

Should we accept the existence of numbers as abstract objects?

Jacob O'Sullivan

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Abstract

This paper argues that we should accept the existence of numbers as abstract objects. I begin by looking at the Indispensability Argument as a positive argument for the existence of mathematical objects. I look at various discussions that challenge the Indispensability Argument, most notably that we can dismiss the doctrine of Confirmational Holism by looking to an argument that we use figurative talk in science, such as idealisations. This is the argument that, like idealisations, mathematical objects are only used in scientific theories figuratively and as such their existence is not confirmed by the success of the scientific theories they are included in. In order for this objection to stand, it needs to explain what figurative uses are and so I look at Mathematical Fictionalism, which aims to explain these uses for mathematical statements. I look at Yablo's fictionalism, as the strongest argument for explaining this, and explain how he hopes to dismiss our commitment to the existence of mathematical objects by arguing that they are 'creatures of existential metaphor'. Finally, I raise issues with this explanation and conclude that, without further argument, the fictionalist position does not convincingly dismiss our commitment to the existence of mathematical objects and so we should accept the existence of numbers as abstract objects.

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Introduction

The following will investigate whether we should accept the existence of numbers as abstract objects.

In part 1, I discuss the indispensability argument. This will be the foundation of my argument for the existence of numbers as abstract objects. I will assume for my argument that naturalism is accepted as our preferred ontological standpoint, but I will also explain briefly Quine's (1960) motivation for accepting naturalism and how it allows the indispensability argument to come out (section 1.1).

Following this naturalist groundwork, I will explain how the indispensability argument leads us to the conclusion that we should be committed to the existence of mathematical objects (section 1.2). In section 1.3, I shall discuss Field's (1980) attempt to show mathematics as dispensable to scientific theory and whether this succeeds in undermining the conclusion of the indispensability argument. I will conclude that he does not do so completely. In section 1.4, I will look at issues taken with the conformational holism premise of the indispensability argument, led by arguments raised by Leng (2010). In this, I will discuss the issues of direct evidence and idealisations and I will give Leng's argument that, just like idealisations, the use of mathematics in scientific theory can be taken to be figurative. Section 1, and the hope of the indispensability argument's conclusion will be shown to rest on whether mathematical statements can be used figuratively in this way, which leads to section 2.

Part 2 discusses fictionalism. I first give an outline of what mathematical fictionalism is and, in section 2.2, I show why fictionalism needs to have some involvement from fiction or a theory of fiction. In section 2.3, I outline the similarities and differences that mathematics and fiction have, and I use these to narrow down those that are important to the question of ontology for mathematical objects. In section 2.4 onwards, I look at Yablo's (2005) explanation of mathematical objects as creatures of metaphor and how he reaches this conception. After showing how he takes seriously the comparison between mathematics and useful fictions, I outline his attempt at explaining the correctness and objectivity of mathematics in terms of metaphor. Whilst not being fully fleshed out, he gives an idea of how this explanation might go. In section 2.5, I raise issues for Yablo's fictionalism. The main issue is that Yablo's account does not explain satisfactorily where mathematical statements get their content from, and seems to lead to a position where they generate their own content. This is unacceptable and is not how fiction or metaphor should be taken to work. Linking this back to section 1.4, I will show how, for this reason, mathematics cannot be the same as idealisations when taken to be used figuratively. Because of this, fictionalism cannot give a full explanation of why we are not committed to the content of our mathematical statements and the conclusion of the indispensability

argument is not undermined. This means that we should accept the existence of numbers as abstract objects.

The first part of this paper will outline and explicate the well-established discussion surrounding the indispensability argument as an argument for this existence of mathematical objects. As a result of this, it will follow closely the arguments made in Leng (2010), as these arguments do great justice to the discourse surrounding this topic. The purpose of this is, first, to show where I agree the strengths of the indispensability argument are and, second, to set up the position that many nominalists (such as Leng) reach. Where relevant, I include references to further discussion in the first part of this paper, but my aim is to reach the position I would like to counter as clearly as possible, and this is done by following and explaining the arguments put forward by Leng.

Part 1 Indispensability

In this section, I shall discuss the indispensability argument as an avenue to an attractive conclusion for the existence of mathematical entities. Following from a naturalist background and the premises of indispensability and conformational holism, the argument is able to conclude that mathematical entities are indispensable to scientific practices. As a result, their existence is confirmed by the success of said theories. I will give an outline of the motivations for the naturalist position and how it allows a move into the indispensability argument. I will explain the argument and show how, if sound, it gives an attractive conclusion for the existence of mathematical entities. Following this, I will discuss some of the issues that have been raised for the indispensability argument, starting with Field's (1980) attempt to make mathematical entities dispensable to scientific practices. Then, after showing that his attempts are not very convincing, I will look at issues taken with conformational holism. One of these issues, raised by Leng (2010), will ultimately try to undermine the indispensability argument by showing that the use of mathematics is analogous to idealisations and similarly can be used figuratively without their existence being confirmed. I will then raise a few questions for this position and ultimately conclude that the indispensability argument still holds and is a good route for someone to take to show that mathematical entities exist.

1.1 Naturalism

When asking the question of what there is, there are many approaches that have been taken. One approach is that of naturalism. For the purposes of this paper, I will be taking this as the approach that says the question of ontology is answered by scientific practice alone; there are no further questions after this. So, according to naturalism, we should believe in the entities posited by our best scientific theories and there are no further questions as to whether we are *really* justified in our belief in these entities. For the naturalist, philosophy should be continuous with scientific enquiry, not some higher-level judge that presides over it. For the purposes of the following, I will be taking naturalism as an assumption

but will also show how Quine motivates the position by showing how he draws a positive ontological conclusion from Carnap's sceptical assumptions.

One motivation to reject 'first philosophy' is Carnap's (1956) quietism. Leng explains as follows:

'he puts forward his famous distinction between internal and external questions regarding the existence claims of a given discourse, or linguistic framework. According to Carnap, if we ask the question 'Do ϕ s exist?', we may mean to ask the question from a perspective internal or external to a given framework. As an internal question, 'Do ϕ s exist?' amounts to the question, 'Is the utterance "There are ϕ s" justified according to the internal rules of the framework?'... What worries philosophers, Carnap thinks, is not these internal, often trivial, existence questions, but rather, the question 'Do ϕ s exist?' understood when asked from a perspective that is external to the framework in question. But the holistic realization that it is only within the context of a theoretical framework that a sentence such as 'There are ϕ s' is given meaning precludes the possibility of there being any meaningful external philosophical question of this sort. The philosopher aims to set aside the presuppositions of a given linguistic framework to ask whether the objects said to exist in the context of that framework really do exist. But in doing so they divorce the question 'Do ϕ s exist?' of any discernible meaning.' (Leng, 2010, c.2, p11)

This motivation has its roots in a scepticism about the possibility of there being a discipline that can further justify already internally justified claims. Carnap's 'quietist' conclusion is to completely abandon the 'philosophical question' of whether we should believe claims justified by internal standards. This is not the conclusion that the naturalist wishes to land on, as it does not allow for science to answer our ontological question of what exists. The sceptical position it takes requires us to abandon these questions entirely. It is worth exploring why Carnap reaches this conclusion, and whether the naturalist has any hope of avoiding it. Carnap's issue with 'first philosophy' is the idea of abandoning all former beliefs and then trying to test each belief to see if it can be justified. However, if everything is abandoned then what do we have left to put our new beliefs to the test? This picture leads to individual hypotheses tested in isolation against foundational beliefs, but Carnap (and Quine, 1951) believe that only in the context of a framework can these hypotheses have any content. Carnap says that it is conventions that we adopt that tests our hypotheses. If they pass these tests, it just shows that they are good against the conventions that we have adopted. As a result, for Carnap, practical use does not equal existence or any reason for us to believe in any entities posited.

The important questions of ontology that I am exploring is one of these 'external questions'. For example, the answer to the internal question, 'Is there a prime number between 6 and 8?' is 'Yes, there is such a number' and is settled by mathematics. Whereas my position is to address the external question of whether

there is such a number and so if Carnap is correct then we have no hope of answering this. Trying to determine what exists outside of the framework we have adopted is not possible, since when the hypotheses are taken outside their framework they lose their content. So, according to Carnap, only internal questions can be answered which are only practical questions, not questions of what exists outside of an established practice. Is there any way for us to move beyond the framework and reach the naturalist conclusion that what is posited by our best scientific theories is what we should believe in outside of any established practice?

Quine's (1960) motivation for naturalism should be able to help us do this. Quine thinks that the practical reasons can serve as evidential and as confirmation of the truth of our scientific utterances. He follows Carnap's scepticism about testing individual hypotheses outside a framework that gives them content. But whilst Carnap draws a strong distinction between conventionally adopted rules and theoretical statements, Quine disagrees:

'According to Carnap, there is a strong distinction to be drawn between the conventionally adopted rules that set up what it means for a statement to be justified according to the internal standards of justification for a given framework, and the theoretical statements that are justified in the light of these conventionally adopted rules, together with empirical evidence. . . Quine's response is just to note that, if there is a difference here, it is a difference in degree rather than character. In each case, we are putting theoretical claims to empirical test, and adopting them to the extent that they contribute, within the context of a theoretical framework, to the efficient organization of our experience.' (Leng, 2010, c.2, p13)

For Quine, we do the same for both the conventionally adopted rules and the theoretical statements that are justified within these adopted rules. We put them to the empirical test and adopt them if they are useful in organizing and explaining our experiences. And so 'If all questions concerning the evidential support there is for a hypothesis involve an element of practicality or convention... the fact that a hypothesis is adopted on practical grounds in no way speaks against our assumption that we have evidence for its truth.' (Leng, 2010, c.2, p14) This is how Quine turns Carnap's 'negative' conclusion into a 'positive' one. We have reason to believe in the entities posited by our adopted precisely because we adopt them.

The question remains: what if we had different frameworks? If this could be the case, then we are aware that what exists is contingent on the conventions that we have adopted and the frameworks that we use. Quine's response is articulated by Leng:

'If all questions concerning the evidential support there is for a hypothesis involve an element of practicality or convention, then, Quine thinks, the fact that a

hypothesis is adopted on practical grounds in no way speaks against our assumption that we have evidence for its truth.’ (Leng, 2010, c.2, p14)

He just accepts this could be the case, but that the best we can do is theorise with what we have. We could find evidence that better describes and organises our experience. This would then be justification to adopt this new evidence and then our ontological framework will improve with each improvement we make to our frameworks and the theoretical statements confirmed within them. The reason we choose science as the guide for our ontological commitments is that it is the best at refining and improving the explanations we have for our experiences. This positive conclusion, which Quine leads us to from the sceptical start he shares with Carnap, lets us reach the naturalist position we need to help us answer ontological questions. What is successful practically is what we have evidence to believe in. This leads us to a position where the indispensability argument can be used as a powerful argument for the existence of mathematical entities, as I will explain in the next section.

1.2 The Indispensability Argument

Although there is no one source for the current standard version of the indispensability argument, it is most commonly rooted in ontological theses argued by Quine. The overall argument can be summed up in the following passage from Putnam:

‘So far I have been developing an argument for realism along roughly the following lines: quantification over mathematical entities is indispensable for science, both formal and physical; therefore we should accept such quantification; but this commits us to accepting the existence of the mathematical entities in question.’ (Putnam, 1971, p374)

The most standard version of the argument today can be spelled out as follows (Cowling, 2017):

‘P1. *Ontological naturalism*. Our best guides to what exists are our best scientific theories, so we ought to believe in the ontological commitments of our best scientific theories.

P2. *Quine’s criterion*. The ontological commitments of a theory are all and only those entities that are the values of bound variables occurring within that theory.

P3. *Indispensability*. Our best scientific theories involve indispensable quantification over mathematical entities.

P4. *Confirmational holism*. Support for a scientific theory accrues holistically to all of its ontological commitments, so belief in a given scientific theory requires belief in all of its ontological commitments.

C1. Therefore... we ought to believe in the existence of mathematical entities.’
(Cowling, 2017, c.1, p49)

It is clear to see that if the premises are accepted then we are committed to a belief in mathematical entities. Of course, these premises are not uncontroversial, and issues have been raised for every one of them for different reasons. I will not discuss further P1, as I have given reasons above in 1.1 for why I believe naturalism is an acceptable (or best) avenue to find answers for our ontological questions. Issues may be taken with P2. A nominalist could accept P2 and give an account of scientific theory that does not have mathematical entities as values of bound variables within said theory. It is possible, and it will arise later, that some nominalists (namely, fictionalists) claim that they can accept P2 but still reject C1. I will look into whether this attempt works or not in Part 2. These premises have a greater bearing on the nominalism vs platonism debate, since undermining either of these will undermine C1 and allow the nominalist to show that we do not have a commitment to the existence of mathematical entities. The nominalist rejection of P3 is to say that mathematical entities are not indispensably quantified over in our best scientific theories. This undermines C1, since if mathematical entities are no longer indispensable to our best scientific theories, then we can dispense with them and no longer be committed to the existence of them. P4 can also be rejected by a nominalist who believes that numbers in some form are indispensable to our best scientific theories, but that this indispensability does not command a commitment to the existence of the numbers used in the theory. In the following (1.3), I will be looking at Field’s (1980) attempt to reject P3. (For further discussion on issues with the Indispensability Argument see Cowling (2017).)

1.3 Nominalising Scientific Theory

If it could be shown that mathematical entities are not indispensably quantified over in our best scientific theories, then the whole indispensability argument would fall apart. Field (1980) attempts to deny P3 and tries to argue that mathematical entities are not indispensable to science. He does this by using the concept of *conservatism*. He argues that mathematical theories need not be true, but only be conservative:

‘Principle C (for ‘conservative’): Let A be any nominalistically storable assertion, and N any body of such assertions; and let S be any mathematical theory. Then A* isn’t a consequence of N* + S + ‘ $\exists x - M(x)$ ’ unless A is a consequence of N [where A* is any nominalistically storable assertion, and N* is any body of such assertions when taken with S].

Why should we believe this principle? Well, it follows from a slightly stronger principle that is perhaps a bit more obvious:

Principle C': Let A be any nominalistically-statable assertion, and N any body of such assertions. Then A* isn't a consequence of N* + S unless it is a consequence of N* alone.

This in turn is equivalent (assuming the underlying logic to be compact) to something still more obvious-sounding:

Principle C'': Let A be any nominalistically-statable assertion. Then A* isn't a consequence of S unless it is logically true' (Field, 1980 p12)

By 'conservative', Field means that adding a mathematical theory to a nominalistic scientific theory does not lead to any conclusions that would not follow from the nominalistic scientific theory alone. He gives a detailed discussion of how the inclusion of mathematical entities is just a conservative extension of an already existing nominalistic theory. The difference between mathematical entities and other theoretical objects that are used in science is, according to Field, that you can add the theoretical objects to an observed case and derive new conclusions. However, with mathematical entities, this is not the case. The conclusions derived could be done so with nominalistic assumptions. Also, if the scientific theory is underdetermined by the evidence, the inclusion of mathematical objects would not lead to any further conclusions that nominalistic evidence would not have reached alone. All that mathematics does in science, according to Field's argument of conservatism, is act as a shortcut to make it easier to get to the conclusion that could already be reached using nominalistic theory alone. As a result, his argument allows that mathematics can be used in science but it need not be. He explains that the reason that it is used is not because of its indispensability but because the inclusion makes the statement of many scientific theories much easier and simpler. The use of mathematics is simply a pragmatic one rather than a necessary one.

To actually prove this, though, Field needs to show how we would go about nominalising the best scientific theories. Of course, it would be too much to nominalise all currently accepted best scientific theories, but applying it to a few would show the method of how to do the rest. To do this, he reparses the theories without quantifying over mathematical entities and attempts to show us that the theory parsed is still attractive. Newton's gravitational theory is the theory he uses in his (1980) to motivate his argument that an attractive nominalisation can be done. If it can be done effectively to this physical theory, he hopes to motivate the view that there is no reason to assume that this could not be done for the rest of science. By doing this, use of the platonist versions of scientific theories is justified, without commitment to their truth, as they are only conservative extensions of nominalistically-statable theories. So, by nominalising this one scientific theory, Field hopes to show that his concept of conservatism can refute the indispensability argument. This is because by showing that empirical theories can be nominalised, he shows that mathematical objects are not

indispensable to our best scientific theories and as a result the truth of mathematical theories is not guaranteed.

1.3.1 Logical Possibility and Consistency

One issue that can be raised for Field's project is how to construe logical possibility. If Field's notion of conservatism appeals to a semantic notion of consequence, then it allows for his notion of logical possibility to be based in set-theoretic models. By adopting set-theoretic models as the basis of logical possibility; Field's attempt to demathematise science in itself relies on mathematical objects, the set-theoretic models. It is clear to see how this is a problem for Field's argument. He no longer has a way of shedding our commitments to all mathematical objects, only those that are conservative extensions of nominalistic theories. If his argument overall still relies on the existence of mathematical objects, then it cannot be accepted as a nominalistic argument, since it would undermine the motivation for his whole project.

To combat this, Field adopts a primitive, modal notion of logical possibility that does not reduce to set-theoretic model versions of possibility. Consistency or logical possibility does not need to rely on the existence of set-theoretic models and so avoids commitment to any mathematical entities. One reason Field gives to accept this is that the set-theoretic model account of logical possibility has things the wrong way around (see Leng (2010, c.3 p10)). The way Field explains this is by saying that certain sets would be able to exist in the set-theoretic model or not, as a result of the primitive modal account of logic. If a set does not exist, then it is because it is logically impossible to exist and this is informed by a more primitive modal account of logic.

Wright and Hale also raise a problem for Field's take on possibility:

'The objection that is to occupy us arises at this point from the consideration that Field's primitive operator of possibility is *object-linguistic*. It has to be, of course. . . . But when the operator is object-linguistic, Field's belief in the conservativeness of, e.g., number theory would appear to commit him to the view that the Peano axioms, say, might have been true as standardly interpreted, and hence that the nonexistence of numbers is a mere *contingency*.' (Wright and Hale, 1992, p113)

They argue that Field's primitive notion of possibility leads to a position where the existence of numbers is a contingency. This would not necessarily be a problem for Field were it not for their further argument that attempts to show us that 'The picture of the realm of contingency as comprising such a unified web seems to go deep enough in our ordinary thinking to ensure that it will be very difficult to vindicate the conception of mathematical objects as strongly contingent merely by description of anything we already find intuitive.' (Wright and Hale, 1992, p134) If their argument holds, then Field's argument leads to a position where the existence of numbers is contingent but that this contingency does not compare at all to our usual notion of contingencies. So it seems Field's

conception requires further argument to show how this conclusion can be resolved.

A further issue for Field is raised by Leng:

‘But one might still object that to claim, of a theory, that it is consistent will embroil Field in some nominalistically unacceptable machinery however he proposes to interpret this claim, even if it is not interpreted as a claim about set-theoretic models. For, however we choose to understand the notion of logical possibility, surely the mere assertion of a theory that it is consistent (logically possible) involves us in a claim about at least one abstract object—a theory?’ (Leng, 2010, c.3 p10)

Field’s account throughout makes use of discussions of theories. The issue that then arises is that the theories themselves that he is discussing are abstract objects. If this is the case, then even if he manages to rid us of our commitments to the existence of mathematical objects, he would fail to reach a completely nominalistic conclusion, since his argument commits us to the existence of theories.

To resist this charge, Field gives a deflationary account of theories, where axioms, when stated to be true, just state something that one says directly when one utters the sentence to state the axiom. Discussions of theories being true are just inflated ways of expressing, in our object language, the axioms of our theory. This account applies to logical possibility as well, so that talk of them as substantial is avoided entirely. They are deflated down to object-level assertions.

Unfortunately for Field, this deflationary account faces another problem. Even if they can be stated by using a deflationist object level account of theory and logical possibility, how is it that mathematics helps in justifying beliefs? By ascending to a mathematical level, even more (nominalistically acceptable) claims can be deduced than the nominalistic assumptions that we start with. Despite this, Field is only really interested in arguing that mathematics needs to be dispensable to formulating our best scientific theories; the role of mathematics and what conclusions can be drawn from their use is somewhat irrelevant to this. So, Field can still get away with arguing against indispensability but it is not very comforting for nominalists who wish to be confident in the conclusions that are drawn from the use of mathematics in scientific theory.

1.3.2 Nominalising Other Scientific Theories

Whilst Field manages to present a nominalistically acceptable version of Newtonian gravitational theory, he may find issues with finding genuinely nominalistic alternatives to other scientific theories. Field’s example of Newtonian gravitational theory quantifies over space-time points and regions of space-time points:

‘talk of such objects as space-time points and regions might be thought to be at least as problematic for a nominalist as talk of points and arbitrary sets of points in \mathbb{R} . For one thing, in Field's nominalistic theory space-time points are held to have precisely the same structure as \mathbb{R} . If Field's nominalization simply builds all the structure of the mathematical theory of \mathbb{R} into physical space, can he really be said to have dispensed with mathematics?’ (Leng, 2010, c.3 p15)

However, even if Field's space-time points satisfy the same axioms as mathematical theory, as long as he does not quantify over mathematical entities, there should be no objection to his rephrased theory from a nominalistic standpoint. The space-time points themselves should be more acceptable to nominalists, since Field distinguishes them from abstract objects by pointing out that our knowledge of them is an empirical matter. Quantifying over regions of space-time points may also seem problematic at first but it is possible to understand this as quantifying over arbitrary mereological sums of space-time points that are located where their parts are located. As they are located, it seems these regions also differ from abstract objects that are usually taken not to have a spatiotemporal location. So, overall, Field's nominalisation of Newtonian gravitational theory does not raise any fatal problems for Field's project.

The main issue for Field is that many scientific theories are not very similar to Newtonian gravitational theory. For example, Newtonian gravitational theory relates space-time points, and Field just reduces the mathematics with non-mathematically defined properties that are instead defined in terms of comparative predicates or relations. Field follows on from Hilbert's axiomatization of geometry (1971), who by using concepts such as point, betweenness and congruence was able to provide a synthetic formulation of geometry. From this system, Hilbert was able to prove a representation theorem, one that linked the platonist system of geometry with his own nominalistic version. Field's aim was to move what Hilbert did for space and apply it to space-time. In order to do this, he needed to formulate Newtonian laws with equivalent comparative predicates. And by doing so Field is able to nominalise Newtonian gravitational theory. He sums this up in the following:

‘we want to come up with a system of ‘intrinsic’ axioms, more or less analogous to Hilbert's but involving somewhat different concepts, and to come up with a representation theorem that explains the legitimacy of coordinatizing space-time and a uniqueness theorem that explains why in the coordinatized treatment of space-time the laws of Newtonian mechanics will be invariant under just the coordinate transformations that they are in fact invariant under.’ (Field, 1980, p51)

But some of the mathematics used in other theories cannot just be replaced in this way. An example of this can be seen in scientific theories such as phase space theories (see Malament (1982)). The theory is a general theory that represents all possible states of particles. It is not as simple in such theories to replace the

mathematics with non-mathematically defined properties, as in doing so we will have to accept commitment to believing in the possible states that are represented in the mathematical theory that we are replacing. Quantifying over collections of possible states would not be the position that Field would like to end up in and it undermines the motivation to dispense with commitments to mathematical objects in the first place if we just replace that commitment to a commitment to a collection of possible states. For nominalists as a whole, this would be problematic only if possibility were understood in terms of abstract objects or if they were only committed to actual objects only. If the nominalist were happy to accept possible states as real, non-actual states, then they could avoid this. However, a further argument would need to be made for this. So, if we approach nominalising theories such as these in the same way that Field approaches nominalising Newtonian gravitational theory, it does not look like we will reach a desired conclusion. However, whilst being very daunting, it is not inconceivable that someday it may be possible to nominalise even theories such as these. So it is true that none of these problems raised for Field's project of shedding our commitment to mathematical objects is conclusive. They do show, however, that any further attempt to continue Field's project will require supremely hard work. It is not conclusive one way or another whether this can be done; it is contingent on whether anyone can show it is possible. This has not been done and, at this stage, it is not convincing that all scientific theories would be able to be completely nominalised in such a way that we are not committed to any objects that nominalists find questionable. As a result of this, the indispensability premise of the indispensability argument has not been shown to be false. If the nominalist has any hope of undermining C1, they will have to look for issues elsewhere.

1.4 Issues for Confirmational Holism

Another point of contention in the indispensability argument is P4. As in 1.2, I argued that P3 should not be rejected. I shall now look at nominalist attempts to refute P4 and undermine C1. P4 states that acceptance of a scientific theory commits us to believing in all entities posited in a theory. So, if an accepted theory posits the existence of electrons, for example, we are committed to believing in their existence because they are posited in our accepted theory. It is clear to see how this premise can be problematic for nominalists. If it is the case that mathematical entities are indispensable to our best scientific theories, then it is P4 that ensures our commitment to believing in their existence. This follows from the naturalism we have started with.

Our starting point for answering ontological questions explained in 1.1 seems to require us to accept P4. Is it possible to reject P4 on a naturalistic basis? To do so would surely somehow drive a wedge between believing our accepted theory, and believing in that thing. If we do this, then we would end up falling into Carnap's sceptical conclusion. This may not be the case, however, because if we remember that the naturalism foundation we are assuming requires us to look no further

than scientific practice to answer our ontological questions, we do not necessarily have to accept P4 if it contrasts with our accepted best scientific practices. In the following, I will look at a couple of ways this contrast between P4 and our best scientific practices can be shown, such as the use of idealisations and scientists' agnosticism towards entities that we do not have any direct evidence for. I will discuss whether these are genuine cases of P4 clashing with our naturalistic framework, because if they are then the nominalist can dispense with it and completely undermine C1.

1.4.1 Mathematics' Special Position in Science

It has been stated that the role of mathematics in science sets it apart, and that the special position it possesses distinguishes it from other objects confirmed by the success of our best scientific theories.

Sober (1993) raises an interesting point that because mathematics is included in all of our scientific theories, both the accepted and the unaccepted ones, they cannot be the entities that participate in the truth of the accepted scientific theories. If they did, then they would not be included in our unaccepted scientific theories, either. Further to this, the mathematical entities and statements used in both accepted and unaccepted theories are often the same mathematical statements, so it is not even that some mathematical entities are confirmed by some accepted scientific theories and not others. This prevents the objection to Sober that accepted mathematical statements can be confirmed against other, unaccepted mathematical statements. Doing so would be unhelpful anyway, as this does not confirm the mathematics themselves. According to Sober, the issue is that there is no way of confirming them in the same way we confirm other posited objects included in our best scientific theories. But is there a way of empirically testing mathematical statements in such a way that they lose this special position? Leng writes:

'Sober considers the possibility of confirming the mathematical hypothesis that ' $2 + 2 = 4$ ' as opposed to all the alternative hypotheses, ' $2 + 2 = n$ ', by counting four apples, first as two pairs, and then as four. ' $2 + 2 = 4$ ' is indeed a hypothesis about mathematical objects, the natural numbers, which, in the light of various assumptions about how to determine the cardinalities of sets of physical objects by counting will have implications for the result of the counting experiment.... [A]lthough this appears to meet Sober's criterion for a genuine contrastive experiment, Sober argues ... that we do not really put the mathematical hypothesis to the test in such cases. The reason Sober gives for this is that, if somehow we counted the apples and ended up counting three rather than four, we would not consider the hypothesis $2 + 2 = 3$ as receiving confirmation (and thus consider the experiment as disconfirming the hypothesis that $2 + 2 = 4$), but would rather look to some mistake elsewhere in our assumptions about the nature of the experiment. But if we never consider the recalcitrant evidence as disconfirmation of the pure mathematical hypothesis, and only as disconfirmation of the bridging

assumptions that allow us to apply that hypothesis, then we cannot, Sober thinks, see such experiments as confirming their mathematical hypotheses when they do succeed.' (Leng, 2010, c.5 p6-7)

Mathematics, then, is not being tested in the same way that other existents are tested in science and so their existence cannot be confirmed in this way. If it is not the case that empirical evidence can disconfirm their existence, then the same empirical evidence is not able to confirm their existence, because no matter the outcome of the empirical test, it seems that mathematical entities will always be confirmed, as any issues in the testing would never be pointed toward mathematical entities (or their lack of existence) but always pointed elsewhere. It is clear to see Sober's objection: mathematics is held in a special position where it is not tested in the same way as other posits in our best scientific theories. As a result of this, mathematical entities that are posited in our theories cannot receive confirmation in the same way that other existents do. Thus, mathematical objects are not covered in P4, their existence is not confirmed by the success or acceptance of our best scientific theory, and so P4 fails to do the work it needs to in order to reach C1.

Indispensability theorists would not accept these conclusions. The objection that mathematics' special position prevents it from being confirmed does not hold. This is because the indispensability theorist, in light of evidence against the requirement of using certain mathematical statements, would be more than happy to drop any commitment to their existence. Along the same lines, if evidence were to stack up against mathematics' indispensability, then the indispensability theorist would have no problem dropping commitment to their existence completely. Mathematics is tested in this way and although there has not yet been an example of evidence stacking up against the use of mathematics, we can conceive of this situation and so mathematics can be confirmed or denied in this way. Colyvan writes:

'Suppose that Hartry Field has completed the nominalisation of Newtonian mechanics but that he and his successors repeatedly fail to nominalise general relativity. Let's also suppose that this failure gives us good reason to believe that general relativity cannot be nominalised. From this we conclude that mathematical entities are indispensable to general relativity, but not to Newtonian mechanics. In this setting, then, can we imagine an experiment to test the hypothesis that there are natural numbers? The answer is yes. Not only can we imagine such an experiment, we can perform it. In fact many such experiments have been performed over the last 80 years or so, for any experiment that confirms general relativity over Newtonian mechanics is such an experiment. In particular, the 1919 Eddington eclipse experiment is such an experiment.'

(Colyvan, 2001, p123–124)

But scientists themselves do not act in this way and are not testing mathematics in the way Colyvan discusses above. So does this go against, once again, the

naturalist desire to follow scientific practices in order to answer our questions of ontology? Scientists, as a whole, are fine to use whatever mathematics they need to in order to prove a hypothesis and so are not concerned about testing the mathematics being used against a sparser mathematics, or against a nominalistic version of the theory. However, this could just be put down to a division of labour. There are those who use mathematics in their scientific practices to prove a hypothesis and then there are mathematicians who spend time reducing the required mathematics to reach the proof. So even if scientists themselves do not compare and test mathematics in this way, our best scientific theories are constantly evaluated and improved by comparing and testing the mathematics used against sparser mathematics by some mathematicians, and, as such, the confirmation of several mathematical statements and entities is lost because of this testing. The supposed special position of mathematics, then, does not prevent it from being confirmed via testing in the way Sober wishes. Thus, P4 has not been shown not to hold for the case of mathematical entities, and thus there is no reason to think the existence of the mathematical entities employed in the expression of those theories is not confirmed in the confirmation of those theories.

1.4.2 Direct Evidence

Another way that P4 might be thought to go against our scientific practices is that scientists themselves are agnostic towards the existence of objects until they have further evidence: mere inclusion in our best scientific theories is not enough for them; so P4 seemingly goes against our naturalist foundations. Leng writes:

‘There are cases where our theories indispensably posit objects of a particular sort, but where scientists hold back from accepting the existence of such objects until they have some more direct evidence of their existence. Maddy's example is of atomic theory, circa 1900: although this successful theory indispensably posited the existence of atoms, it was only when Jean Perrin's Brownian motion experiments provided some more direct evidence of the existence of such objects that many scientists became convinced of their reality. Similar behaviour can be found amongst modern scientists: cloud chambers and particle accelerators are constructed in order to detect, and thereby confirm, the existence of the subatomic particles posited by our theories, even though the assumption that there are such particles already appears indispensable to those theories. It seems, then, that indispensable occurrence in a successful theory isn't always enough to convince scientists that they have reason to believe in the objects posited by our theories. In at least some cases, a more direct kind of evidence is required.’ (Leng, 2010, c.5, p24)

Indispensability and inclusion in our best scientific theories is not enough for scientists to be convinced of an entity's existence. As a result, P4 seems to go against scientific practice and as such goes against the naturalist foundation of the indispensability argument. Direct evidence seems important too. So, for

example, a lepton-like hypothesis is preferred to a no-lepton hypothesis because without the inclusion of leptons we are left without an explanation for the events that occur in cloud chambers. This at first seems to suggest the idea of direct evidence requiring a causal element. But given our concern is about acausal mathematical objects, to define direct evidence in this way would be question begging. The causal story is important for leptons, as explanatory power of cloud chambers is lost without their existence. Whether direct evidence is causal or not, the most important point to take away is that direct evidence is another example of how being indispensable is not the final justification for existence for scientists. The difference between mathematical entities and other posits in scientific theories could be considered here. The fact that we are able to test for direct evidence of leptons using cloud chambers means that, without that direct evidence, it may make sense for scientists to hold back from commitment to their existence. However, scientists do not seem to be agnostic about mathematical entities in this way. As stated in 1.3.1, they are happy to use any mathematical entities needed to reach their conclusion. It is possible that because mathematical entities differ in kind to concrete entities, they are confirmed in different ways. As a result, I do not believe the example of agnosticism and direct evidence is enough to completely refute P4 for mathematical entities and as a result C1 still holds.

1.4.3 Idealisations

Another way that P4 might be thought to go against our common scientific practice is in the use of idealisations. Scientists often make use of idealisations to help come to successful scientific conclusion. They momentarily take for granted the truth of statements they know independently to be false. Leng writes:

‘Many of our actual scientific theories do not consist of bodies of straightforward truths about ordinary objects, but rather include hypotheses that, if interpreted as assumptions about such objects, are explicitly known to be false. Thus, for example, in our theoretical account of the trajectories of projectiles, for ease of calculation we may assume, as is known to be false, that air resistance is not a factor. In accounting for economic trends in societies we may assume, as is surely false, that individual agents are fully rational utility maximizers. And in order to have a tractable theory of the dynamic behaviour of fluids, we may assume, as is known to be false, that fluids are continuous substances... If we apply our idealizing assumptions to actual projectiles, economies, fluids, etc., we are able to get on reasonably well, making good predictions about aspects of the behaviour of each. And given that a degree of falsification can still lead to successful theoretical predictions (and might even be necessary in order to make any predictions), it may indeed be rational for us to adopt such literally false hypotheses in the context of our theorizing, even though we do not believe those hypotheses.’ (Leng, 2010, c.5, p13-14)

These false hypotheses can and do lead to successful theoretical predictions and so whilst it is rational to adopt them for our theorising, we do actually have independent evidence that they are in fact false. As we know these hypotheses to be false independently of our theorising, it cannot be the case that the acceptance of the theories they are part of confirms the truth of the hypotheses. So it is possible for us to have hypotheses that are used in successful theories that are not confirmed by their participation. If this is the case for not having air resistance, frictionless surfaces and treating liquids as continuous, then surely the same could be said of our mathematical theories. Mathematical statements used in our accepted scientific theories could have the same status as these idealisations, so that their participation in the successful theory does not commit us to believing in them.

It could be said, though, that these idealisations are not false assumptions about real things. The hypothesis about liquids being continuous is not a false assumption about actual physical liquids, but just assumptions about their behaviour on a macro-level. They are actually true assumptions about the nature of ideal fluids, which would be abstract objects themselves. The existence of such abstract objects are then posited by our use of them in theories, and are confirmed in the same way, just like the indispensability argument concludes for mathematical objects. This would refute the objection and actually turn it against the nominalist by allowing for the confirmation for even more abstract objects. So instead of undermining P4, this objection could be strengthening the platonist position by adding more abstract objects to the list confirmed by our best scientific theories.

Quine would not be happy with this move. For Quine, we often use abbreviations and idealisations in our best scientific theories. These also do in fact help us reach successful predictions about the real world and so are very useful because of this. But Quine does not take this to confirm the existence of these idealisations, precisely because they are eliminable and what is said with them could also be said in a way that does not quantify over them (see, for example, Quine, 1960, p249). They are just convenient myths that we are able to use without being committed to their existence because we are able to still reach the same scientific theories without use of them. What is really asserted in use of these idealisations, such as frictionless planes and lack of air resistance, is the behaviour of objects when the variable (such as friction or air resistance) approaches 0. P4 survives, then, because the success of our theories confirms the existence of posited objects in our best formulated theories (i.e. those that are left when all eliminable assumptions are done away with). So, again, mathematical entities have not been shown not to be confirmed in accordance with P4 because they remain indispensable to our best scientific theories.

If this is the case for Quine, then he may need to commit himself to more abstract objects than he hopes for. Leng suggests that Quine's reason for allowing these idealisations to be eliminable is that they are just cases of variables approaching 0

(See Leng, 2010, c4, p19). However, other idealisations are not of this form. Fluids being continuous, for example, are not a matter of the distance between particles in a liquid approaching zero. The liquid would become more solid in this case. So the idealisation of liquid being continuous is different in type to limit myths, where the idealisation is simply an example of the variable approaching zero (See Leng, 2010, c4, p20). If this is the case, then liquids being continuous may be an essential idealisation that cannot be eliminated for the same reason as other idealisations, since we currently have no believed account of ordinary fluids that does the job of continuous fluids in theorising. Because of this, it seems like continuous fluids may be confirmed by their inclusion in our best scientific theories.

Is there any hope for the nominalist to argue against this and refute P4? One answer may be in the following:

‘According to Quine, speaking figuratively as if there are point masses can serve the theoretical purpose of representing the behaviour of extended massive objects as being thus and so, regardless of whether there really are any point masses. ...[G]iven their similarities, we might wonder whether the utility of postulating continuous ideal fluids is more like the utility of postulating point masses than the utility of postulating (say) electrons. Here too, we might think, we should see ourselves as speaking merely figuratively, and not literally, when we adopt the hypothesis that there are such things, in order to take advantage of the representational value of that hypothesis in allowing us to paint a picture of how things are with real fluids.’ (Leng, 2010, c.5, p22)

The nominalist’s hope, then, relies on saying that our use of these idealisations are figurative uses and not to be taken literally. The nominalist hopes that, because of this, P4 fails; since, if by using false hypotheses figuratively, then there is no reason why they have to be confirmed by the success of the theory they are used in. And the hope then is that mathematical statements can be used in the same figurative way that idealisations are. When using mathematical statements in our best scientific theories, we are simply acting as if they exist, in order to reach our predictions. This position is often known as Mathematical Fictionalism and, because of this, we are not committed to the existence of the objects posited. Or so the argument goes.

Fortunately for the indispensability theorist, there is no reason to agree with Quine that essential idealisations cannot be the case. As a result of this, they need not accept the route to fictionalism. Indispensability theorists as a whole should have no problem accepting that it is possible that liquids could exist in a way that they are continuous. So there is no reason for the indispensability theorist to abandon this idea for figurative uses. It could be the case that there is a world where liquids are continuous. In this case, our talk of these idealisations is not just representations of non-existing objects (the fictionalist position), but referring to the existing idealisations. Quine may not wish to accept this

conclusion but that is no reason for any other indispensability theorist to reject this explanation via possibility or abstract objects. Just like mathematical objects, the indispensability theorist can accept the existence of essential idealisations until such a time they are eliminable from our best scientific theories and at that time we would no longer be committed to their existence, and instead only committed to the objects that are indispensable to our best theories.

In order to undermine C1, the nominalist must motivate the position of figurative speech. This position is that our scientific discussion of idealisations and mathematical statements should only be taken to be talk of what would be the case if they did exist, because they in fact do not. If they are able to motivate this position they would be able to undermine C1, not by showing that mathematical entities are dispensable to science, but that their inclusion in our best theories does not require their existence. Unfortunately, it may be problematic for idealisations and mathematical statements to be conceived of as figurative. The nominalist in this case still needs to explain where the content of these statements comes from, and if taken as a fiction, they need to explain what exactly gives the fiction its content. I will be discussing this position in more depth in part 2, but at this stage it does not seem convincing that P4 and conformational holism can be disposed of completely, without first giving a reason to prefer idealisations to be used figuratively over their existence, independently of any nominalistic justification. If this can be done, the nominalist must also explain where their figurative uses derives its content. In the next section, I will be exploring the ways nominalists have tried to use fiction to remove our commitment to the existence of abstract objects. I will look at the importance of theories of fiction for these nominalists. I will look at serious attempts to explain mathematic statements as make-believe games, and whether they are able to motivate the position and explain where the content comes from. If successful, they would successfully undermine our commitments to mathematical entities, but I will conclude that they fail to do so.

Part 2 Fictionalism

In this part, I will investigate mathematical fictionalism as an avenue for nominalists to take to undermine the existence of mathematical objects. Initially, I will outline the fictionalist position and discuss the motivations for the position. I will then discuss how some fictionalists attempt to remove the need for a theory of fiction and will show why this ultimately leaves their position unconvincing. After showing that fiction should be involved in any fictionalist account, I will look at the ways fiction and mathematics has been compared and whether this has any impact on the ontological question of the existence of mathematical objects. I will point to how some similarities discussed between the two do not shed any light on the ontological question, but then move on to how some fictionalists use a fictional framework to explain the application of mathematics to scientific theory. I will ask whether such a fictional framework can correctly explain why some mathematical theory is taken to be correct whilst others are taken to be incorrect.

I will give some time to Yablo's (2005) attempt to begin an understanding of this, although the response is not fully fleshed out. I will finally raise an issue of how the representational aids used in the fictionalist framework gets any content, as it seems their framework relies on the fiction generating its own content. This is not generally how fiction works and so without a supporting theory of fiction that explains this, I will conclude that the fictionalist framework does not succeed in explaining away the existence of abstract objects.

2.1 What is Fictionalism?

One way of characterising fictionalism is in the following:

'What characterises a fictionalist approach to subject matter X is the suggestion that X can be understood by appeal to the notion of fiction. Otherwise, fictionalism does not deserve its name. Something about the features of fiction leads fictionalists to think that it provides a model for engaging in a way of talking about X without incurring the commitments of a realist approach to X.' (Bourne & Caddick Bourne, 2018a).

Mathematical fictionalism, then, is the claim that mathematics, mathematical objects and statements, can be understood by appealing to the notion of fiction. The fictionalist takes fiction to have certain features that allows them to engage with mathematical statements without having any commitment to the existence of the objects seemingly referred to.

Burgess (2004, p18) explains this in another way by dividing nominalism in the following way: 'my colleague Gideon Rosen and I distinguished a negative or destructive side of nominalism, which tells us not to believe what mathematics appears to say, from a positive or reconstructive side, which aims to give us something else to believe instead.' It is clear the reconstructive side points to nominalists, such as Field, who tried to refute platonism by showing how mathematical statements could be reduced to nominalist statements, and put considerable work into showing how this could be done. Burgess goes on to say, 'if nothing else was clear from the work of Hartry Field, Charles Chihara, Geoffrey Hellman, and other reconstructive nominalists whose work we surveyed, it was clear that the amount of honest toil that would be required for a nominalistic reconstrual or reconstruction of mathematics would be quite considerable' and so 'almost everything that has come forth since from the nominalist camp has represented the light-fingered larcenous variety, which helps itself to the utility of mathematics, while refusing to pay the price either of acknowledging that what mathematics appears to say is true, or of providing any reconstrual or reconstruction that would make it true. The usual label for this variety of nominalism is '[mathematical] fictionalism'.'

So mathematical fictionalism is the idea that mathematical statements can still be useful in the way they are commonly taken to be by platonists, but that they are not true, and the objects seemingly referred to do not exist. The fictionalist's hope to use features of fiction to show why mathematical statements can be useful but false, and that we can continue to apply them in the ways we do now without being committed to the existence of the content of the statements, i.e. mathematical objects.

So fictionalists now overcome what Burgess calls the 'reconstructive side' of fictionalism by not having to bother themselves with the difficulties of creating concrete versions of mathematical statements. Added to this, their conclusion does not fall to another platonist objection as they agree with the platonist argument for applicability of mathematics other than believing that this entails the existence of mathematical objects. All platonists who take issue with reconstructive nominalism because it disagrees with how mathematical statements are used correctly in scientific theory cannot raise that issue with fictionalism. Yablo (2005) explains:

'Where the standard line offers little *other* than truth to explain usefulness, Field lays great stress on the notion that mathematical theories are *conservative* over nominalistic ones, i.e., any nominalistic conclusions that can be proved with mathematics can also be proven (albeit often much less easily) without it. The utility of mathematics lies in the no-risk deductive assistance that it provides to the beleaguered theorist ... This leaves more or less untouched, however, the problem of how mathematics *does* manage to be useful without being true. It is not as though it benefits only practitioners of Field's qualitative science (it does not benefit Field-style scientists at all; there aren't any). The people whose activities we are trying to understand are practicing regular old platonic science.' (Yablo, 2005, p 91).

Fictionalism does not face this issue. Fictionalists agree with platonists about the usefulness of mathematics to science and so can explain the activities of the people practicing regular old platonic science. By comparing mathematics to fiction, the fictionalist hopes to continue using mathematics in scientific practice but without having any commitment to the content of the mathematical statements, as they take it we have no commitment to the content of fictional statements. For fictionalists to hold their theory in this regard, they must fully explain how mathematical statements are like fictional statements, so that we can accept the usefulness of the statements without accepting any commitment to the statement's content. In the following, I will look at the different ways mathematics and fiction are compared, and I will see if the fictionalist theory has a way of getting to their desired aim.

2.2 *The role of Fiction in Fictionalism*

Given the label 'fictionalism', one would take it that a theory of fiction would be important for the fictionalist. But some fictionalists deny the need to have a

supporting theory of fiction. They claim that as they are not attempting to compare mathematics and fiction fully, they are only comparing them insofar that the statements in both are untrue because the objects referred to in the statements do not exist. As Balaguer (2009, p135-136) says:

‘As I have defined the view here, mathematical fictionalism is a view about mathematics only; in particular, it is the view that

(i) platonists are right that mathematical sentences like ‘4 is even’ should be read as being about (or purporting to be about) abstract objects; but

(ii) there are no such things as abstract objects (e.g., there is no such thing as the number 4); and so

(iii) sentences like ‘4 is even’ are not literally true.

That’s it. It does not say anything at all about fictional discourse, and so it is not committed to the claim that there are no important disanalogies between mathematics and fiction.’

Even these fictionalists (if they can be called that) face trouble from some issues. For example, they would need to explain what makes mathematical statements correct and incorrect in the real world. One response from Balaguer (2009) is that:

‘In order to respond to this objection, fictionalists need a different theory of what the story of mathematics consists in. The fictionalist view I want to develop is based on the following claim:

The story of mathematics consists in the claim that there actually exist abstract mathematical objects of the kinds that platonists have in mind—i.e., the kinds that our mathematical theories are about, or at least purport to be about.

This view gives rise to a corresponding view of fictionalistic mathematical correctness, which can be put like this:

A pure mathematical sentence is correct, or fictionalistically correct, iff it is true in the story of mathematics, as defined in the above way; or, equivalently, iff it would have been true if there had actually existed abstract mathematical objects of the kinds that platonists have in mind, i.e., the kinds that our mathematical theories purport to be about.’

The story of mathematics, from what has been said by Balaguer, seems best to be taken as a fiction, according to which mathematical objects actually exist and so, in this fiction, the statements that purport to be about these objects are true. This is what makes them correct in our world.

In this case, the story of mathematics is explained like a fiction and so a theory of fiction is still relevant and will still have implications for their position. It has to go further than just storytelling, as mathematical statements are the same even

when told at different times, in different places and with different mediums. Even if the story is taken as a useful one and that usefulness is the factor that determines whether statements are taken to be correct or incorrect in this world, the idea of a story of mathematics still faces the problem of conceptualising mathematical characters as fictional characters. This is because it relies on the assumption that fictional characters do not exist in any way. This is how they hope this argument leads to the non-existence of mathematical characters. However, they need a supporting theory of fiction to argue for this position on fictional characters. Fictionalists also face a problem further to this too. If they argue from the position that mathematical statements are false, since mathematical objects do not exist, then it is hard to see how they would use a story of mathematics to solve the issue of correctness. If mathematical objects do not exist, then what would a world where they do exist be like? Balaguer claims that a mathematical statement is true iff there actually existed abstract mathematical objects. What would this mean to a nominalist, who claims there are no such things and would not want to concede that there could be? In order to do this, they would need a supporting theory of fiction to help show what mathematical objects are or are comparable to. Only then could they start to explain why some mathematical statements are taken to be correct and others incorrect.

Mathematical realists are able to explain the correctness as they are able to explain correct statements as the statements that are made true by the existence of the mathematical objects that the statements refer to. The incorrect statements on the other hand are, of course, the statements that are made false by the objects the statements refer to. Fictionalists on the other hand are unable to explain the correctness of statements in this way as they cannot rely on the existence of the mathematical objects to make certain statements true or false. They would therefore need to come up with a way of explaining how some mathematical statements are taken to be correct and others are taken to be incorrect in this world.

Further to this, it is hard to see without a fully formed theory of fiction that supports the fictionalist position how they can avoid introducing other abstract objects whilst trying to rid us of our commitments to the more traditional mathematical objects:

‘One option within the semantics of fiction is to view fictional names as referring to abstract things, “fictional characters”. If all fictions are like this, fictionalism about arithmetic would not be well motivated by a desire to avoid commitment to abstracta. ... Another example. Fiction operators like “according to Sherlock Holmes stories ...” typically involve apparent reference to stories, or, more generally, fictional discourses. These seem not to be concrete things. ... If they are abstract things, then ontological scruples about abstracta would not be well served by a fictionalism which relies upon fiction operators.’ (Sainsbury, 2010, p3)

What 'fiction' is and what 'fictional characters' are is still being debated, and there are many theories that take them to be abstracta themselves, such as the theory of abstract artifacts (see Sainsbury, 2010 and Thomasson 1999, 2003). Without a supporting theory of fiction, the fictionalist has no way of explaining the issues pointed at above and they have no way of guaranteeing that what they explain mathematics away with is not abstracta itself. In the next sections, I will look at fictionalist positions that do take fiction itself to be important to the position to see whether they avoid commitment to abstract objects.

2.3 Similarities and Differences between Mathematics and Fiction

For a theory to be a fictionalist one, it must at least appeal to the notion of fiction and thus invoke a supporting theory of fiction. For a mathematical fictionalist, then, there must be some comparison between mathematics and fiction, and there must be some features they share that enables the fictionalist conclusion. I will look at a discussion by Burgess (2004) that examines similarities and differences between mathematics and fiction. The following outlines varying comparisons between mathematics and fiction in order to show that any comparison is not helpful to the fictionalist discussion. Whilst his conclusion overall is that fictionalism does not hold (for different reasons than I shall conclude) his discussion is still very useful at outlining what fictionalism must do if it is to succeed, namely: to show comparisons between mathematics and fiction that explains where mathematical statements get their content in order to answer the question of ontology of mathematical objects. Burgess discusses clear ways that mathematics is non-fiction:

'The compilers of the New York Times best-seller list will never put any mathematical work, however wonderful, at the top of the fiction column, and not just because nothing even by Andrew Wiles will ever sell like Stephen King. Nor will any librarian catalogue, say, the Proceedings of the Cabal Seminar, as an 'anthology of short stories based on the characters created by Georg Cantor'.'

It seems clear to me that Burgess's point that mathematics is non-fiction or is *not* fiction illustrates an issue for fictionalists. For them to do away with the content of mathematical statements whilst still using them, the fictionalist has at least two things to do. First, they must show that mathematics is sufficiently like fiction so that they can do away with their commitment to the content of the statements. Second, they must outline a theory of fiction that shows that we use fictional statements without any commitment to the content of the statements. I will address the first of these initially and then expand on the second in section 2.4.

As Burgess points out, mathematics is clearly non-fiction. So, to show that mathematics is sufficiently similar to fiction, he looks into fiction in a more specific way. 'So the question is: in which respects is mathematics like, and in which respects is it unlike, fiction? That in part depends on the species of the genus fiction one considers.' (Burgess, 2004) The idea is that whilst mathematics is not similar enough to fiction to be counted as a fiction itself, by distinguishing

between different 'genus' of fiction, the fictionalist may be able to draw a strong enough comparison between mathematics and a type of fiction. This is not the best fictionalist position, but it is important to show what exactly is needed by the fictionalist in order for their position to stand. Not any similarity between mathematics and fiction will do. After listing a few types of fictions that have been discussed in regard to their likeness to mathematics, Burgess lands on fables as the most apt genus of fiction to compare mathematics to:

'I believe the comparison with fables is the most apt of the candidates I have considered, and comparison with novels the least so. Novels almost always are attributable to identifiable individual authors, Proust or Flaubert, Trollope or Dickens. Some fables are attributable to such authors, Lafontaine for instance, others are traditional. Mathematics also consists of both traditional elements and elements with identifiable authors. Novels are almost always unique. Fables tend to be retold over and over in variant versions by different writers, so that we have Aesop's version, Lafontaine's version, and many latter-day retellings of the fox and the crow, for instance. Mathematics likewise gets retold by textbook writer after textbook writer. The characters in one novel seldom reappear in another, and even those who do reappear, like Swann or Palliser, do so only in comparatively few stories, all by the same author. This is so with some characters of fable, but many, like the clever fox, reappear in whole cycles of tales. The same mathematicalia, π and e , the sine and cosine functions, 0 and 1 and 2, and so on, reappear throughout whole libraries of mathematical works. Again, characters encountered in novels are generally of the same species as those encountered in daily life, while those in fables are, as one dictionary definition reminds us, beings of a different order, 'animals that talk and behave like human beings'. Mathematics, too, has objects even more unlike those of any other subject, and it is for precisely that reason that there is thought to be a philosophical problem about them.' (Burgess, 2004, p 21-22)

So unlike novels, fables are the best type of fiction to compare to mathematics for Burgess. He gives numerous examples of how they are similar in the way they do not have a particular author to attribute their creation to, they are retold over and over, and their characters are often shared between many of the works, as well as these characters being very different from those encountered in real life. Whilst there are cases of these examples being correct, there will always be counter examples such that the comparisons do not hold. Furthermore, none of these comparisons explains the crucial fictionalist point, which is that we can make use of mathematical statements without having commitment to the content of the statements. If it is the case that we do not have commitment to the content of the fables, such that we are not committed ontologically to their worlds and characters etc., it would not be because they do not have a particular author to be attributed to, or because of them being retold over and over in many ways. As a result, these comparisons may show that mathematics can be like fiction in a way, but they do not show any likeness in any way that matters for any ontological

argument. They do not explain where fiction gets its content from and so cannot be used to explain the same for mathematics.

It is likely from what follows in his discussion that Burgess never intended to argue that these similarities are sufficient bases on which to build a model of fictionalism. However, it is still important to note that not any likeness between mathematics and fiction will do for the fictionalist. They must be compared in some way that has some bearing on the ontology of the contents of the fiction. Whilst Burgess' comparisons do not quite hit the mark, he points to another comparison for the fictionalist to make with fables that makes them the best genus of fiction to compare mathematics to:

'Yet more important is the matter of application, which in literature typically takes the form of a 'message'. The fable typically though not invariably has a 'moral', while to demand one of the novel is virtually the definition of Philistinism. ... The question of applications is crucial in the case of mathematics, because though it would be a kind of Philistinism to demand that every piece of mathematics have one, many do; and it is precisely because many do that many philosophers have opposed nominalism, this being the least common denominator of all 'indispensability arguments'.' (p 22)

Fables are therefore the best genus of fiction to compare to mathematics because, unlike novels, they have an application, a moral. This is a useful comparison because, as Burgess states, the applicability of mathematics is the basis on which many philosophers oppose nominalism and support platonism. This is because it would be a miracle for mathematics to be so applicable to scientific theory if it were an accident; so, for many platonists, this is taken to be proof of the truthfulness of mathematical statements. However, much like the comparisons outlined previously, the fact alone that fables have an application is not a sufficient comparison to the applications of mathematics to draw the fictionalist conclusion. As the applications are themselves different, a fable is not applied to scientific theory in the same way mathematics is. So even if it was the fact that fables have applications that led fictionalists to deny commitment to their content, because this application is different to the applications of mathematics it is hard to see why this would carry over to it from this comparison alone.

However, it is not even the case that fables' applications lead us to deny commitment to their content. It requires a further argument to explain why we are not committed to their content; it is not their message that leads to this conclusion. So not only is their application different to the application of mathematics, their application does not lead to us not being committed to their content. The analogy with fables here could be useful in that we might take their role to be similar to that of metaphors, but I will be looking into that discussion in section 2.4.

Up to now the similarities that hold between mathematics and fiction that I have discussed have yet to show convincingly why fictionalists are able to continue to

use mathematical statements whilst discarding any commitment to the content of their statements. What, then, is the main comparison to fiction that drives the fictionalist conclusion? Burgess summarises it nicely:

‘Still yet more important, however, is a feature common to all genres of fiction. The most important single respect in which fictionalists hold mathematics to be like novels or fables or whatever is in being a body of falsehoods. Especially the existence theorems of mathematics are supposed to be untrue: these say there exist, for instance, prime numbers greater than 10, whereas according to mathematical fictionalists, and indeed all nominalists, there are no such things as numbers at all.’

So, the main comparison that fictionalists would like to make between mathematics and fiction is that they are a body of falsehoods, and it is because the statements are false that in both cases we are not committed to their contents.

Burgess is wrong, however, that fiction is a body of falsehoods. This is because there are plenty of propositions in a lot of fiction that are accepted as true, e.g. grass is green, people die when hit by speeding trains, etc. So, it is important to see how the fictionalist attempts to explain how mathematics is not only the same as fiction but also explain why they are like the false statements in fiction, rather than the accepted true ones. To do so, I shall focus Yablo’s account, which is the most developed account of mathematics which draws on an established theory of fiction in a way that tries to explain where their content comes from and why we should not be committed to the existence of this content.

2.4 Mathematics as a Useful Fiction

Fictionalism hopes to retain the usefulness of mathematics that platonists explain with truth. By comparing mathematics to fiction, they hope to do this without appeal to truth. But a lot of fiction is not useful, and so the comparison to fiction does not necessarily help mathematics. This can be illuminated in the following parallel in discussing mental fictionalism:

‘... the mental fictionalist aim of retaining a representation without its commitments is often expressed by envisaging [folk psychology] as a ‘useful fiction’. But fiction is not an obvious model for the ‘useful’ representation required by the fictionalist, since identifying something as fiction leaves open the possibility of it being unreliable in various ways. Thus, to make good on the notion of a useful fiction, the fictionalist needs to specify the ways in which [folk psychology] can be both useful and a fiction’ (Bourne and Caddick Bourne, 2018a)

The fictionalist needs to have a theory of fiction to support their claims. Fictionalists need to show how mathematics can be shown to be like fiction in the sense of allowing us to let go of any commitment to the content of the statements. In addition, the applicability of mathematics has to be explained, for

this is one of the main reasons that some philosophers take an anti-nominalist position regarding mathematical objects. As Yablo says:

‘The first point people make is that since applicability would be a miracle if the mathematics involved were not true, it is evidence that mathematics *is* true. The second thing that gets said (what on some theories of evidence is a corollary of the first) is that applicability is explained in part by truth. ... The most that can be said in *general* about why mathematics applies is that it is true.’ (Yablo, 2005, p 89-90)

Yablo denies that the applicability of mathematics is explained by its truth. For Yablo, and for fictionalists, the help mathematics gives to science ‘is a kind it could give *even if it were false*.’ (p 90) This ‘usefulness-without-truth’ stance to mathematics is common between the majority of fictionalist arguments, but they often derive this usefulness-without-truth via different ideas. Field (1980), for example, has an account of how mathematics is useful without being true. As explained above in section 1.3, his idea is that mathematical theories get their usefulness by being conservative over nominalist theories. However, Yablo has two issues with Field’s approach to mathematics’ usefulness-without-truth:

‘I do not doubt that Field has shown us a way in which mathematics *can* be useful without being true. ... This leaves more or less untouched, however, the problem of how mathematics *does* manage to be useful without being true. ... Field might think that the role of mathematics in the *non*-nominalistic theories that scientists really use is *analogous* to its role in connection with custom-built nominalistic theories. ... If that were Field’s view, then one suspects he would have done more to develop the analogy. Is the view, then, that he has *not* explained (or justified) actual applications of mathematics – but that is OK because, come the revolution, these actual applications will be supplanted by the new-style applications of which he *has* treated? This stands our usual approach to recalcitrant phenomena on its head.’ (Yablo, 2005, p 91–92)

Whilst Field accounts for how mathematics is able to be useful without being true, it does not help at all to show how it does so for those applying mathematics in the way that they commonly do. Scientists who do ‘regular old platonic science’ actually apply mathematical theory; they do not just use them as shortcuts in the way Field suggests. So, in order to explain this, Field either needs this application of mathematics to be analogous for the role mathematics has in connection to the nominalistic theories Field discusses, or these actual applications will be supplanted by the new-style applications. Yablo suggests that Field has not done enough work in his discussion of mathematics’ usefulness-without-truth for either of these suggestions to come to fruition. As it stands, then, Field’s attempt to show mathematics as useful without being true does not fully explain how it does so for actual applications of mathematics.

The reason Field does not address the application of mathematics as well as it requires is because his argument is more designed to address indispensability over applicability. Yablo targets applicability over indispensability:

‘How is the Fieldian nominalist to explain the usefulness-without-truth of mathematics in *ordinary*, quantitative, science? More important, though, suppose that an explanation can be given. Then *indispensability becomes a red herring*. Why should we be asked to *demathematize* science ... Putting both of these together: The point of nominalizing a theory is not achieved unless a further condition is met, given which condition there is no longer any need to nominalize the theory.’ (Yablo, 2005, p 93)

Yablo’s hope is to show that if mathematics can be applied to science in the way that it is actually done so, but can be shown to do so without being true, then there is no need to dispense of the use of mathematics in science. As a result, for Yablo, mathematics can be indispensable to science, but if its application does not guarantee the truth of mathematical statements, then we would still not be required to believe in the existence of mathematical objects. Yablo’s approach to the usefulness-without-truth conclusion is to argue that mathematics does not need to be true to be applied to science in the way that it is. To do this, he argues that numbers can serve as representational aids. As representational aids, they are used to state something which has nothing to do with numbers. He gives the following example to illustrate his point. A physicist discovers the following:

‘(A) A projectile fired at so many meters per second from the surface of a planetary sphere so many kilograms in mass and so many meters in diameter will (will not) escape its gravitational field.’

But without quantifying over mathematical objects, she would run into problems recording facts of this kind:

‘One is that since velocities range along a continuum, she will have to write uncountably many sentences. ... Second, almost all reals are ‘random’ in the sense of encoding an irreducibly infinite amount of information. So, unless we think there is room in English for uncountably many semantic primitives, almost all of the uncountably many sentences will have to be infinite in length.’

In order to escape this issue, Yablo puts it that we just sum up the facts as:

‘(B) For all positive real numbers M and R , the escape velocity from a sphere of mass M and diameter $2R$ is the square root of $2GM/R$, where G is the gravitational constant.’ (Yablo, 2005, p 94)

The role of numbers in this example is as representational aids. What is trying to be expressed has nothing to do with mathematical objects. Their purpose is just to state finitely that which otherwise could not be stated so. Do numbers need to exist in order to play this representational role? Yablo does not think so:

'That (B) succeeds in gathering together into a single content infinitely many facts of form (A) owes nothing whatever to the real existence of numbers. It is enough that *we understand what (B) asks of the non-numerical world, the numerical world taken momentarily for granted.*' (p 95)

So, he argues, numbers do not need to exist in order for (B) to achieve its aim of summing up the infinitely many facts, because we need only take the numerical world for granted *momentarily*. And this is how Yablo hopes to reach the usefulness-without-truth of mathematics. If numbers are merely representational aids and we need only momentarily take for granted the numerical world to use them as representations, then they can continue to be applied to scientific theories without the requirement of existing. This avoids the issue that Yablo raised for Field, as it clearly does not clash with how scientists actually use mathematics.

However, it is not clear what the Yablo-style fictionalist means when they refer to a 'numerical world' that is momentarily taken for granted, since, according to them, it does not exist. Presumably, they believe some conception of fiction explains how we can in some way refer to something, even momentarily, without it existing. The theory Yablo endorses, as well as other prominent fictionalists, is that of make believe games. Through this, Yablo gives an account of how numbers can be used as representational aids that can be applied to science, as well as helping us learn more about the representations themselves.

2.4.1 Make Believe Games

Make believe games can be understood as pretending that something is the case. An example would be when playing at working in a shop. We can pretend that a table is a shop counter, that household items are the items in the store and that we are the employees and the customers in the shop. The items and roles that are imagined make up the content of the game.

'[T]o elaborate and adapt oneself to the game's content is typically the game's very point. An alternative point suggests itself, though, when we reflect that all but the most boring games are played with *props*, whose game-independent properties help to determine what it is that players are supposed to imagine.'

(Yablo, 2005, p 96)

The props we use also have an impact on the game. So, in the case of our pretend shop, the items in the store do not follow just from the content of the game (that is the imagined items and roles), it is the household items we have at hand determines this also. The idea follows from this that there are two ways of viewing these pretend games. Ordinarily, the props are important to the extent that they influence the content of the game, but it could also be the case that the content of the game sheds light on the props themselves. Walton (1993) calls these 'content-oriented' and 'prop-oriented' games of make-believe. Content-oriented make-believe is when the focus of the game is on the content. Prop-

oriented make-believe in contrast, is when we focus on the props as the aim of the game, using the game for the purpose of illuminating the props used. Walton gives a few cases of this:

“Where in Italy is the town of Crotona?” I ask. You explain that it is on the arch of the Italian boot. “See that thundercloud over there – the big, angry face near the horizon”, you say; “it is headed this way” ... We think of Italy and the thunderclouds as something like pictures. Italy (or a map of Italy) depicts a boot. The cloud is a prop which makes it fictional that there is an angry face.’ (Walton, 1993, p 40-41)

In the case of the pretend shop, the props assist in helping us elaborate the content of the game and allow us to engage in the game. The toys and objects around the house are gathered in a way to represent products in a shop and they are brought up to the table (counter) and purchased (exchanged for pretend currency). In this game, it is clear that the props are helping to elaborate the content of the game for us, but in Walton’s examples, which illustrate prop-oriented make-believe, it is the other way around. We learn more about the props in the game (Italy and the thundercloud) from the content of the game of make-believe. We are able to point to where in Italy (or where on a map of Italy) Crotona is, and we are able to point to a thundercloud in a sky of clouds as a result of the content of our make-believe. It is because of the imagery of the boot and the angry face that we are able to learn more about the props themselves. This is similar to what Yablo wishes to claim about numbers.

By having mathematics serve as a fiction that is useful because it helps elucidate conditions of the real world, Yablo is able to make a comparison between mathematics and fiction that seems to show mathematics to be a useful fiction. Yablo writes:

‘numbers as they figure in applied mathematics are *creatures of existential metaphor*. They are part of a realm that we play along with because the pretense affords a desirable – sometimes irreplaceable – mode of access to certain real-world conditions, viz. the conditions that make a pretense like that appropriate in the relevant game.’ (p 98) Continuing with Walton’s theory, a metaphor is ‘an utterance that represents its objects as being *like so*: the way that they *need* to be to make the utterance ‘correct’ in a game that it itself suggests.’ (Yablo, 2005).

Similarly, to the examples given above, numbers are the content that we use in a pretense to learn more about the prop. In the case where the props are the entire world, prop-oriented games become world-oriented games. So:

‘as we make as if, e.g., people have associated with them stores of something called ‘luck’, so as to be able to describe some of them metaphorically as individuals whose luck is ‘running out’, we make as if pluralities have associated with them things called ‘numbers’, so as to be able to express an (otherwise hard

to express because) infinitely disjunctive fact about relative cardinalities like so: The number of *Fs* is divisible by the number of *Gs*.' (p 98)

Using this conception of fiction allows Yablo to explain the usefulness-without-truth of mathematics in the following way. When we use mathematics, we are engaging in a make believe game. Under the pretence of the game, we pretend that the mathematical objects that make up the content of the make believe game exist, similarly to how when playing shopping, we imagine that the products of the shop exist and the counter that we take them to exist. But instead of being content-oriented, like the game of shop, applied mathematics is prop-oriented and, in the case of applied mathematics the world is the prop that we are focusing on. Yablo calls this world-oriented. Like the example of Italy (or the map of Italy) being a boot, it is the content of the game (that Italy is a boot) that allows us to learn about the prop. By engaging in this pretence, the numbers can help us to express real world conditions that otherwise we would not be able to. That explains the usefulness of mathematics in the way that we actually apply them, unlike in Field's attempt.

A question remains, though, once we rid mathematical statements of any truth. How can one explain the correctness of mathematical statements? It is clear that some mathematical statements are correct, and others are taken to not be so. The platonist is able to account for this correctness because, for them, the correct statement is the true one and the incorrect statement is the false one. If the fictionalist is successful in ridding themselves of the ontological burden of mathematical objects, they must also explain how it is that the application of some mathematical statements is correct and others incorrect.

2.4.2 Objectivity and Correctness

Can the fictionalist who takes prop-oriented make-believe as an explanation for mathematics' usefulness-without-truth explain why some mathematical statements are taken to be correct whilst others are not? Yablo does not give a definitive account but he has an idea where an answer could begin. Importantly he compares mathematics to metaphor where 'a distinction is often drawn between *true* metaphors and metaphors that are *apt*.' (p 100-101) The claim is that a metaphor can be true without being *apt* and vice versa. He gives examples of true but not apt metaphors such as 'Tooth Decay: America's Silent Dental Killer' and 'South America: Sleeping Giant on Our Doorstep'. Whilst these metaphors aim at truth, they miss out on the important part of metaphors that really makes them apt. Yablo suggests that this aptness is also part of prop-oriented make-believe games:

'Aptness is *at least* a feature of prop-oriented make-believe games; a game is apt relative to such and such a subject-matter to the extent that it lends itself to the expression of truths about that subject matter. A particular metaphorical *utterance* is apt to the extent that (a) it is a move in an apt game, and (b) it makes impressive use of the resources that game provides.' (p 102)

The aptness of our prop-oriented make-believe game of Italy being a boot is determined by how it helps our knowing about the prop. It is apt, considering how in Walton's example it helped us learn the location of Crotona, and could be used further to learn about other locations in Italy, or Italy's location relative to other places. (Albania is located on the right hand side of the heel.) The example of the angry face being a thundercloud is another apt metaphor, but perhaps less apt than Italy being a boot. Whilst it lends itself to the expression of some truths (like which cloud is being pointed to, and the shape it takes) it seems to lend itself less so to the expression of truths than the example of Italy being a boot. Overall, we can determine the aptness of a prop-oriented make-believe game relative to its subject-matter by the extent to which it lends itself to the expression of truths about that subject matter. After accepting this move, it is a simple step to the idea that to show correctness in mathematics is to demonstrate aptness in prop-oriented games of make-believe where mathematical objects are the props. Yablo sums this up in the following:

'I want to say that a proposed new axiom A strikes us as correct roughly to the extent that a theory incorporating A seems to us to make for an *apter game* – a game that lends itself to the expression of more metaphorical truths – than a theory that omitted A , or incorporated its negation. To call A correct is to single it out as possessed of a great deal of 'cognitive promise'.' (p 102)

In the case of world-oriented make-believe games that involve mathematics being applied to a theory, the numbers or mathematical statements involved are the content of the make-believe game whilst the world itself is the prop of the game. For a mathematical theory to be seen as correct, it must lend itself to the expression of truths about the world better than not using it or using its negation. Further to this, we must take into account the history of mathematical application and currently accepted mathematical theory. This is because some mathematical statements are correct not because they express truths about the world better than not using them, but simply because they follow from other mathematical statements that have already been taken to be correct. Yablo's account of correctness then 'has two parts. Sometimes a statement is correct because it is true according to an implicitly understood background story ... sometimes though there is no well-enough understood background story ... the second kind of correctness goes with a statement's 'cognitive promise', that is, its being suited to figure in especially apt pretend games.' (p 103). Yablo does not provide this as a definitive account of correctness in mathematics but just how it could be the case. Following this, though, fictionalists have at least an outline of an account of how it is that some mathematical statements are taken to be more correct than others, whilst utilising the notion of fiction.

2.5 Problems with Yablo's account

2.5.1 Problem of Content and Ontology

The whole fictionalist project relies on the idea that by comparing mathematics to fiction, they can rid us of our ontological commitments that the truth of mathematical statements burdens us with. However, there is no guarantee that this would be the case. Bourne and Caddick Bourne write:

'It is easy to assume that there can be content without ontological commitment for the very reason that (it is often assumed) fiction can be contentful without being committal. But a theory of fiction might hold that what it is for a fiction to have content is for there to exist the things the fiction is about. ... Mere appeal to fiction does not, without a supporting theory of fiction, guarantee avoiding commitment to whatever the fiction is about. Neither does an approach to fictional truth in terms of imagination or games of make-believe automatically allow us, as might be assumed, a non-committal account of fictional content. One might agree that it is prescriptions to imagine (for example) which determine that the fiction is about this rather than that, whilst still holding that what allows it to be about anything is that there exist things that it is about.' (2018a)

Suppose we grant that Walton's example of descriptive prop-oriented make-believe games allows Yablo to attempt to explain mathematics in the same way. Without a further theory of fiction that rids us of our commitments to the contents of fiction, any fictionalist attempt to remove commitments to the content of mathematics by comparing it to fiction would not work. All that would do is describe the nature of mathematics to be that of fiction, whether that be as possibilities, non-existents or even abstract entities (See Sainsbury, 2001 for further discussion). If the content of fiction is itself abstract, then the comparison to fiction that fictionalists attempt would only be helping to show how mathematical objects are abstract objects. The fictionalist comparison of mathematics to fiction alone does not do enough to show mathematics usefulness-without-truth that they wish to conclude and as a result they do not show why we can let go of our commitment to the existence of abstract objects.

In part 1 I concluded that, in order to undermine conformational holism (and the indispensability argument), the nominalist needs to explain away the use of idealisations as being used figuratively. They also needed to show that the use of mathematics in science is analogous to these figurative uses of idealisations. But even further to this, they need to explain why we have no ontological commitment to these idealisations and mathematical statements. I have shown above that fictionalism requires a supporting theory of fiction, in order to explain where the content of these figurative mathematical statements comes from. A comparison to fiction does not do this alone without a further argument to show how fiction can have content without requiring any ontological commitments. Because of this, it does not seem that the conclusion of the indispensability argument can be undermined by way of fictionalism without this further explanation of non-

committal content. Yablo tries to solve this problem by arguing mathematical statements are related to fiction via metaphor and this is where the content comes from. However, the next section will look at issues with this.

2.5.2 Problem of Contentful Metaphor

Walton's examples of Italy and the thundercloud show how fiction can be used for descriptive purposes. In these cases, it is because we have an understanding of the content of the game that allows for us to learn more about the props. Because we know the shape of a boot, it is useful to pretend Italy is one so that we can learn about the prop (Italy), and because we know what an angry face looks like, we are able to point to a thundercloud that resembles it when we pretend it is one. However, we lack this independent understanding in relation to mathematical objects. It is not as clear how we can ever come to have this understanding of mathematical objects, if they do not exist. This is a similar issue that can be taken with Balaguer's attempt to dismiss the inclusion of fiction. Without a supporting theory of fiction, we are unable to gain a clear enough understanding of what mathematical objects would be like, so we would not be able to explain how we are able to use them in the way Yablo thinks we can. Yablo's account does go further than Balaguer, however, and gives a direct comparison when he compares the metaphor of luck running out with our use of numbers in scientific theory.

The example of luck running out is less obvious as the boot and the angry face. Yablo thinks that he can show with this example that we can talk of such things as 'luck' without there being any such thing, i.e. without the talk having any content. But as I have discussed throughout Part 2, it needs some sort of content, or it is hard to make sense of what it means when people use the word 'luck'. One way it would be possible to give talk of luck the content it needs is to imagine a world that is run by some kind of normative law, such that people have stores of what is called 'luck'. These laws determine that for each fortuitous thing that happens this store is reduced. In such a world, this is known as their luck 'running out'. We can then draw useful comparisons between such worlds and our own, articulating features of ours in terms of luck, without having to believe that luck is a feature of our world.¹ It is not difficult to understand a possibility like this and so the contents of the prop-oriented make-believe game of saying someone's luck is 'running out' like in Yablo's example is not so different from the examples from Walton. And through this way of giving content to the talk of luck, we have a way of grasping all of the components in talk of luck.

This is not the case for the idea of associating numbers to pluralities. If numbers are just 'creatures of existential metaphor' then Yablo is suggesting their content comes from metaphors. A metaphor works by comparing existing things that we are able to grasp, as each side of the comparison has independent content. In the

¹ For further discussion of how such normative features like this might be 'seen in' non-normative aspects of the world, see Bourne and Caddick Bourne (2018b).

case of the thundercloud being an angry face, the thundercloud gets its content from our experience of thunderclouds (or in the case of seeing the cloud and then stating the metaphor, its content comes from the thundercloud we see), and the angry face is graspable as we have an understanding of angry faces, and it gets its content from this. The metaphor would not land or work if we were unable to grasp it, if we were unaware of the where either of the side of the comparison gets its content. An example of this, could be one close friend saying to the other that they are Shaggy and Scooby Doo, when the other friend has no knowledge of Scooby Doo. In this case, the metaphor would not land, but it still makes sense as a metaphor because both sides of the comparison derive their content independently of the metaphor itself. But, the content of 'mathematical' metaphors would be generated solely from the metaphor itself. This could not be the case, as how would we ever first gain the understanding of the mathematical object in order to use it in the metaphors it only exists in? As a result, mathematical metaphors would not only not land but would not make sense at all. Therefore, arguing that numbers are 'creatures of existential metaphor' which get their content from these metaphors does not line up neatly with Yablo's comparison of luck running out. Without showing how we could have a metaphor that works without a side of the comparison having content independent of the metaphor, Yablo fails to convincingly show how numbers can just be 'creatures of existential metaphor.' Further to this, the use of metaphor itself would not be a way to absolve us of any ontological commitment; rather, it would invoke it because of its requirement of comparing existing things with independent content.

So, it seems the attempt to remove our commitments to the existence of mathematical objects via the fictionalist avenue of invoking fiction to illuminate how mathematics represents is not convincing. One of the best attempts at this is seen in Yablo's reconstruction of mathematics as a form of Walton's idea of metaphor as prop-oriented make-believe. The suggestion that numbers are 'creatures of existential metaphor' seems well constructed on the face of it. However, it does not successfully give any indication of how the content of the mathematical metaphor is initially generated, since if they only exist as part of the metaphor, we would never come to understand them before using the metaphor (and because of this we would never be able to first use the metaphor). This leaves the fictionalist project as a whole a major issue, as without a supporting theory of fiction that shows we can use fictional statements without any commitment to the existence of their content, there is no reason to assume we can do the same for mathematical statements. Without any progress toward solving these issues, the fictionalist claim that we are not committed to the existence of abstract objects is not convincing.

Conclusion

In the first section, I gave a framework for the naturalist position and how it leads us to the indispensability argument. This argument states that we should be

committed to the existence of mathematical objects because they are indispensable to our best scientific theories and their existence is confirmed by the success of those theories. I looked at arguments raised against this argument, starting with Field's attempt to show mathematics as dispensable. However, as it stands, no one has done the work needed to make Field's position fully convincing. I then looked at problems raised for the conformational holism premise of the indispensability argument that hoped to undermine the indispensability argument's conclusion by removing the confirmation the existence of mathematical objects receives from the success of our best scientific theories. This led to resting on the idea that mathematical statements could be used figuratively. The idea that mathematical statements can be used figuratively comes from a discussion of idealisations, in which it is argued that talk of frictionless surfaces and continuous liquids are just used in science figuratively and their use is not meant to confirm the existence of such ideals. And so it is argued in the same way that mathematical statements are just used in science figuratively, and their use in scientific theories does not confirm their existence. If this is the case, then the existence of mathematical objects would not be confirmed by the success of our best scientific theory and the conclusion of the indispensability argument would not stand.

So, to explore this claim further, I looked into mathematical fictionalism. This is the idea that, through some comparison with fiction, we can get the usefulness of mathematics without being committed to any truths. I showed why it is vital to the fictionalist that they have some supporting theory of fiction to argue their point. I explored Yablo's fictionalism as one that takes the inclusion of a theory of fiction seriously and showed how he compares mathematics to useful fiction and arrives at the conclusion that mathematical objects are creatures of metaphor. I showed how Yablo uses this to explain the correctness and objectivity of mathematics, a position that is not fully fleshed out but does justice to these features of mathematics. I showed that Yablo's account still faces the issue that his supporting theory of fiction does not fully account for where the content of his fictional mathematical statements comes from. If mathematical objects are creatures of metaphor, then the content of the metaphors that use them is generated from the metaphors themselves. So fictionalists who attempt to treat mathematical statements figuratively have more to do to show that this strategy could work in a more satisfactory way. Because of this, there is no reason to reject the conclusion of the indispensability argument, that we should be committed to the existence of mathematical objects stands, and is good reason to accept it.

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