Developing independent learners in maths using threshold concepts: transitioning from A-level to first year university maths

Abstract

One of the key roles of a university is to create independent learners. To achieve this students need guidance on how to learn independently, help to build their academic confidence, and direction on learning the discipline. This article investigates ways in which teachers can develop independent learners, specifically in the discipline of maths, with the focus on activities for first year university students. The activities proposed can be implemented within a standard university setting. Specific barriers to student learning are the threshold concepts of the discipline. Teaching students how to recognise and work through the threshold concepts can help them to develop as independent learners as well as speed up their learning progress. The following activities have been explored within a first year university maths environment:

1. A group presentation to enable students to get to know each other and to start developing their communication skills.
2. Students explain a threshold concept to their peers.
3. Students create their own maths problems and peer-review problems.
4. Weekly guidance on tasks expected of students to show them how to learn independently.

By making several changes to my teaching practice and assessments it has been possible to increase the focus on developing independent learners. Providing a more diverse range of activities has resulted in increased student satisfaction.

1 Why do we want to develop independent learners?

We are at a unique time in history; the way education is being delivered is rapidly changing due to the Covid19 pandemic. Rather than university education stopping during the pandemic it has been provided using online learning. The widespread use of the internet and personal computing devices, the vast amount of knowledge and educational material on the world wide web, and now the mass use of these tools for education during the pandemic means that we can harness this opportunity to change how we teach and learn.

We have the ability to make our teaching work as an individual experience for the learner, the key question is how. This article addresses ways in which we can encourage individual learning by developing students who are independent learners able to take full advantage of the knowledge and learning materials online.

A key question when changing to more online teaching is what is the purpose of face-to-face activities at universities? [Barnett, R., 2004] Do we need students to be in the same building for them to learn? This is a scary question to ask of teachers because it potentially challenges our jobs but it is an important question to ask. I believe that we can improve the way we teach using individualised online learning while, at the same time, still require the need for human interaction to explain, encourage, praise and provide forward signposting. When learning there is the need for discussion of the key concepts and the sticking points. The way we teach now, in lectures with large numbers of students, is not the best way of allowing students time with the 'expert' to overcome their personal bottlenecks. Although every student is different and requires individualised teaching, there are certain concepts that many students struggle with that the teacher can focus on to enable student progression. These are called the threshold concepts.

A key for developing independent learning (or any learning) is that students need the 'will to learn' [Barnett, R., 2007]. As Barnett says "*Our job as teachers is to sustain and develop the students will to learn*" and "*To provide diverse activities that enable learners at 'whatever level' they are*". I report on my investigation into the use of a range of diverse activities, that are based on developing threshold skills and concepts, and to develop independent learners who can communicate and work collaboratively in maths.

2 Guiding students to be independent learners

Guiding students to be independent learners does not mean leave them to make their own notes, learning plans and to study what they have been told to study. It means showing them what they can do to learn how to study independently. In their first year students need more direction and help in order for them to learn how they can study on their own [Alcock, L., 2012, 2018a]. It is essential to 'educate' the students how to learn independently

so that they can move successfully into the second year of their degree where concepts get harder. The better prepared they are in the first year then the more likely the student is to complete their degree successfully.

Alcock, L. [2018b] starts from the premise that the "*lecturer guides and orchestrates student activity*" and uses four practices that provides structure for the students' learning: routine, announcements, breaks and notes.

1. Have a clear weekly routine showing the student what is expected of them.
2. Be well organised and tell the students, e.g. put up 'handwritten' announcements each lecture.
3. Two minute breaks in lectures every 25 minutes: work to their concentration span and allow reflection time.
4. 'Gappy' notes given out each week (4 pages per lecture plus problem sheet) – this shows the students what they need to learn (gives the 'feed forward') and they can fill in the gaps themselves. The lecturer is giving them a start to their note-taking.

Providing a clear structure develops the students' own study routines and habits and teaches them how to become an independent learner.

2.1 Using online learning to develop independent learners

Some of the noticeable impacts of working significantly online during the Covid19 pandemic is that students are saying they are lonely, that they get screen fatigue and that they find it more difficult to concentrate so the workload seems overwhelming. Academic isolation should be considered here. It is the role of universities to guide the student through the paradigm of the discipline making sure that they have properly understood the key concepts and have got through the threshold concepts so can progress to the next level. It is equally important for the universities to provide the opportunity for the student to interact with others to discuss the academic subject and to provide an academic community, otherwise what is the purpose of the university. The students learn by being part of that academic community and it is even more important to provide that during online teaching through as much live online interaction and group activities as possible.

3 Teaching to the threshold concepts

Meyer, J., and Land R., [2003, 2006] defined a threshold concept as knowledge that is "*central to the mastery of their subject*", and can be transformative, counter-intuitive and troublesome [Perkins , D., 1999]. A set of threshold concepts and skills for first year maths are shown in Table 1. The threshold skills are taken from Budd, C. [2020] and the threshold concepts are those that are from Meyer, J., and Land R. [2003] and from my own experience.

TABLE 1: THRESHOLD CONCEPTS AND SKILLS IN MATHS FOR FIRST YEAR UNIVERSITY STUDENTS

THRESHOLD CONCEPTS		**THRESHOLD SKILLS**
Transposing equations		Procedural fluency
Calculus: Partial & implicit differentiation Deciding the method for solving integration problems (particularly by substitution and by parts)		How to set up equations for a problem
Complex numbers and the concept of 'mathematical space'		Mathematical proofs
Transforming co-ordinates		Communicating maths (to mathematicians and non-mathematicians); use of precise language – written and verbal skills
Limits of functions Convergence and divergence of series Approximating functions		Creative problem solving Applying the maths to real problems

Land, R., Cousin, G., Meyer, J.H.F. and Davies, P. [2005] advocate a focus on threshold concepts and Breen, S., and O'Shea, A. [2016] suggested that *"placing an emphasis on the threshold concepts involved in a course can enable teachers and students to focus on what is fundamental to the study and mastery of their subject*". Cousin, G., [2006] says that "*a focus on threshold concepts enables teachers to make refined decisions about what is*

fundamental to a grasp of the subject they are teaching. It is a 'less is more' approach to curriculum design."

All too often we use exams and tests to check the student's knowledge. Getting the student to discuss and talk about the concept could be a better way of assessing. Research from China [Yiming, C., 2019] showed that increasing student participation, student-teacher and student-peer interactions enables more students to understand threshold concepts and so to progress quicker. This can be achieved by increasing the discussion and presentation based activities [Zhao, W., Mok, I. A. C., and Cao, Y., 2016]. It does not mean just asking questions in the class, it is about encouraging students to talk to each other, to the tutor and to be confident giving solutions on the board. Participation needs to be gently nurtured in the students to build their confidence. Learning is an emotional thing [Cousin, G., 2006]. If the student feels nervous, anxious, shamed, scared, humiliated then they will stop engaging. Maths students are particularly reluctant to talk in class if it is done in the wrong way. Slowly building the students communication skills in a compassionate, encouraging environment creates a safe space where students can learn without fear or failure.

It could be said that providing step-by-step guides and modifying the teaching to match the students' misconceptions is spoon-feeding the students and is not developing their independence. But what is independent learning? Allowing the students to stay with their misconceptions is not teaching the students. The role of the teacher is to identify where the students get stuck and get them through these threshold concepts by providing the students with the mathematical tools and skills. As the students learn how to identify what concepts are preventing them moving forwards and learn how they can move through these thresholds then they become independent learners. The role of the teacher then becomes that of a sign-poster rather than a spoon-feeder of the knowledge. Signposting requires feeding forward to show the students what they need to know next and directing the students to where to find the information and showing the students how they can assess their own progress. Continuing with a single plan of teaching regardless of what the students are learning is the spoon-feeding.

The lecturer must do work to break down the topics into steps. I have found that the students ask for the mathematical steps – they are instinctively wanting this. This is 'decoding the discipline' and can be difficult for the expert because some of the steps have become instinctive and tacit knowledge for them so they have forgotten what they need to tell the students [Houston, K., 2009, 2018]. There is a reluctance by lecturers to provide these steps because they want the students to work them out for themselves, however, it is important to recognise that the steps give the students their first taste of the task they need to do. As they practise these steps they begin to see the patterns and gain the procedural fluency. This allows them to cross the threshold. The 'laid out steps' helps the students to cross the threshold. Without this help they may take longer to reach the understanding. The aim is to get them to the threshold understanding as quickly as possible so that they can start using the concept, solving problems and go to the next level. 'Feed-forward' allows the students to see where they can go next once they cross that threshold. So rather than let the students struggle to find the steps they need, the lecturer provides the steps and then the student uses them in the exercises.

This way of designing what is done in the classroom is backward design [Wiggens, G. and McTighe, J., 2005]. It identifies the desired outcomes and it identifies those students who are crossing the threshold concepts and developing the right skills. This is a good way to find out the individual understanding but can be time consuming for the tutor so we generally do this via exams. Exams are not necessarily the best way to find out whether the student has crossed the threshold because they could have just learned how to solve that one type of problem. What we want to assess is whether they can apply their knowledge in new ways. This creativity is difficult to test in maths. Testing it with new problems each year can be a restriction because there may be a few particular problems that are good at testing the skill and then once used cannot be reused in following years.

4 Changes to my teaching practice

To implement these ideas for developing independent learners using threshold concepts I modified my teaching practice and changed several of the first year maths assessments.

4.1 Changes to assessment activities

I focused on assessments where the students have to communicate the maths. The first activity was a group presentation on the history and application of a calculus topic. The second activity was for the student to explain a threshold topic to their peers. The third activity was for students to create their own maths problems. All activities were easily introduced within a typical university setting. The aim of these assessment activities was;

1. to promote discussion and understanding of the threshold concepts,
2. to increase student-teacher and student-peer interaction,
3. to build the students' confidence to talk about maths.

In addition, the activities were designed to allow students to extend their knowledge beyond what was taught in the lectures particularly for those who had already covered the topic at A-level. The activities added variety to the assessment tasks making the course more interesting for the students and promoting engagement.

4.1.1 Activity 1: Group presentation

Students prepared and presented a group talk on a calculus topic of the students' choosing that had to include an historical element, a mathematical explanation and an application. The assessment was held during the first term to provide an opportunity for students to get to know other students, particularly during the pandemic lockdown, and to establish early on that communication is important. It was the start of building their confidence to talk about maths. The presentations were given within a small group to reduce the anxiety for students. Guidance was given to the students about how to prepare and deliver a presentation and on how to work together. In terms of teaching time, setting the activity took less time than a standard assignment of set questions but it took more time organising groups than anticipated and required more discussions with students. This interaction with the students had the added benefit of an additional opportunity to get to know them individually. Not all groups were successful though and some students did end up working on their own (where the other students did not engage).

Sixty four students were involved in the activity with 75% of them making a presentation. They were split into groups of up to six students and each group presented within their

small group tutorials. Student feedback from the activity was collected by the tutors at the end of each presentation session. In addition, I asked tutors to give me their views on the activity. Twelve students said they enjoyed the task, particularly researching the topic and the history of the topic. Nine students said they found it helped their maths and motivated them. Several of the presentations were on topics that were beyond what was taught in the lectures with one student saying that it was more useful than a numerical assignment. Interestingly, the students also said that the activity made them appreciate that working in groups and making presentations are important skills, with one student saying that it helped them in "overcoming shyness". The main difficulty the students mentioned was communicating as a group due to lockdown restrictions preventing face-to-face meetings and they would have liked more time to work on the task. The feedback from the tutors was that they "enjoyed listening to the presentations", it was a "useful exercise" and that it helped the students' confidence. About half the tutors who commented said the level of maths was as expected and about half said it extended the students' maths knowledge. Overall, the activity engaged the students, developed their confidence to talk about maths and for some it extended their understanding of the maths.

4.1.2 Activity 2: Explain the maths

This activity required the students to study a concept from a threshold topic and then explain that concept to their peers within their small group tutorial. Students picked a theorem or formula from the threshold topics and prepared an explanation that included a proof or derivation with a description of the type of proof used. The objective was for the student to focus on the proof and in that way extend their understanding of the topic as well as develop a threshold skill. The activity had the added benefit that the students learned from listening to their peers. It also provided an opportunity for the tutor to find out where misconceptions existed. The assessment was held in the second term and built on communication skills developed in the group presentation. Seventy per cent of students participated in the assignment.

The students varied in their feedback; some found it challenging, some enjoyable and some found it easy. The majority of tutors felt it was a useful exercise and that the students did improve their presentation skills compared to the group presentation. Some students

extended their knowledge beyond what was in the lectures while others did not include a proof or derivation and did not extend themselves mathematically. It was apparent to the tutors that some students did not understand what they were presenting. This highlighted where students did not understand a threshold concept.

There are several changes that can be made to this activity to improve it. Due to the lockdown the explanations were made online as presentations rather than using a whiteboard. This was not the intention of the activity so the first change is to require the students to use the whiteboard when explaining the maths. This will develop the students' confidence in solving problems at the whiteboard and to talk about maths in front of their peers (as shown by Yimming, C. [2016] to aid learning progression). The second change is to provide more in-depth instruction to the students on different types of mathematical proof. As a threshold topic, some students required additional help to understand the task.

4.1.3 Activity 3: PeerWise - create maths problems

The third activity was designed to promote peer-interaction and develop threshold skills. It required students to create their own maths problems and solutions and then to solve and comment on the problems that their peers created. An online application called 'PeerWise' [Denny, P., Hammer, J., Luxton-Reilly, A., and Purchase, H., 2008] was used for students to create and store their questions. As the teacher, I could track how frequently the students participated in the activity enabling evaluation of student progression and engagement. As Dreyfus, T., [1991] said "Students can use the standard procedures but struggle to use them in a flexible manner". This activity addressed this and was a way for the students to practise the threshold skills of procedural fluency, setting up equations and using precise language.

I set up PeerWise as a formative activity to encourage participation. However, the opposite happened. Without the reward of achieving grades the students did not participate. In future, this activity will be a graded assignment to ensure that the students engage and practise creating their own maths problems.

4.2 Routine changes to my teaching practice

The transfer to online teaching due to the pandemic required changes in teaching practice. The student feedback at mid-term resulted in two important requests: to get personal progress updates and to have live lectures. In response to the students I introduced changes that also focused on independent learning. Once these changes were put in place the student feedback changed to be more positive; there were no negative comments in the end-of-term feedback.

4.2.1 Providing structured guidance

To support the students to become independent learners I introduced the practice recommended by Alcock, L., [2018a] to provide structure and organisational advice to students on a weekly basis. The weekly announcements that I was previously making became more focused on directing the students to the tasks that they could do each week to develop their own learning skills and guidance for their own study routines.

4.2.2 Weekly topic quizzes

To enable students to see their personal progression weekly online topic quizzes were introduced. Before the lockdown, the students brought in written solutions to set questions into the tutorial which the tutor then graded. Moving to online teaching took away that weekly interaction with the tutor which, although brief, was still a point of personal feedback for the student. The introduction of a weekly short quiz, some formative and some summative, allowed the students to see their progression and also practice exam style questions. The quizzes also allowed the teacher to see individual student progression and identify those students that were not participating. These topic quizzes will continue when face-to-face teaching recommences since they provide structure to the weekly maths practice and tracks progression.

4.2.3 Live online lectures

When the lectures moved online at the beginning of lockdown the lectures were pre-recorded. Many students like this flipped learning approach but some students commented that they felt isolated. Live lectures were then put in place that were recorded so that the

students had a choice of watching live or recorded. Feeling part of a learning community, and particularly developing that community for first years, is an important role of a university whether that is online or on campus. Live online lectures provided the students with an opportunity to ask questions, to hear questions from their peers and to feel part of the academic community.

5 Conclusions

Assessments that required student-interaction were trialled as part of an adjusted programme of maths activities to develop independent learners in a first year university environment. By making a few changes to the teaching practice, including weekly guidance and changing several assessment activities, it has been possible to increase the focus on developing independent learners and on teaching to the threshold concepts. Comparison of student feedback before and after the changes showed that student satisfaction improved. It is too soon to know the impact of the changes on students' ability to learn independently.

In future, the activities that have been trialled will be continued; particularly the group presentation assignment to develop communication skills, weekly announcements providing organisation and routine, topic quizzes to provide personalised progress, and step-by-step teaching in live lectures to master the threshold concepts. The assessment activities will be improved. Using PeerWise to develop threshold skills will be tried again as a summative assignment. The 'Explain the maths' assignment will be modified so that students have to use the white board to solve a maths problem with the aim to increase their confidence to do maths in front of their peers.

This work has shown that a focus on developing independent learners requires a multi-faceted approach, including the design of the assessments, the focus on threshold concepts in lectures and the teacher providing habit-forming routines and forward-signposting to the students. Such a focus can increase students willingness to learn and speed up their progress by tackling threshold skills and concepts.

6 References

Alcock, L., 2012, “How to study for a mathematics degree”, Oxford University Press

Alcock, L., 2018a, Slides from Lara Alcock lecture on “Tilting the Classroom: Engaging Students in Large Lectures” https://warwick.ac.uk/fac/sci/statistics/news/workshops/workshop/alcock_warwick_2018.pdf

Alcock, L., 2018b, “Tilting the classroom”, London Mathematical Society Newsletter, 474, pp 22-27

Allan, C., Campbell, C., and Crough, J., 2019, “Blended learning designs in STEM higher education”, Springer

Barnett, R., 2004, “The purposes of higher education and the changing face of academia“, London Review of Education, 2, 1, pp 61-73 (13)

Barnett, R., 2007, “A will to learn: being a student in an age of uncertainty”, Open University Press

Breen, S. and O’Shea, A., 2016, “Threshold concepts and undergraduate mathematics teaching”, PRIMUS (Problems, resources and issues in mathematics undergraduate studies), 26, 9, pp 837-847

Budd, C. , 2020, online interview with Chris Budd on knowledge transfer in maths https://ima.org.uk/15134/interview-with-professor-chris-budd-obe-cmath-fima/

Cousin, G., 2006, “An introduction to threshold concepts ”, Planet, 17, pp 4-5

Denny, P., Hammer, J., Luxton-Reilly, A. and Purchase, H., 2008, ICER '08: Proceedings of the Fourth international Workshop on Computing Education Research, pp 51-58 https://doi.org/10.1145/1404520.1404526

Dreyfus, T., 1991, “Advanced mathematical thinking processes” in Tall, D. (Ed.) “Advanced mathematical thinking”, Dordrecht: Kluwer Academic Publishers, pp 25-4

Houston, K., 2009, “How to think like a mathematician”, Cambridge University Press

Houston, K. 2018, Slides from Kevin Houston lecture on “The Transition To Thinking Like A Mathematician” https://warwick.ac.uk/fac/sci/statistics/news/workshops/workshop/transitions.pdf

Land, R., Cousin, G., Meyer, J.H.F. and Davies, P., 2005, “Threshold concepts and troublesome knowledge: implications for course design and evaluation”, in Rust, C. (Ed.), “Improving student learning diversity and inclusivity”, Oxford: OCSLD, pp 53-64

Meyer, J. and Land, R., 2003, “ Threshold concepts and troublesome knowledge: linkages to ways of thinking and practising within the disciplines”, in Rust, C. (Ed.), “Improving student learning – ten years on”, Oxford: OCSLD, pp 412-424

Meyer, J. and Land, R., 2006, “Overcoming barriers to students understanding: threshold concepts and troublesome knowledge”, Routledge

Perkins, D., 1999, “The many faces of constructivism”, Educational Leadership, 57 (3)

Wiggens, G. and McTighe, J., 2005, “Understanding by design”, Association for supervision and curriculum development

Yiming, C., 2019, Article by Yiming Cao on “Investigation of mathematics teaching and learning in China” https://researchfeatures.com/wp-content/uploads/2019/08/Yiming-Cao.pdf

Zhao, W., Mok, I. A. C. and Cao, Y., 2016, “Curriculum reform in China: Student participation in classrooms using a reformed instructional model”, International Journal of Educational Research, 75, pp 88–101