A PSO Method for Wavelet Approximation

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Abstract: For implementing wavelet transform in analog hardware systems with very low power consumption and small size, a particle swarm optimization method is employed in the approximating of wavelet functions. First, utilizing the least-squares error criterion, a general mathematical model for approximating wavelet functions is established. Then the technique of $L_1$ approximation based on Particle Swarm Optimization (PSO) is presented, which is more attractive. These techniques are compared by means of a worked example, involving some wavelet approximation. The $L_2$ approximation approach based on PSO is shown to exhibit superior performance.

Key words: Wavelet transforms, $L_1$ approximation, Particle Swarm Optimization (PSO), analog circuits

INTRODUCTION

The Wavelet Transform (WT) is a merited technique for analysis of non-stationary signals like cardiac signals. Being a multiscale analysis technique, it offers the possibility of selective noise filtering and reliable parameter estimation. Often WT systems employ the discrete wavelet transform, implemented in a digital signal processor. However, in ultra low-power applications such as biomedical implantable devices, it is not suitable to implement the WT by means of digital circuitry due to the high power consumption associated with the required A/D converter. For power consumption considerations it therefore is preferable to perform WT in the analog domain.

Low-power analog realization of the wavelet transform with the technique of analog circuits (Karel et al., 2005, 2012; Gurrola-Navarro et al., 2010; Haddad et al., 2005) has been introduced. The quality of such implementations depends on the accuracy of the corresponding wavelet approximations. Previous approaches reported for wavelet approximations include mainly pade approximation method (Haddad et al., 2005) and $L_2$ approximation method (Karel et al., 2005; Haddad et al., 2005). The Laplace transforms of wavelet functions were approximated by rational functions in the Laplace domain with Pade approximation (Baker, 1975; Bultheel and Barel, 1986). However, there are some problems which limit the practical applicability of Pade approximation. One important issue concerns stability. The stable transfer function of wavelet filter does not automatically result from the Pade approximation technique. If the selection of the point $s_0$ is improper, the result of approximation will be unstable. Some poles of the Morlet wavelet transfer function obtained by this method in Haddad et al. (2005) lie in the right half of the s-plane, which indicates the transfer function is not stable. Another important drawback is: the quality of the approximation of the wavelet is not measured directly in the time domain but in the Laplace domain, which results in a larger error of approximation. The performance of implementing WT in analog domain depends largely on the accuracy of the approximations involved in this approach. Karel et al. (2005), an alternative approach, based on $L_2$ approximation that works directly in the time domain, was introduced. A drawback of this approach is that the numerical optimization of objective function usually ends in local, non-global optimization when a starting point is not exactly selected. To find a good starting point for $L_2$ approximation, a method involving high-order FIR approximation and balance-and-truncate model reduction is used. However, it is a computational complex method and limited to approximate such low order wavelet approximation methods.
functions as the Gaussian and Daubechies. This method has also some convergence problems when one tries to approximate a function with many oscillations (high order wavelet), such as the Morlet wavelet. So far, there is a lack of the effective method to approximate various low or high order wavelet functions, which is an obstacle for implementing WT in analog domain.

In this study, we focus on the wavelet approximation for implementation in analog circuits. The innovative aspect of the present work is threefold. First, by extending pioneering work (Karel et al., 2005; Hongmin et al., 2008; Li et al., 2010; Gurrola-Navarro et al., 2010), we propose a generalized optimization mathematical model of approximating various wavelet functions, which is based on the L_q approximation. Second, the Particle Swarm Optimization (PSO) (Kennedy and Eberhart, 1995) algorithm is introduced to solve the optimization problem. The PSO algorithm is one of the most powerful methods for solving global optimization problems and is effective, efficient and fairly robust to initial conditions. This method overcomes these shortcomings of approximation technique in Haddad et al. (2005) and Karel et al. (2005). Using PSO algorithm, we have successfully approximated various wavelet functions, especially the Morlet wavelet (a high order wavelet).

**WAVELET TRANSFORM**

The wavelet transform provides a time-frequency representation of the signal (Mallat, 1999; Walbank, 2004). It was developed to overcome the short coming of the Short Time Fourier Transform (STFT), which can also be used to analyze non-stationary signals. While STFT gives a constant resolution at all frequencies, the wavelet transform uses multi-resolution technique by which different frequencies are analyzed with different resolutions.

The definition of the Continuous Wavelet Transform (CWT) for a real valued time signal x(t) is given as (Mallat, 1999):

\[
\text{WT}_a (a, \tau) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t) \psi^* \left( \frac{t-\tau}{a} \right) dt \tag{1}
\]

where, a is scale parameter (a \in (0, R)) and \tau is translation parameter (\tau \in R). The base function \Psi (t) (\Psi (t) \in L(R)) is called the mother wavelet. The mother wavelet used to generate all the basis functions is designed based on some desired characteristics associated with that function. The translation parameter \tau relates to the location of the wavelet function as it is shifted through the signal. Thus, it corresponds to the time information in the Wavelet Transform. The scale parameter a is defined as |1/frequency| and corresponds to frequency information. Scaling either dilates (expands) or compresses a signal. Large scales (low frequencies) dilate the signal and provide detailed information hidden in the signal, while small scales (high frequencies) compress the signal and provide global information about the signal. The above equation shows that the wavelet transform performs the convolution operation of the signal and the basis function.

The mother wavelet must satisfy two restriction conditions. One is:

\[
\int_{-\infty}^{\infty} \Psi(t) dt = 0 \tag{2}
\]

This ensures the mother wavelet has no DC component and is fast in decaying rate. The other is the admissibility condition, i.e.,

\[
\int_{-\infty}^{\infty} \frac{|\Psi(w)|^2}{|w|} dw < \infty \tag{3}
\]

where, \Psi (w) is the Fourier transform of the mother wavelet \Psi (t). The second restriction in Eq. 3 is stronger than the first one. The reason for requiring this condition is to guarantee that the reconstruction of the original time signal from the continuous wavelet transform is possible. Wavelet transforms usually cannot be implemented exactly in analog electronic hardware. If a time signal x(t) is passed through a linear system, then x(t) is convolved with the impulse response h(t) of that linear system, producing the output signal:

\[
\int_{-\infty}^{\infty} x(t) h(t-\tau) d\tau
\]

On the other hand, from the definition of WT given by Eq. 1, the analog computation of WT_a (a, t) (scale a) can be achieved through the implementation of a linear filter of which the impulse response satisfies:

\[
h(t) = \frac{1}{\sqrt{a}} \Psi(-t/a) \tag{4}
\]

For obvious physical reasons only the hardware implementation of (strictly) causal stable filters is feasible. In other words, a linear filter will have a (strictly) proper rational transfer function H(s) that has all its poles in the complex left half plane. Because the h(t) will then be zero for negative t, any mother wavelet \Psi (t) which does not have this property must be time-shifted to facilitate an accurate approximation of its
(correspondingly time-shifted) wavelet transform $\tilde{W}_t$, $(a, t)$. This may result in a truncation error for a wavelet that does not have compact support, such as the Gaussian wavelet. Note that an approximation error will also be due to the fact that a wavelet does not usually possess a rational Laplace transform.

**L₂ APPROXIMATION OF WAVELET FUNCTIONS BASED ON PSO ALGORITHM**

Generalized optimization model of $L_1$ approximation of wavelet functions: The theory of $L_1$ approximation (Karel et al., 2005) provides an alternative framework for studying the problem of wavelet approximation which offers a number of advantages. On the conceptual level it is quite appropriate to use the $L_2$ norm to measure the quality of an approximation $h(t)$ of the function $\tilde{\psi}(t)$ ($\tilde{\psi}(t) = \psi(t - t_0)$). Indeed, the very definition of the wavelet transform itself involves the $L_2$ inner product between the signal $x(t)$ and the mother wavelet $\Psi(t)$. It is also desirable that the approximation $h(t)$ of $\tilde{\psi}(t)$ behaves equally well for all time instances $t$ since $h(t)$ is used as a convolution kernel with any arbitrary shift. This property holds naturally for $L_2$ approximation but it is not supported by the Padé approximation approach. Another advantage of $L_2$ approximation is that it allows for a description in the time domain as well as in the Laplace domain, so that both frameworks can be exploited to develop further insight. According to Parseval's identity the squared $L_2$ norm of the difference between $\tilde{\psi}(t)$ and $h(t)$ can be expressed as:

$$\|\tilde{\psi}(t) - h(t)\|^2 = \int_0^\infty |\tilde{\psi}(w)|^2 dw - \int_0^\infty |h(w)|^2 dw$$

Minimization of $\|\tilde{\psi}(t) - h(t)\|^2$ is therefore equivalent to minimization of the $L_2$ norm of the difference between the Laplace transforms $\tilde{\psi}(t)$ and $H(s)$ over the imaginary axis $s = iw$.

Particularly in the case of low order approximation, the $L_2$ approximation problem can be approached in a simple and straightforward way in the time domain. As is well known from linear systems theory any strictly causal linear filter of finite order $n$ can be represented in the time domain by the impulse response function $h(t)$ (its Laplace transform $H(s)$). For the generic situation of stable systems with distinct poles, the impulse response function $h(t)$ is a linear combination of damped exponentials and exponentially damped harmonics. For low order systems, this makes it possible to propose an explicitly parameterized class of impulse response functions among which to search for a good approximation of $\tilde{\psi}(t)$. For instance, if a $N$th order approximation is attempted, this parameterized class of functions $h(t)$ may typically have the following form:

$$h(t) = \sum_{m=0}^{N-1} c_m e^{\lambda t} + \sum_{j=0}^{J} \left[ c_j e^{\gamma_j t} \sin(\tau_j t) + d_j e^{\gamma_j t} \cos(\tau_j t) \right]$$

where the parameters $b_j$ and $d_j$ must be strictly negative for reasons of stability. When the expression of wavelet functions includes sine term $A \cos(\Omega t)$, such as the Morlet Wavelet, the $h(t)$ may be given by:

$$h(t) = \sum_{m=0}^{N-1} c_m e^{\lambda t} + \sum_{j=0}^{J} \left[ c_j e^{\gamma_j t} \sin(\tau_j t) + d_j e^{\gamma_j t} \cos(\tau_j t) \right]$$

Note that wavelets typically are oscillatory functions so that a good fit requires the contribution of sufficiently many damped harmonics, which further explains the structure of this class. Given the explicit form of the wavelet $\tilde{\psi}(t)$ and the parameterized class of functions $h(t)$, the $L_2$ norm of the difference $\|\tilde{\psi}(t) - h(t)\|^2$ can now be minimized in a straightforward way using standard numerical optimization techniques and software. The negativity constraints on $b_j$ and $d_j$ which enforce stability are not difficult to handle.

One common property of a wavelet function $\tilde{\psi}(t)$ that wasn't discussed so far is that its integral is usually equal to zero:

$$\int_0^\infty \tilde{\psi}(t) dt = 0$$

If this property is not shared by the approximation $h(t)$, this will cause an unwanted bias in the approximation of the wavelet transform. So we have that:

$$\int_0^\infty h(t) dt = 0$$

This yields the explicit nonlinear condition, if such an extra nonlinear condition is not conveniently handled by the optimization software, then it can easily be used to eliminate one of the variables from the problem. Based on the analysis above, a generalized optimization mathematical model of approximating various wavelet functions is then given by:
\[ \begin{align*}
\min \quad & E(a, b, c, d, f, g) = \sum_{n=0}^{N-1} [h(n\Delta T) - \tilde{h}(n\Delta T)]^2 \\
\text{s.t.} \quad & b_i < 0, d_i < 0, i = 1, 2, \ldots, k, j = 1, 2, \ldots, m, \\
& \int_{-\infty}^{\infty} h(t) dt = 0,
\end{align*} \tag{9} \]

This is a typical nonlinear and constrained optimization question. It is difficult to obtain the global optimal solution using common numerical optimization techniques, which in general provide no global optimality guarantee and give different local optima with different starting points.

**Particle swarm optimization (PSO) algorithm:** The PSO algorithm is one of the most powerful methods for solving global optimization problems and is effective, efficient and fairly robust to initial conditions. In order to optimize parameters of \( h(t) \), we use the Particle Swarm Optimization (PSO) algorithm to solve the optimization question in (9), search the whole parameters space effectively and globally.

Particle swarm optimization algorithms (Kennedy and Eberhart, 1995) are evolutionary computations. The particle swarm optimizer algorithms find optimal regions of complex search space through the interaction of individuals in a population of particles. The rapid speed of calculation and simple realization are its excellent performance. Precision is not good. PSO is a population-based, bio-inspired optimization method. It was originally inspired in the way crowds of individuals move towards predefined objectives, but it is better viewed using a social metaphor. PSO is similar to the other evolutionary algorithms in that the system is initialized with a population of random solutions. However, each potential solution is also assigned a randomized velocity and the potential solutions, called particles, corresponding to individuals. Each particle in PSO flies in the D-dimensional problem space with a velocity which is dynamically adjusted according to the flying experiences of its own and its colleagues. The location of the \( i \)th particle is represented as \( X_i = (x_{i1}, \ldots, x_{id}, \ldots, x_{in}) \), where \( x_{id} \in [l_{id}, u_{id}] \), \( d \in [1, D] \), \( l_{id}, u_{id} \) are the lower and upper bounds for the \( d \)th dimension, respectively. The best previous position (which giving the best fitness value) of the \( i \)th particle is recorded and represented as \( P_i = (p_{i1}, \ldots, p_{id}, \ldots, p_{in}) \), which is also called best. The index of the best particle among all the particles in the population is represented by the symbol \( g \). The location \( P_g \) is also called best. The velocity for the \( i \)th particle is represented as \( V_i = (v_{i1}, \ldots, v_{id}, \ldots, v_{in}) \), is clamped to a maximum velocity \( V_{\text{max}} = (V_{\text{max1}}, \ldots, V_{\text{maxd}}, \ldots, V_{\text{maxn}}) \), which is specified by the user. The particle swarm optimization concept consists of, at each time step, changing the velocity and location of each particle toward its best and best locations according to the Eq 10 and 11, respectively:

\[ v_{id} = w \cdot v_{id} + c_1 \cdot \text{rand()} \cdot (p_{id} - x_{id}) \]
\[ + c_2 \cdot \text{rand()} \cdot (g_{id} - x_{id}) \]
\[ x_{id} = x_{id} + v_{id} \]

where, \( w \) is inertia weight, \( c_1 \) and \( c_2 \) are acceleration constants and \( \text{rand()} \) is a random function in the range [0, 1]. For Eq 10, the first part represents the inertia of pervious velocity; the second part is the “cognition” part, which represents the private thinking by itself; the third part is the “social” part, which represents the cooperation among the particles. If the sum of accelerations would cause the velocity \( v_{id} \) on that dimension to exceed \( V_{\text{maxid}} \) then \( v_{id} \) is limited to \( V_{\text{maxid}} \).

**APPROXIMATION OF THE COMMON WAVELET FUNCTIONS**

**Approximation of gaussian and morlet wavelet:** To demonstrate the proposed method, we first discuss how to approximate Marr wavelet base. Marr wavelet is a favorite choice in many signal processing applications. The Marr wavelet \( \Psi(t) \) is the second derivative of a Gaussian probability density function:

\[ \psi(t) = (1 - t^2)e^{-\frac{t^2}{2}}, -\infty < t < \infty \]

Select the time-shift \( t_s = 4 \), get time-reversed and time-shifted Marr wavelet \( \Psi(4-t) \). Let \( h(t) \) be the impulse response of Marr wavelet filter to be designed and the order of wavelet filter \( N \) be 9, then the parameterized class of functions \( h(t) \) given by:

\[ h(t) = a_1 e^{x_1} + a_2 e^{x_2} \sin(a_3 t_i) + a_3 e^{x_3} \cos(a_4 t_i) + a_4 e^{x_4} \sin(a_5 t_i) + a_5 e^{x_5} \cos(a_6 t_i) \]
\[ + a_6 e^{x_6} \cos(a_7 t_i) + a_7 e^{x_7} \sin(a_8 t_i) + a_8 e^{x_8} \cos(a_9 t_i) \]
\[ (t \geq 0) \]
Note that choice of order of wavelet filter involves an important trade-off between optimal solution and complexity of filter circuits. If N is chosen too small, the h(t) may be far away from the versatile wavelet. On the other hand, if N is chosen too large, a more complex analog IC is demanded to realize wavelet transform. We define the distance between h(t) and Ψ(t-4):

$$se(a)=\frac{1}{6}\left[\int h(t)-\Psi(t-4)\right]^2 dt$$  

where, \(a = (a_1, a_2, a_3, \ldots, a_n)^T\) is an undetermined parameter vector. To sample \(D(a)\), the fitness function is given:

$$\min E(a) = \min_{a} \sum_{i=0}^{\infty} [\Psi(a_i) - \Psi(a_i - 4T)]^2$$  

The optimization model of approximating Marr wavelet function is described as:

$$\min E(a)$$

s.t.  
$$a_1 < 0, a_3 < 0, a_4 < 0, a_6 < 0, a_8 < 0$$

$$a_1 + a_4 + a_5 a_6 + a_7 a_8$$

$$a_2 a_5 + a_6 a_7 + a_8 a_9$$

$$a_3 + a_4 a_7 + a_5 a_8$$

$$a_4 + a_5 a_8 + a_6 a_9$$

$$a = (a_1, a_2, a_3, \ldots, a_n)^T, a_n \in \mathbb{R}$$  

This is a nonlinear constrained optimization question. Using the proposed hybrid algorithm to solve Eq. 16, the parameters for PSO are set as: Population size i = 100, Inertia weight factor \(\omega_{\max} = 0.4\), \(\omega_{\min} = 0.9\), acceleration constant \(c_1 = c_2 = 2\), maximum iteration \(N = 9000\). The position and the velocity of the i th particle and the fitness function of corresponding sampling point in the nth iteration are denoted by \(a_i^{(n)} = (a_{i1}, a_{i2}, \ldots, a_{in})\). Its local best position and the global best position of the particle swarm are denoted by \(P_i\) and \(P_g\), respectively. The PSO optimization program is run in MATLAB 7.1 and the search process of PSO is shown in Fig. 1. Because PSO is a stochastic algorithm, it is difficult to guarantee a global optimal solution only by a certain experiment. Here, the number of experiments is set to 25. After finishing many times test, the best global solution are selected, which is shown in Table 2. To replace the parameters in Eq. 13 with a in Table 1, the following Marr wavelet filter transfer function can be obtained:

$$h(t) = -1.28e^{-0.55t} - 0.78e^{-0.55t} \sin(-1.54t) + 10.16e^{-0.55t} \cos(-1.54t) + 5.52e^{-0.55t} \sin(0.82t) + 5.46e^{-0.55t} \cos(0.82t) + 0.64e^{-0.55t} \sin(3.12t) + 0.66e^{-0.55t} \cos(3.12t) + 3.1e^{-1.67t} \sin(-2.28t) - 3.49e^{-0.55t} \cos(-2.28t) (t > 0)$$
where, $H(s)$ is the wavelet filter to realize $W_{Marr}(1, \tau)$ under scale 1. By the theory of Laplace transform, the transfer function of analog wavelet filter under certain scale $\alpha$ is expressed as $\sqrt{\alpha}H(\omega \alpha)$. The time-domain waveform of approximated Marr wavelet in Fig. 2 it inherits the excellent qualities of Marr wavelet.

Now, we consider approximation of the Morlet wavelet. Morlet wavelet $\psi(t)$ is defined:

$$\psi(t) = \cos(5t)e^{-\frac{t^2}{2}} \quad (-\infty < t < \infty)$$

Choosing the time-shift $t_0 = 3$, this gives rise to the following time-reversed and time-shifted wavelet function:

$$\tilde{\psi}(t) = \cos(5(3-t))e^{-5(3-t)^2} \quad (-\infty < t < \infty)$$

To obtain a stable 10th order approximation of $\tilde{\psi}(t)$, the $L_2$ approximation technique was applied using the parameterized class of functions $h(t)$ given by Eq. (7):

$$h(t) = \left[ a_p e^{\delta t} + a_p e^{\delta t} \sin(a_t) + a_p e^{\delta t} \cos(a_t) \right] \cdot \cos(5(3-t)) t > 0$$

where, the parameters $a_2$, $a_8$, and $a_{10}$ must be strictly negative for reasons of stability. Given the explicit form of $\tilde{\psi}(t)$ and the parameterized class of functions $h(t)$, the squared $L_2$ norm of the difference between $\tilde{\psi}(t)$ and $h(t)$ is expressed as:

$$se(a) = \|\tilde{\psi}(t) - h(t)\|^2 = \frac{1}{2} \int[h(t) - \tilde{\psi}(t)]^2 dt$$

where, $a = (a_2, a_8, a_{10}, \ldots, a_{19})^T$ is parameter vector. The sum of squares of error of nonlinear functions is expressed by $E(a)$:

$$E(a) = \sum_{n=1}^{20} \| h(n\Delta t) - \tilde{\psi}(n\Delta t) \|^2$$

where, the sampling time interval equal 0.01 ($\Delta t = 0.01$). The optimization problem is described as:

$$\min E(a) \quad \text{s. t.} \quad a_2 < 0, a_8 < 0, a_{10} < 0$$

$$\int h(t) dt = 0$$

From:

$$\int_0^\infty h(t) dt = 0$$

this yields the constrained condition:

$$\begin{align*}
15\sin(15\Delta t) - a_{18} \cos(15\Delta t) &+ a_{19}\Delta t^2 + (a_{18} - 0.05)\cos(15\Delta t) + 0.05\sin(15\Delta t) \\
-0.05\cos(15\Delta t) &+ a_{18} + a_{19}\Delta t^2 + (a_{18} - 0.05)\sin(15\Delta t) + 0.05\cos(15\Delta t)
\end{align*}$$

(25)

Using the Particle Swarm Optimization (PSO) algorithm to solve Eq. 26, the parameters for carrying out PSO are Population size $I = 80$, Inertia weight factor $w_{\min} = 0.4$, $w_{\max} = 0.9$, acceleration constants $c_1 = c_2 = 2$, maximum iteration $N = 6000$. The optimum parameter $a = (a_2, a_8, a_{10}, \ldots, a_{19})^T$ was obtained in Table 2.

The following filter was obtained:

$$H(\omega) = \{ -0.01 \omega^2 + 0.15 \omega^2 - 2.88\omega^2 + 18.77\omega^2 - 74.75\omega^2 + 254.39\omega^2 + 4.88 \cdot 10^3 \omega^2 \\
-1.05 \cdot 10^3 \omega^2 + 3.43 \omega^2 + 6.97\omega^2 + 155.55\omega^2 + 787.28\omega^2 + 8.58 \cdot 10^3 \omega^2 \\
+ 3.06 \cdot 10^3 \omega^2 + 2.86 \cdot 10^3 \omega^2 + 4.82 \cdot 10^3 \omega^2 + 2.18 \cdot 10^3 \omega^2 + 2.54 \cdot 10^3 \omega^2 + 10^3 \omega^2 \}$$

(26)

The corresponding wavelet approximation $h(t)$ is shown in Fig. 3, yielding an $L_2$ approximation error equal to 0.0015.

**APPROXIMATION PERFORMANCE COMPARISON**

To determine the quality of the wavelet approximations obtained with the three kinds of techniques: padé approximation (Haddad et al., 2005), L2 approximation (Karel et al., 2005) and L2 approximation
Table 3: Comparison of approximation performance

<table>
<thead>
<tr>
<th>Wavelet</th>
<th>Pade approximation</th>
<th>L2 approximation</th>
<th>Log approximation</th>
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</thead>
<tbody>
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<td>Gaussian</td>
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<tr>
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<td>0.0294</td>
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<tr>
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<td>N/A</td>
</tr>
<tr>
<td>wavelet</td>
<td>12th</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Fig. 3: Approximation of the Morlet wavelet

Optimization (PSO) algorithm is used to solve the optimization problem. Because the PSO algorithm is one of the most powerful methods for solving global optimization problems and is effective, efficient and fairly robust to initial conditions, the proposed method overcomes these shortcomings of approximation technique in Haddad et al. (2005) and Karel et al. (2005).

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