Title: All models are wrong, but the *critical power model* is useful.

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This is a letter to the editor in response to this paper:

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We thank Drake et al. (2023) for an interesting paper in this issue of the European Journal of Applied Physiology. The authors addressed the long-standing question of which mathematical model best describes the relationship between the intensity of exercise, and the duration such intensities can be sustained before task failure (P-D relationship). The authors argued that, over a wide range of intensities, the P-D relationship is better fitted by a power law, instead of the more commonly used hyperbolic model. Specifically, the key argument presented by Drake et al. was that, for the 'central' part of the P-D, there are no real differences between power and hyperbolic functions. However, both at very high intensities and thus short durations, typically <2 min, and particularly at lower intensities with extended durations, typically >45 min, the hyperbolic model of the P-D relationship fails to predict the observed values. The power law function, therefore, is suggested by the authors to provide a better fitting of the P-D relationship over a wide range of intensities and durations.

Power functions are characterised by a constant (referred as 'speed' parameter) and exponent (referred as 'endurance' parameter). These parameters, however, appear to lack a defined physiological significance. For example, the 'endurance' parameter has been described as 'an expression of endurance capability' (Vandewalle 2018), but it is not clear what this means, or its physiological significance. The hyperbolic P-D model is also described by two parameters: the asymptote of the hyperbola and the curvature constant. Crucially, and in contrast to the power law model, these parameters appear to describe a physiological function, and dictate key physiological responses to exercise. The asymptote of the hyperbola is termed critical power (CP) and has been argued to represent the boundary of the heavy and severe domains of exercise. However, Drake at al. (2023) argued that such asymptote is an artefact derived from fitting a hyperbolic function to the P-D relationship. Irrespective of whether CP is a mathematical artefact or not, we would like to stress that fitting the P-D relationship with a hyperbolic function results in quantitatively distinct physiological responses to exercise performed above vs. below CP. Indeed, CP represents a key feature of aerobic function, with implications for both endurance performance and health (Goulding and Marwood 2023). The second parameter of the hyperbolic model is the curvature constant, termed work prime (W′), which represents exercise capacity for exercise at intensities above CP. Physiologically, W′ has been linked to the accumulation of metabolites derived from an increased contribution of substrate level phosphorylation. One such metabolite accumulating during supra-CP exercise is inorganic phosphate, which contributes to the rapid development of fatigue and increases until a "peak" (i.e., limiting) concentration of inorganic phosphate is reached, and exercise is terminated (Goulding and Marwood 2023).

We acknowledge, however, that the hyperbolic model to describe the P-D relationship is limited for long durations (e.g. >45 min) and short durations (<2 min), as argued by Drake et al. (2023) and others (e.g. Vandewalle 2018). However, we highlight the need to integrate physiologically relevant parameters to mathematical functions describing the P-D relationship. For example, there is evidence that CP, as well as other markers of endurance performance, do not remain static during exercise, but deteriorate during prolonged exercise (Jones 2023). The ability to endure such changes over time has been recently described as durability, or resilience, and suggested to be a key parameter of endurance performance (Jones 2023). Indeed, a mathematical model combining the running analogous of CP, critical speed, and a marker of durability, has been shown to improve the accuracy of predictions of marathon performance compared to a purely hyperbolic function (Smyth et al. 2022). The results from this study suggest that it may be plausible to resolve this conundrum by incorporating durability to traditional markers of endurance performance (Jones 2023). It is unclear whether adjusting for the decline in CP and/or W′ during prolonged exercise results in a comparable fitting to that observed by a power law function. However, such approach may result be physiologically sound, whilst producing reasonably accurate mathematical models of performance.

Our goal with this letter was to highlight the need to consider the physiological significance of the parameters of mathematical functions describing the P-D relationship, instead of only paying attention to how well the mathematical model fits the data. In summary, paraphrasing the famous quote by George Box: All models are wrong, but the CP model remains a useful approach to explain the P-D relationship, due to the physiological significance of its parameters, and its ability to demarcate the heavy and severe domains of exercise.

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