad , \; j=1,\ldots,n \qquad (2)$$

$$\text{and } u_{irk} > \varepsilon \quad , \quad r=1,\ldots,s \qquad (3)$$

Where ε is a small positive number, n is the number of DMUs and u_{irk} is the weight attached to output r when evaluating the efficiency of input i of DMU k . As with DEA each DMU has its own set of weights which show it in its best light within the constraints of the method (which merely ensure that efficiency scores do not exceed unity). The key difference between this and the DEA formulation is that here each linear program only deals with a single input rather than a weighted sum of all inputs.

The interpretation of the efficiency scores is best understood in terms of target values. Consider a branch which has a score of 0.8 in relation to one of its inputs, this means it could aim to produce the same outputs as before but using only 80% of the current level of that input. This is because there is a combination of the other branches which could achieve the same outputs using only 80% of the input at the branch being studied.

METHOD USING COMMON WEIGHTS

Once again we study each input resource separately but using a common set of weights to be used on all DMUs. These weights will determine the position of the efficient frontier, hence in estimating them we must first discard any DMUs which are seen to be dominated (by linear combinations of other DMUs) in their usage of the given input; this has already been done for us by our first method since a score below 100% when there is complete flexibility of weights implies that the DMU is dominated by others. We are then left with the non-dominated set (unit efficiency and zero slacks) ; we now try to find a relationship which shows how these DMUs disperse the input amongst their relevant outputs. Hence we might try to fit an expression of the form :

$$I_i = \sum_{r=1}^{s} c_{ir} O_r \qquad (4)$$

The parameters (c), can be interpreted as the amount of input i used per unit of output j (i.e.they are resource consumption rates). Hence each term on the right of (4) estimates the amount of input contributed to that output by an efficient firm with the given output mix. Such an expression is a departure from the usual econometric approach in which aggregate output is expressed as a function of inputs; however we believe that this approach may have the benefits of being easier to comprehend and use. (In econometrics one might try various production functions (Cobb- Douglas,translog,CES,etc) whose form is far from being intuitive, particularly to the average manager.)

The parameter values c in (4) are determined separately for each input. They can be found using least-squares regression subject to the constraints that they be non-negative and that

$$I_i \geq \sum_{r=1}^{s} c_{ir} O_r \qquad (5)$$

This says that the input used cannot be lower than the efficient level, i.e. it ensures the efficiency scores do not exceed unity.

When the values of the parameters are found, the efficiency scores using common weights are simply the ratios of the right hand side of (5) to its left hand side i.e. the ratio of the efficient or target input value to the actual input value. Notice that these "weights" are derived without any subjective choice and are based only on those DMUs which display some indication of good practice in their usage of the given resource. Note that the number of parameters must not exceed the number of non-dominated DMUs arising out of our first method otherwise they are not uniquely determined. In practice this is not much of a restriction by comparison to what is needed for DEA - Charnes and Cooper [2] state that, as a minimum, the number of DMUs should exceed three times the sum of the number of input and output measures. Although the notion of a common set of weights is perhaps foreign to users of DEA , it is likely that in some situations central managers will feel that conditions at each DMU are sufficiently similar for a common basis of comparison to be justifiable.

TEST RESULTS

By using data generated from a known production model it is possible not only to compare our methods with DEA but also to see how closely each method reproduces the true efficiency scores according to the production model. The data set is taken from Bowlin et al.[1] and is reproduced in Table 1. There are 15 hospitals (H1 to H15) each with three inputs: the number of full- time-equivalent staff, the number of hospital bed-days available in the year, and the expenditure on supplies (I_1 to I_3 respectively). The three outputs are the number of people receiving training at the hospital, the number of regular patients treated in the year, and the number of severe patients treated in the year (O_1 to O_3 respectively). The production model that was used is as follows :

Staff: $I_1 = 0.03\ O_1 + 0.004\ O_2 + 0.005\ O_3$ (6)

Bed-days: $I_2 = (7/0.95)\ O_2 + (9/0.95)\ O_3$ (7)

Supplies: $I_3 = 200\ O_1 + 20\ O_2 + 30\ O_3$ (8)

By inserting values for the outputs into these equations one can find what the efficient input values should be. This was done for the first seven hospitals. Whereas for hospitals 8 to 15 the input levels were chosen to be larger than the efficient levels i.e. these hospitals were set up to be inefficient in at least one input. Dividing the efficient input value from the model by the actual input value one obtains the true efficiency score.

Table 2 compares the true results with those of the methods we are considering. All scores are of the form 'target input divided by actual or observed input'. The DEA results are based on an input minimisation model ; our DEA results are actually different from those in Bowlin et al [1], ours make DEA appear closer to the true scores (we suspect the difference may be due to using different lower bounds on the weights). All three methods obtained the correct scores (to two decimal places) for the first nine hospitals so those results are not shown. However DEA has incorrectly rated two of the inefficient hospitals (H10 and H13) as being efficient in all three inputs; in fact for hospital 10 DEA is out by as much as 31 and 18 percentage points for inputs 1 and 3 respectively. The individual weights method actually deduced precisely the correct score in most cases; in the few remaining cases it is still closer to the true result than DEA. Errors will always be in the direction of an overestimate with this method for the same reason that this occurs in DEA , namely that an optimal set of weights is being found for that branch. However the amount of over-estimation will not be as great as for DEA because there are fewer weights to be manipulated.

Even more remarkable are the results from our second method. For inputs 1 and 2 the true scores were reproduced exactly for all 15 hospitals. For input 3 the results were always correct to at least two decimal places and the largest error was 0.004 . Hence, as these results were the same as the true scores (to at least two decimal places) there was no need to include another identical column in Table 2. Due to the less restrictive nature of our first method, it will provide scores which are at least as great as those of the method using common weights. If there were a large difference between the scores from our two methods for a given DMU it could be due to the weight flexibility of the first method disguising deficiencies in performance (one would check for near-zero weights). However it may be due to justifiable reasons relating to the particular circumstances at that DMU - further investigation would be called for. From the above comparisons it would seem that the approaches presented show considerable promise and are therefore worthy of further investigation.

FURTHER DEVELOPMENTS

(1) When two inputs are known to act as substitutes for each other then it may be desirable to assess their efficiency jointly rather than separately. If the substitution rate is known then the value of the denominator in the objective function (1) and in the constraints can be replaced by the value of the aggregate input. If the substitution rate is not known then one would resort to conventional DEA but using only those two inputs in the formulation.

(2) We have assumed a linear relationship between each input and the outputs , this assumption could be relaxed to take account of factors such as economies of scale. Also, interactive terms (i.e. cross-products in the outputs) could be included to deal with economies of scope.

CONCLUSION

This paper has dealt with the efficiency of utilisation of individual resource inputs in organisations with multiple outputs. We have shown how these can be calculated using a simple adaptation of the widely explored DEA technique. For those managers who want an assessment based on a common standard we showed how this could be achieved using a combination of DEA and regression. This involved generating a best-practice model where the common set of weights is based only on the best-practice DMUs.

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