

A PERSPECTIVE ON STRUCTURAL HEALTH AND USAGE MONITORING IN AEROSPACE APPLICATIONS

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ABSTRACT

There is great interest in the benefits of Structural Health and Usage Monitoring in the Aerospace Industry both from a safety point of view and because of the possibility of extending the life of aerospace structural components. Although fail-safe and damage tolerance approaches to design are extensively used and have great advantages, there are never the less components and circumstances where a safe life approach remains appropriate. This leads to an approach to fatigue clearance whereby a component will be taken out of service after a certain number of hours usage irrespective of the environment it has experienced having been cleared based on very conservative loading assumptions. If the actual loads experienced by critical parts of a structure can be derived from a Structural Health and Usage Monitoring System (SHUMS), this then leads to the possibility of extending the time for which the component can remain in service with consequent cost savings. In this paper, a number of fundamental approaches to loads prediction using data available from a Structural Health and Usage Monitoring Systems are reviewed, with the particular application in mind being that of an air-carried guided weapon. Approaches considered will include time-domain and frequency-domain based methods making use of a structural model, together with machine learning based approaches. Their different strengths, weaknesses and pitfalls will be highlighted together with ways to overcome them. Practical aspects of their possible implementation will also be addressed.

NOMENCLATURE

a_i	Generalise mass
F_k	Force acting at location k
I_{cg}	Missile pitching moment of inertia
M	Missile mass
q_i	Generalised coordinate
t	Time
x	Distance from datum along missile
x_k	Distance from datum along missile of location k .
x_{cg}	Missile centre of gravity location
z	Bending displacement
\mathbf{B}	Matrix relating \mathbf{F} and $\dot{\mathbf{Z}}$
\mathbf{F}	Vector of loads at a set of times and locations
\mathbf{U}	Left-hand matrix in singular value decomposition of \mathbf{B}
\mathbf{V}	Right-hand matrix in singular value decomposition of \mathbf{B}
\mathbf{Z}	Vector of displacements at a set of times and locations
Σ	Matrix of singular values
Σ_k	Modified matrix of singular values in regularisation
α	Tikhonov Regularisation parameter
ϕ_i	Mode shape
ζ_i	Modal critical damping ratio
σ_i	i th singular value

ω	Frequency
ω_i	Natural frequency
ω_{di}	Damped natural frequency
τ	Time

1. INTRODUCTION

There is great interest in the benefits of Structural Health and Usage Monitoring in the Aerospace Industry both from a safety point of view and because of the possibility of extending the life of aerospace structural components. In the defence sector, this technology is potentially of particular relevance to air-carried guided weapons. At present, a safe life approach is adopted with a missile taken out of service after a certain number of hours of air carriage irrespective of the load environment it has experienced. These hours are determined based on a conservative fatigue clearance approach. However, it is anticipated that the actual loads experienced will generally be much lower than those used in the fatigue clearance. Hence if these loads can be predicted, then air carry life can potentially be extended. A major issue is that structural responses at critical locations cannot easily be directly measured, and hence an alternative is to predict the input forces on the missile based on structural responses that can be

measured. This paper considers two approaches to doing this.

The first approach involves a structural model relating input loads to measured structural responses at known locations – for example through the use of accelerometers. The model is then used to predict the loads given the responses. This may be done either in the time domain or frequency domain. Examples of the former include [1] – [7] and examples of the latter include [8] – [12]. A feature of these kinds of approaches to loads prediction is that they lead to ill-conditioned problems, which require the use of regularisation methods to address the issue, and this is highlighted in many of [1] – [12]. The second approach is through machine learning, whereby a relationship is established between measurements and input forces based on a ‘training’ process such as Gaussian Process Regression. This has the advantage of dispensing with the need for a structural model. Examples of this approach are [13] - [15]. This is but a sample of the extensive literature in the field, and a broad review of loads reconstruction techniques is to be found in [16].

The layout of this paper is as follows. Section 2 considers the structural modelling method including a description of the model, the loads prediction and regularisation approaches, after which illustrative results are presented using both the time domain and frequency domain methodologies. Section 3 considers the application of Gaussian Process Regression and presents results for the same case consider in Section 2. Concluding remarks are given in Section 4 in which the wider aspects of the possible implementation of these methods will also be discussed.

2. STRUCTURAL MODELLING METHOD

2.1 STRUCTURAL MODEL

A guided weapon in external air carriage is subject to loads transmitted to it via its attachments to the carrier aircraft, sometimes referred to as hangers, together with aerodynamic loadings. For the studies considered here, the missile structure is modelled as a beam and the applied loads are represented as discretised forces applied on the missile centre line

as shown in Fig. 1. It should be noted that it is assumed that the missile is restrained on the launcher by simple supports, and hence there are no rotational constraints. Motion in a plane only is considered, although extension to more general scenarios is readily achievable.

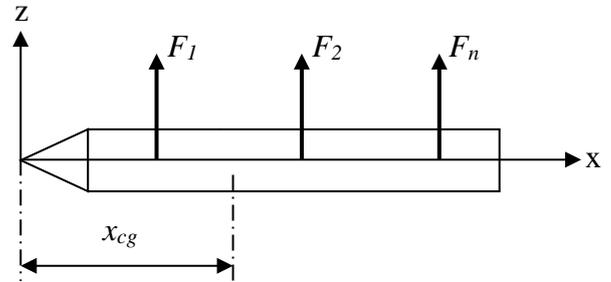


Fig. 1 Missile Under External Loading

Analysis is carried out on the basis of modal analysis. Thus, the free-free modes $\phi_i(x)$, natural frequencies ω_i generalised masses a_i and stiffnesses are first determined (where $i = 1, 2, \dots$). Hence the equations of motion may be written as:

$$\ddot{q}_i + 2\zeta_i\omega_i\dot{q}_i + \omega_i^2q_i = \sum_{k=1}^n \frac{F_k}{a_i} \phi_i(x_k) \quad (1)$$

where n is the number of applied forces, F_k is the k th force which is applied at station x_k , and ζ_i is the critical damping ratio of the i th mode. It may be noted that as free-free modes are used, the first two modes are rigid body modes and it is possible to write these, together with the corresponding natural frequencies and generalise masses as:

$$\begin{aligned} \phi_1(x) &= 1; & \phi_2(x) &= x - x_{cg} \\ \omega_1 &= \omega_2 = 0 \\ a_1 &= M & a_2 &= I_{cg} \end{aligned} \quad (2)$$

where M is the missile mass, and I_{cg} the missile pitching moment of inertia about the Centre of Gravity whose location is x_{cg} . The model may be generated through finite element analysis or through modal testing. In the study considered here, loads prediction is assumed to be based on measurements from accelerometers placed along the length of the missile. The acceleration at any point on the missile centre-line may then be written:

$$\ddot{z}(x, t) = \sum_{i=1}^{\infty} \phi_i(x) \ddot{q}_i(t) \quad (3)$$

In the time domain, the acceleration response at any point along the missile may be related to force input through the Equation:

$$\ddot{q}_i(t) = \int_0^t \sum_{k=1}^n \frac{F_k(\tau)}{a_i} \phi_i(x_i) K_i(t, \tau) d\tau + \sum_{k=1}^n \phi_i(x) \frac{F_k(t)}{a_i} \quad (4)$$

where:

$$K(t, \tau) = -\frac{1}{\omega_{di}} \left\{ 2\zeta_i \omega_i \omega_{di} \cos \omega_{di} (t - \tau) - \omega_i^2 \zeta_i^2 \sin \omega_{di} (t - \tau) + \omega_{di}^2 \sin \omega_{di} (t - \tau) \right\} \times \exp\{-\zeta_i \omega_i (t - \tau)\} \quad (5)$$

and:

$$\omega_{di} = \omega_i \sqrt{1 - \zeta_i^2} \quad (6)$$

for $i \geq 3$. For $i = 1, 2, :$

$$\ddot{q}_i(t) = \sum_{k=1}^n \phi_i(x) \frac{F_k(t)}{a_i} \quad (7)$$

In the frequency domain, the acceleration response at any point along the missile may be related to force input through the Equations:

$$\begin{aligned} & \mathcal{F}(\ddot{z}(x, t)) \\ &= -\sum_{k=1}^{\infty} \omega^2 \left\{ \frac{1}{a_k (-\omega^2 + 2i\zeta_k \omega \omega_k + \omega_k^2)} \right\} \\ & \times \sum_{j=1}^n \mathcal{F}(F_k(t)) \phi_k(x_k) \phi_k(x) \end{aligned} \quad (8)$$

where \mathcal{F} denotes a Fourier transform.

2.2 LOADS PREDICTION APPROACH

Loads prediction based on the mathematical model of Section 2.1 may be carried out in either the time or frequency domain.

When working in the time domain, given a set of accelerations at times t_1, t_2, \dots, t_m at a number of locations on the missile, loads are obtained using Equations (3) and (4). The integral in (4) may be written as a sum of integrals over intervals $[t_j, t_{j+1}]$ for $j = 1, \dots, m-1$. Each of these integrals may be approximated so that they become functions of $F_k(t_j)$ and $F_k(t_{j+1})$. This may be done in a variety of ways such as, for example, the trapezium rule or zero or first order hold. This then leads to a system of linear simultaneous equations enabling each F_k to be determined at each time t_j . The order of the system depends on the number n of forces to be obtained, the number of accelerometers, and number of samples, m , in the time interval.

When working in the frequency domain, the set of acceleration time histories are converted into the frequency domain by Fast Fourier Transform. Frequency response functions such as those given in Equation (8) are then determined so that at each frequency within a chosen range, the aim is to find the Fourier transform of the applied loads. This leads to a large set of small systems of linear simultaneous equations at each frequency in the range of interest, the size of which depends on the number of accelerometers used and the number of forces to be predicted. The resultant Fourier transforms of the forces are then inverted to obtain loads in the time domain.

Whether working in the time domain or the frequency domain, the applied forces, characterised by a vector \mathbf{F} , are determined, given the acceleration measurements, from an equation of the form:

$$\ddot{\mathbf{Z}} = \mathbf{B}\mathbf{F} \quad (9)$$

where the left hand side of (9) denotes a column vector of acceleration measurements, either in the time or frequency domain and \mathbf{B} is a matrix derived from Equations (3) and (4) for time domain analysis and Equation (8) for frequency domain analysis. Typically, the dimension of the left hand side of (9) is greater than that of \mathbf{F} (ie the number locations

where acceleration is measured is greater than the number of input forces) so that a least squares problem is to be solved. This then results in the following equation for \mathbf{F} :

$$\mathbf{F} = (\mathbf{B}'\mathbf{B})^{-1}\mathbf{B}'\ddot{\mathbf{Z}} \quad (10)$$

2.3. REGULARISATION

It is not usually as straightforward to determine \mathbf{F} as might at first appear because in inverse problems of this type, the matrix $\mathbf{B}'\mathbf{B}$ is often nearly singular so that the problem is ill-conditioned – thus small errors, from whatever source, can lead to large changes to \mathbf{F} . This issue may be addressed by regularisation.

A well-known approach is Tikhonov regularisation, which involves solving the following problem:

$$\mathbf{F} = (\mathbf{B}'\mathbf{B} + \alpha\mathbf{I})^{-1}\mathbf{B}'\ddot{\mathbf{Z}} \quad (11)$$

α is the regularisation parameter and needs to be chosen carefully. Its effect is to produce a less singular matrix so that the effect of errors is significantly reduced. However, it has to be chosen appropriately – if α is too small, the problem will still be ill-conditioned. If α is too large, the term $\alpha\mathbf{I}$ will swamp the other terms. Various methods for choosing α have been developed, with the L-curve method perhaps being one of the best known and this will be one of the approaches adopted here.

2.4. RESULTS

In this section, results are presented using both the time domain and frequency domain approaches. In each case, a uniform beam representative of a missile structure is used having the following characteristics:

- Mass: 90 kg; Length: 3.1 m
- Flexural Rigidity: 350 kNm²
- CG position: 1.55 m from nose
- Pitching moment of inertia: 72.075 kgm²
- Accelerometer locations: 0.2 m, 0.6 m, 1.5 m, 2.0 m, 2.4 m, 2.9 m
- Force locations:
 - Two Forces – 1.2 m, 2.4 m
 - Three Forces – 0.2 m, 1.3 m, 2.4 m

with the locations being measured from the nose. The number of force locations taken reflects a missile in external carriage being supported at either two or three points. In these studies, a beam element finite model is used to predict acceleration responses for given force inputs and the loads prediction methodology is then used to recover the loads which may then be compared with the originally defined forces. Section 2.4(a) presents results from time domain analysis and 2.4(b) results from the frequency domain approach.

2.4 (a) Time Domain Prediction

An example of two force prediction is first presented. The following force input is used:

$$F_1 = 100\sin(3.25\pi t) + 150\cos(7.1\pi t) \quad (12)$$

$$F_2 = 200\sin(4.2\pi t) + 150\cos(9.2\pi t)$$

over a time interval of 5 seconds. The geometry, inertial and stiffness data is as given above in this Section and with a critical damping ratio ζ_i of 0.05 taken for all elastic modes. 5 modes, including the two rigid body modes, were used in the analysis. No regularisation has been applied. Discretisation of Equations (3) and (4) is by zero order hold. The results are shown below in Figs. 2 and 3, and the load prediction is good. This might be anticipated given that this problem is statically determinate.

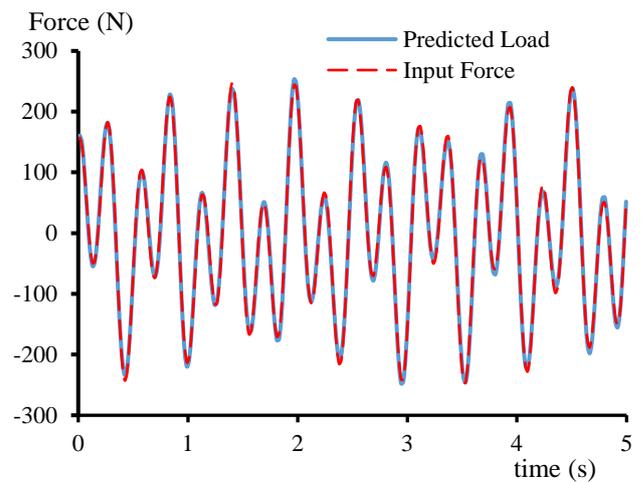


Fig. 2. Load Prediction – Force 1

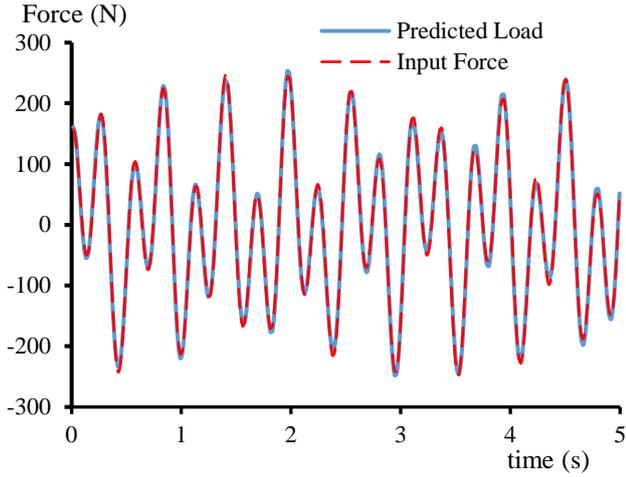


Fig. 3. Load Prediction – Force 2

An example of three force prediction is now presented. The following force input is used:

$$\begin{aligned}
 F_1 &= 100\sin(20\pi t) + 200\cos(8\pi t) \\
 F_2 &= 200\sin(20\pi t) + 100\cos(8\pi t) \\
 F_3 &= 150\sin(20\pi t) + 250\cos(8\pi t)
 \end{aligned} \quad (13)$$

over a time interval of 1 second. The geometry, inertial, stiffness and damping data is as given above for the two input force case. Again, 5 modes, including the two rigid body modes were used in the analysis. Results in the absence of regularisation are first shown below in Figs. 4 to 6. While there is no evidence of numerical instability, the predictions are poor at some points. As this is now a statically indeterminate system this might be expected.

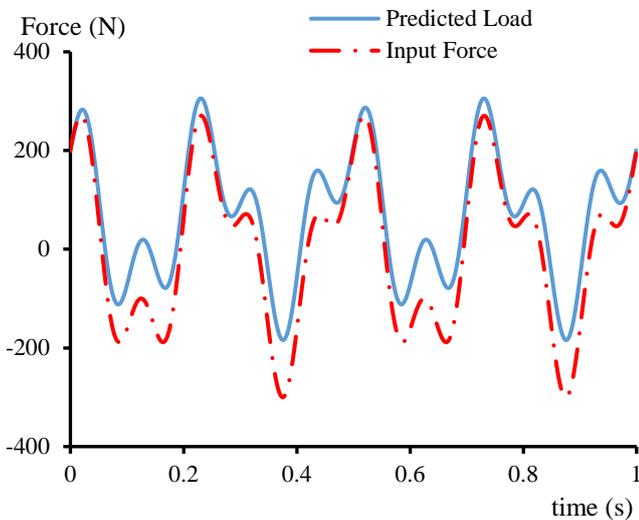


Fig. 4. Load Prediction – Force 1 – No Regularisation

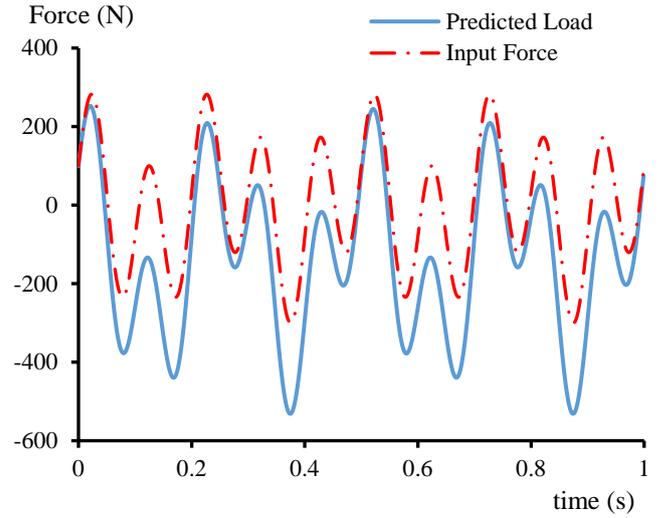


Fig. 5. Load Prediction – Force 2 – No Regularisation

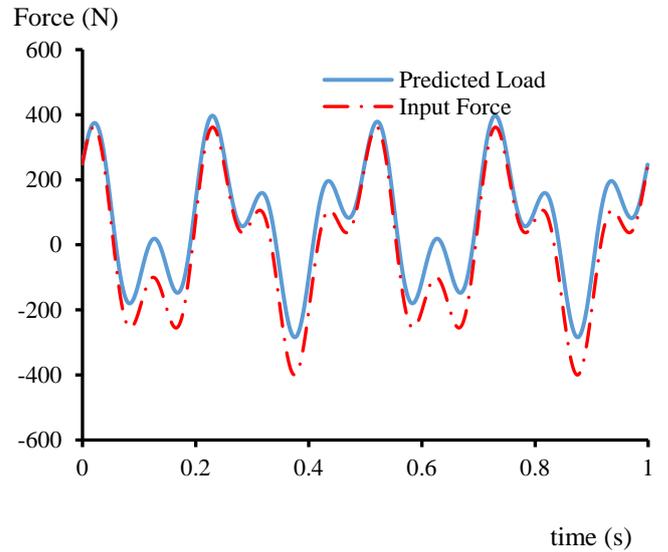


Fig. 6. Load Prediction – Force 3 – No Regularisation

Regularisation is now applied to improve the results. The approach adopted is through truncated singular value decomposition. Thus, \mathbf{B} in Equation (9) is decomposed as follows:

$$\mathbf{B} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}' \quad (14)$$

where:

$$\mathbf{\Sigma} = \text{diag}\{\sigma_1, \sigma_2, \dots, \sigma_n\} \quad (15)$$

with $\sigma_1 > \sigma_2 > \dots > \sigma_n$. Small values of σ_j can give rise to ill-conditioning of $\mathbf{B}'\mathbf{B}$. Regularisation is

carried out by solving Equation (10) with Σ given by:

$$\Sigma_k = \text{diag}\{\sigma_1, \sigma_2, \dots, \sigma_{n-k}, \dots, \sigma_{n-k}\} \quad (16)$$

for $k = 1, 2, 3, \dots$. The effect of this is to improve the conditioning of the matrix \mathbf{B} as k is increased. An L-curve is then determined by plotting

$$\log\|\ddot{\mathbf{Z}} - \mathbf{BF}\|$$

against

$$\log\|\mathbf{F}\|$$

For $k = 1, 2, 3, \dots$. The resulting curve is shown in Fig. 7 for k from 1 to 20. An appropriate value of k will correspond to the corner of the L-curve, and in this case, $k = 8$ was identified by determining where the radius of curvature of the L-curve was smallest.

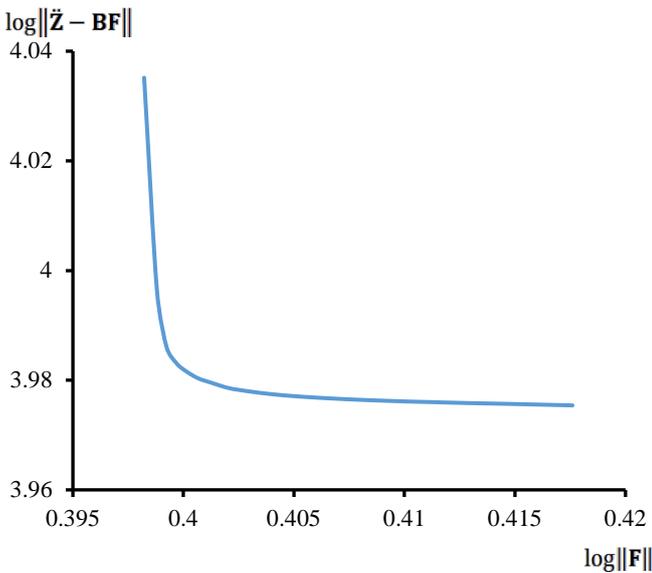


Fig. 7. L-Curve for Load Prediction with 3 Forces

The effect of regularisation is then shown in Figs. 8 to 10 and show that the force predictions are improved by the regularisation process.

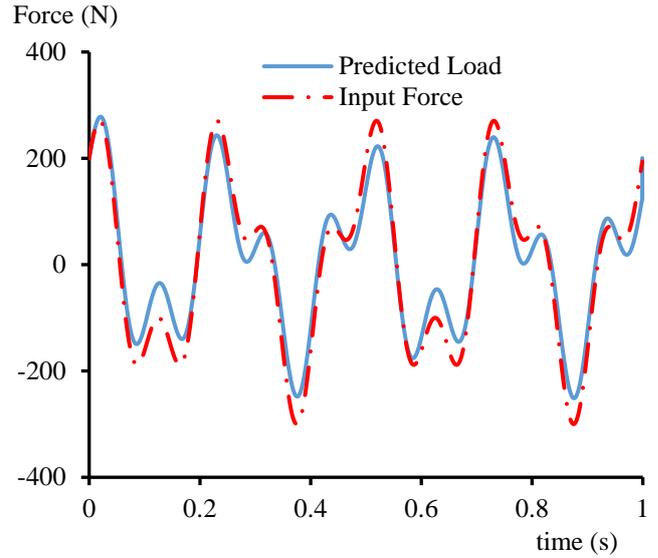


Fig. 8. Load Prediction – Force 1 – With Regularisation

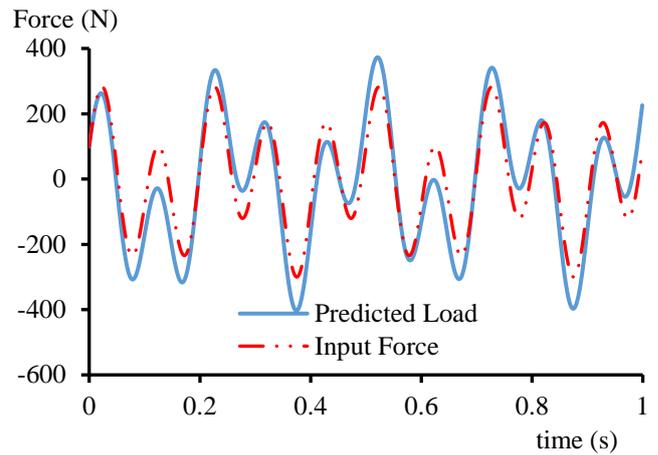


Fig. 9. Load Prediction – Force 2 – With Regularisation

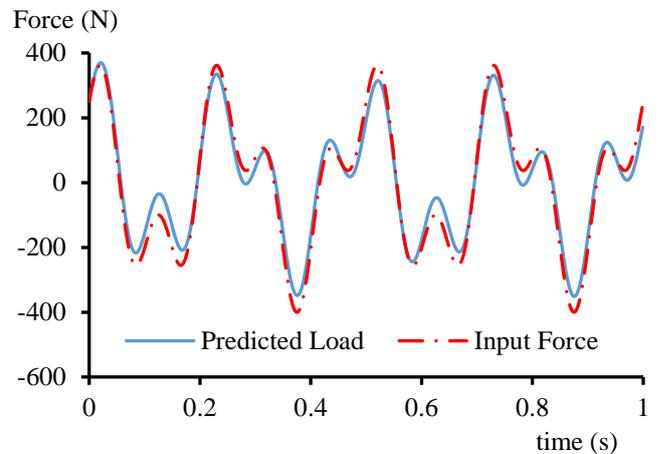


Fig. 10. Load Prediction – Force 3 – With Regularisation

2.4 (b) Frequency Domain Prediction

Two force prediction for input forces given by (12) is now carried out in the frequency domain over a time interval of 5 seconds. No regularisation has been applied. The results are shown below in Figs. 11 and 12, and as in the time domain case, the load prediction is good.

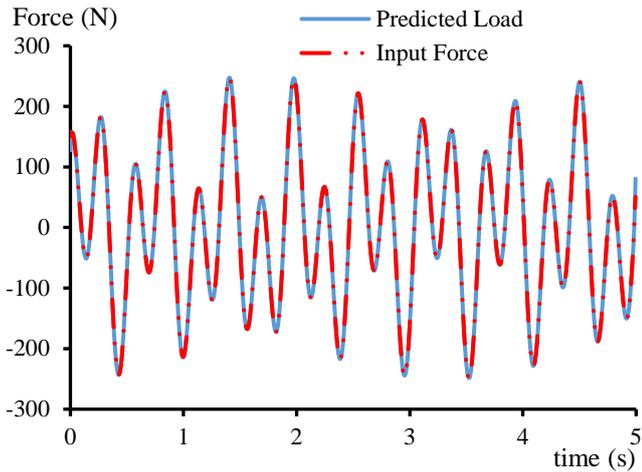


Fig. 11. Load Prediction – Force 1

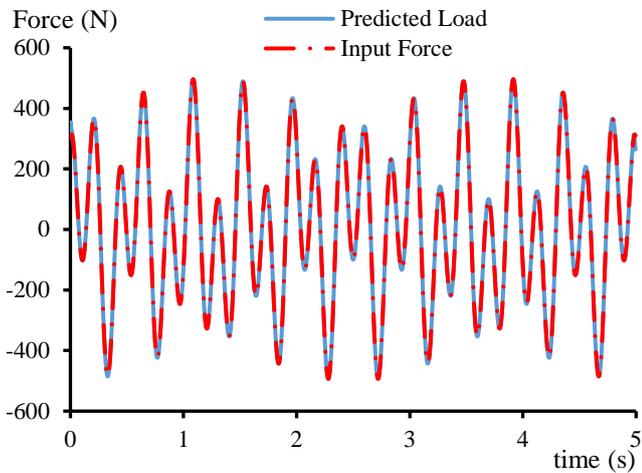


Fig. 12. Load Prediction – Force 2

Three force prediction for input forces given by (13) is now carried out in the frequency domain over a time interval of 1 second. Without regularisation, significant numerical instability occurs, and this can be attributed to a very high condition number when solving Equation (10) for zero frequency. This might be expected because of the static indeterminacy of the problem. It was found that an effective form of regularisation was to obtain an L-curve for the first non-zero frequency and then choosing the regularisation parameter α to correspond to the corner of that L-

curve. This regularisation parameter was then applied for all frequencies. The results of this regularisation process are shown in Figs. 13 to 15 and results in good agreement between input force and predicted force.

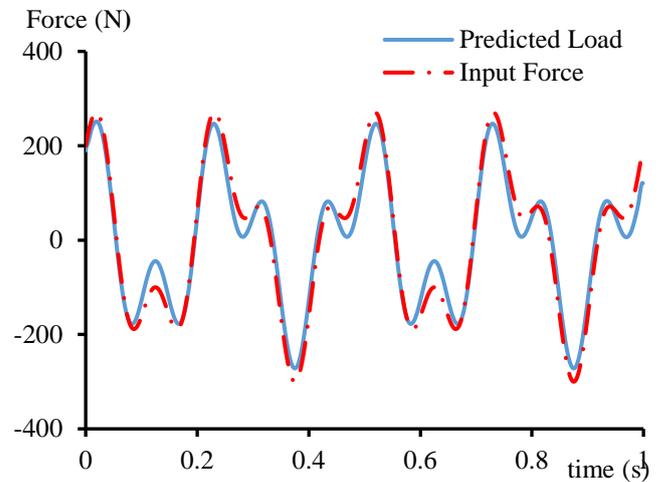


Fig. 13. Load Prediction – Force 1

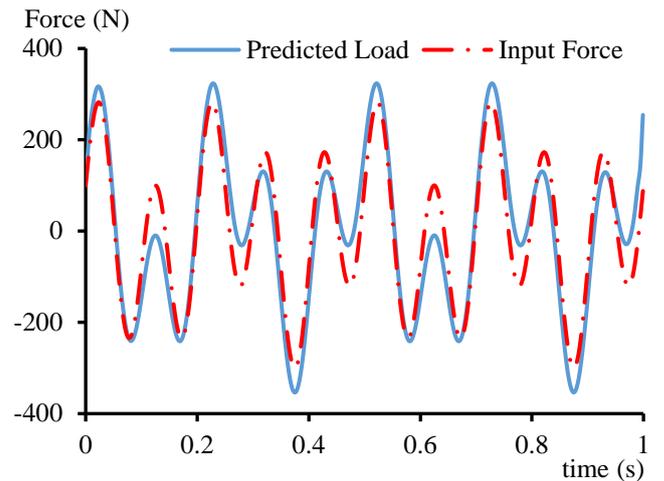


Fig. 14. Load Prediction – Force 2

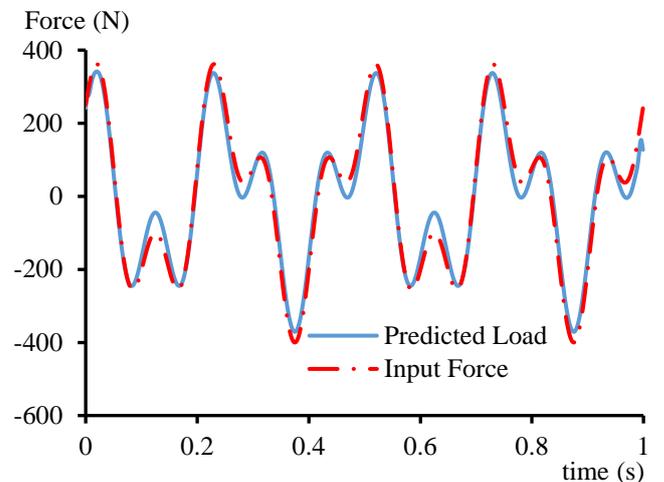


Fig. 15. Load Prediction – Force 3

3. GAUSSIAN PROCESS REGRESSION

In this Section, the three loads prediction problem considered in Section 2 is tackled using Gaussian Process Regression [15]. The process works by having a training dataset which contains observed response data; in this case the applied forces, and the input which is taken as accelerations. The algorithm used for this project was developed by Rasmussen [17]. Figs. 16 to 18 represent the predicted loads generated using Gaussian Process Regression at the three locations and are compared with the original input forces used to generate the acceleration response. It may be seen that the agreement between input and predicted forces is very good. Additionally, confidence intervals for the predictions are shown in green. These indicate less confidence in the predictions for Force 2 than for Forces 1 and 3. It is interesting to note that Force 2 predictions were poorer than those for Forces 1 and 3 when using the structural modelling approach in Section 2.

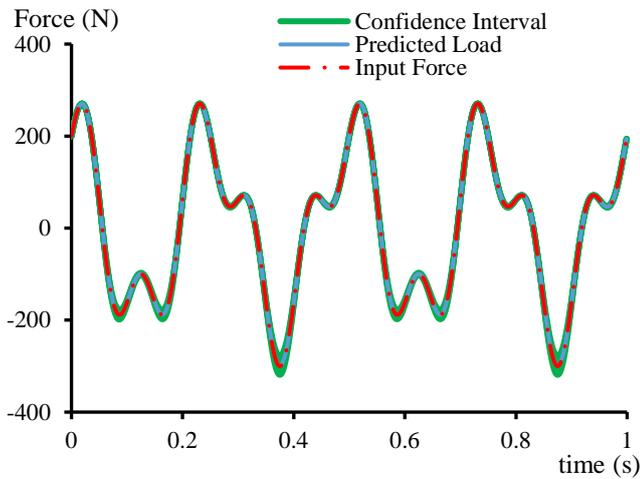


Fig.16. Load Prediction – Force 1

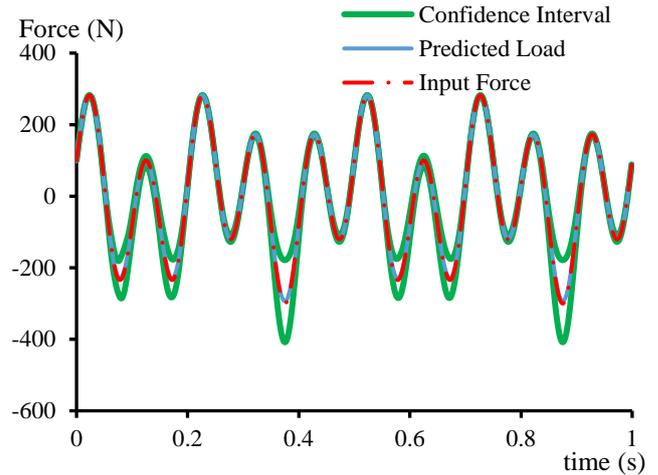


Fig.17. Load Prediction – Force 2

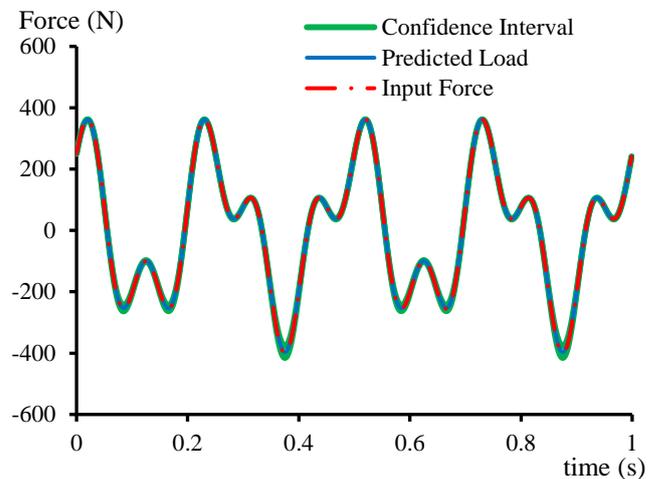


Fig.18. Load Prediction – Force 3

4. CONCLUSIONS

In this paper, two kinds of approaches have been presented for loads prediction on a guided weapon in air carriage, namely, a structural modelling approach and Gaussian Process Regression. The structural modelling approach may be sub-divided into two methods, based on time domain and frequency domain analysis respectively. For the illustrative example given, each method has been shown to be successful, with Gaussian Process Regression giving the best predictions. Each approach has its own strengths and drawbacks.

The structural modelling approach relies on a model of the missile, which can be provided by Finite Element Analysis or Structural Testing data. As a consequence, assumptions could be made

which may not be warranted, for example with regard to the attachments of the missile to the aircraft. Both time domain and frequency domain techniques have been demonstrated to be effective. In both cases, regularisation was applied. It was interesting to note that in the time domain case, there was no evidence of numerical instability without regularisation, but regularisation via Singular Value Decomposition could be used to improve the predictions, while in the frequency domain case, regularisation was essential to avoid numerical instability. The frequency domain approach may be more efficient as it involves the solution of a series of small least squares problems, rather than a single large one as in the case of the time domain approach. The structural modelling approach can use sensor inputs from accelerometers, gyros or strain gauges – potentially either from the missile or carrier aircraft. Although, only attachment loads have been considered here, aerodynamic loads could potentially be included also, augmented by appropriate assumptions about the aerodynamic load distribution along the length of the missile.

Gaussian Process Regression is founded on the training data used, and hence makes no explicit modelling assumptions. The method can potentially use a wide range of sensors and input variables from both missile and carrier aircraft, ie not only from accelerometers, gyros or strain gauges, but also aerodynamic data such as dynamic pressure readings, flight Mach number, angles of incidence and sideslip, if available. The method also generates a confidence interval providing an indication of reliability of the predictions. There are drawbacks – for example, the training phase can be computationally expensive and may need to account for missile location on aircraft and effects of different store configurations.

Beyond the study reported in this paper, a number of other studies have been carried out and issues addressed in this research. These include theoretical studies looking at a range of input load types including effect of frequency and effect of noise in accelerometer measurements. The methods have been applied in two experimental studies for the two force inputs case and one of these studies is reported in [15]. In the structural modelling approach, the sensitivity of the predictions with

respect to the structural model has been investigated – in particular the impact of using a simple theoretical model, rather than a model based on test measurements. Some investigation of the effect of modelling assumptions has also been carried out – in particular, the effect of assuming a rigid launcher in three load input studies, which introduces an additional constraint. The impact of drop-out of a sensor signal has also been investigated.

Whichever approach is chosen, the ultimate aim is for loads predictions to be used to estimate fatigue damage accumulation. In any practical implementation of a loads prediction methodology, efficiencies may be achievable on the basis that fatigue damage accumulates at some determined rate below a threshold level of force magnitude and the monitoring system is only triggered beyond this. In practice, it will also be necessary to extending the method to handle loads and responses in more than one plane.

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