State Ownership, Credit Risk and Bank Competition: A Mixed Oligopoly Approach

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State Ownership, Credit Risk and Bank Competition: A Mixed Oligopoly Approach*

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Abstract: Recent events have led many governments to buy equity in banks leading to a situation of mixed oligopoly in banking markets. We model such a case where a partially state-owned bank competes with a private bank in collecting deposits. The government is purely a welfare maximiser while the private bank maximises profits. Both banks face risks in the loan market. We show that if the risk of default is sufficiently high and there is limited liability, then the state-owned bank tries to mitigate depositors’ losses by mobilising less deposit leading to contraction of aggregate deposit. This contradicts the standard mixed oligopoly results in the literature.

Keywords: Banking, mixed duopoly, default risk

JEL classification: G21, L13, L33.

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1 Introduction

During the recent financial crisis policy makers around the world have turned their attention to state ownership of banks. Ironically as recently as a decade ago there was an apparent consensus in favour of bank privatisation, and governments in developing countries were compelled to divest a significant proportion of the equity of the state-owned banks. This was seen as a cure for non-performing assets, a chronic problem that was associated with public sector banks. But in a dramatic reversal of fortune, it is now the private sector banks that are at the centre stage of recent meltdown and have even been blamed in the media for their pursuit of profit. While economists are divided in their prescription for a solution, policy makers have rushed to bail out distressed private banks, and even (significantly) nationalised many of them. One immediate benefit of nationalisation is recapitalisation. But how will it affect bank competition in the deposit market and in the loan market? The banking literature has studied issues ranging from financial contagion (Allen and Gale, 2000), bank runs and liquidity crisis (Smith, 1991; Diamond and Rajan, 2005), to competition and risk-shifting (Boyd and De Nicolo, 2005); but it has mainly considered interactions among private banks, and therefore may not be adequately helpful in understanding interactions among public and private banks. This paper studies the outcome of bank competition in the context of nationalisation. Specifically we show that if credit risk is sufficiently high and there is limited liability, then a nationalised bank would try to protect depositors from potential losses by mobilising less deposit leading to contraction of industry deposit.

There is a separate body of work in industrial organisation known as mixed oligopoly that exclusively studies competition between public and private firms (De Fraja and Delbono, 1989; Matsumura, 1998). The underlying view of mixed competition is that state ownership provides an indirect means of regulating the conduct of private firms using market instruments rather than using administrative-judicial interventions like the US Anti-trust system. A general result of this literature is that the publicly owned firm will act more aggressively and have greater market share than the private firms; but
social welfare may not always rise, though in most cases will, especially if the public firm’s
degree of state ownership is optimally chosen. Our aim is to adapt the mixed oligopoly
approach to bank competition and see how state ownership affects bank interactions.
There is a significant difference between banks and ordinary firms. In the industrial
organisation literature firms mainly interact in the output market and have no or little
interactions in the input market. Banks, on the other hand, have to interact in both the
deposit (input) market, and the loan (output) market; moreover by lending to each other
they add another dimension to their relationship. Above all, unlike ordinary firms banks
have to deal with credit risk e.g. loan default risk and its knock-on effects on different
aspects of the banking business. In this paper we explicitly allow for such features which
distinguish banks from standard non-financial firms that are studied in the industrial
organisation literature.

Apart from the above-mentioned theoretical issues, there are good empirical reasons
as to why banking should be modeled using the mixed oligopoly approach. Despite
wide spread banking deregulation, entry of private and foreign banks has by and large
remained a subject of state control in many countries. In fact, state presence is a common
feature in banking systems all over the world. See Barth *et al* (2001) for evidence on
and India. It is, therefore, reasonable to expect that the banking industry in many
economies (especially emerging) resembles mixed oligopoly with strategic interactions
among banks occurring at many dimensions. There have been only a few articles that
have considered mixed ownership or non-profit maximizing behavior of banks. Purroy
and Salas (2000) studied competition between a private bank and a savings institution,
and Saha and Sensarma (2004) between a private bank and a public bank. Both articles
have demonstrated that the mixed duopoly approach can be useful; but they share a
common limitation of focussing only on deposit competition and not credit risk, which
we aim to overcome in this paper.

In modeling public banks, our first hurdle is to define the objective of a public bank.
Following the mixed oligopoly literature we assume that it maximises social welfare,
which is the sum of the payoffs earned by all participants in the banking industry, which
principally are banks’ shareholders, borrowers (who are essentially entrepreneurs seeking
funds for risky projects) and depositors. But by no means is this the only possibility.
One can add other objectives, specifically linked to loan or deposit market, that might
better describe the circumstances and motivation for state intervention in a particular
country.

Throughout our analysis we will consider only a duopoly setup. We model deposit
competition between a partially state-owned bank and a private bank. We introduce
a loan market and allow uncertainty in the form of default risk. The government’s
objective in this model is entirely social welfare maximisation and the private bank is
a profit-maximiser. Both banks have equity capital along with deposit liabilities. We
first consider the case where the banks bear the entire risk; they pay depositors the
due amount, even if the borrowers have defaulted. In this case the public bank acts
aggressively, and this leads to greater deposit mobilisation. Then we introduce limited
liability, which essentially puts the depositors at risk; if borrowers default, depositors lose
money as banks (like their borrowers) are protected by limited liability. We show that
the public bank in this case takes into account the depositors’ loss, and if the default risk
is significant it will act conservatively and collect less deposits. Though this will benefit
the private bank in their deposit mobilisation, aggregate deposits can fall below the pure
duopoly level (which corresponds to the case of competition between two private banks).
This result is quite opposite to what standard mixed duopoly models predict.

Standard mixed duopoly models, which usually focus on output competition, predict
more aggressive behaviour of the public firm largely because it puts a positive weight
on social welfare, and in the usual way greater output leads to greater welfare. This
argument largely holds for bank competition as well, if depositors do not face any risk
(due to either lack of limited liability or complete deposit insurance). But as has been
seen in recent crises, limited liability on the part of the entrepreneur-borrowers and
banks put depositors at risk. A social welfare oriented public bank takes into account
the potential loss that depositors might face when default occurs. If this risk is not small,
the public bank will try to restrict its deposit mobilisation below a profit-maximizing level. The overall outcome may be a contraction of total deposits.

Our result can be related to a well established literature that has studied the effect of competition on risk taking. Both the theoretical and empirical results are mixed, though policy makers tend to believe that competition has destabilising effects on risk-taking by the bankers. Boyd and De Nicolo (2005) provide an overview of this literature, and argues that the negative view of competition is largely based on the theories that consider mainly single market interactions, viz. the deposit market (see also Hellman et al, 2000). If a loan market is introduced, they argue, bank competition will reduce the interest rate on loans and thus reduce the risk of default. In our paper, the state owned bank has a similar effect on aggregate deposit as competition does in a setup of private banks. We show that if the probability of default is below a critical level, then the Boyd and De Nicolo (2005) type of positive effect emerges, but if the probability of default is above this critical level, then the negative effects may dominate.

The paper is organised as follows. Section 2 sets up the model of deposit competition with default risk in the lending side. In section 3 we consider the benchmark case of unlimited liability where the depositors are not exposed to the effects of credit risk. Section 4 explores the case of limited liability, which shifts the risk onto depositors. Here we derive our main results on the effects of state ownership. Section 5 concludes.

2 The Model

The public bank is indexed 0, and the private bank indexed 1. Depositors earn interest rate \( r_D \) by the following rule

\[
r_D = b(D_0 + D_1), \quad b > 0.
\]

The public bank is jointly owned by the government and a private partner, and the choice of the volume of deposit is made by the bank’s board of management consisting
of a government representative and the private partner. It maximises the following objective function

\[ Z = \theta SW + (1 - \theta)\pi_0. \]  

(1)

where \( \theta (\theta \in [0, 1]) \) is the degree of public ownership.\(^1\) The resultant deposit choice gives rise to the public bank’s reaction function.

There is an alternative approach to public firm suggested by Fershtman (1990), in which the same weights (\( \theta \) and \( (1 - \theta) \)) can be applied to a fully public bank’s and a fully private bank’s reaction function to arrive at the partially public firm’s reaction function. Saha and Sensarma (2004) followed this approach. Both approaches offer qualitatively similar conclusions. In fact, Kumar and Saha (2008) have shown that both approaches are formally equivalent.

The private bank’s manager chooses \( D_1 \) to maximise \( \pi_1 \). The manager’s choice of \( D_1 \) gives the private bank’s deposit reaction function.

We now introduce a loan market where the interest rate is determined by the following loan demand curve

\[ r_L = \bar{c} - cL, \quad c > 0, \]  

where \( L \) is the total loans made. The banks give out loans from not only the deposits raised but also from equity (\( E \)) raised from the capital market. Total lending by a bank equals the sum of deposits and shareholder equity, i.e. \( L = D + E \). Left to themselves the shareholders might not put in enough of their own funds into the bank leading to a highly leveraged bank (where the leverage ratio is \( D/L \)). Therefore, through capital regulation the regulator has to enforce a minimum level of shareholder equity (Milne, 2006). Let \( \gamma \) be the capital to asset ratio imposed by the regulator (similar to risk weighted capital adequacy ratio e.g. 8 % under Basel-I norms) such that \( \gamma = E/L \). We can then rewrite the balance sheet equality as \( L = kD \) where \( k = \frac{1}{1-\gamma} \).

We also assume that the entrepreneur-borrowers face uncertainty in the realisation of the value of their project. There are two states of nature: good and bad. The good state occurs with probability \( p_1 \) and bad state with probability \( p_2 \). In the good state the

---

\(^1\)While the ownership exceeds 50 percent is very important, it cannot be denied that any change in \( \theta \) will have some effect on the bank behavior.
borrowers are able to pay back both principal and interest, but in the bad state they
pay back nothing. However, the bank can liquidate the firm and recover the principal,
but the interest is lost. Alternatively the borrowing can be assumed to be collateralised.
In the bad state, as the bank loses interest earnings, its ability to repay the depositors is
affected. The depositors get back the principal, but lose out on the interest payments.
However, since the bank has equity \((k > 1)\) it will be able to meet the interest obligations
to the extent it has funds. Thus, the depositors stand to lose in the bad state. Note
that if there is complete deposit insurance or if the banks are sufficiently capitalised \((k
large)\) then the depositors do not lose. But we do not consider deposit insurance here
and instead allow the depositors to bear the risk by allowing limited liability on the part
of the banks and entrepreneur-borrowers.\(^2\)

3 Bank competition: Unlimited liability case

As a benchmark case we consider here deposits as debt obligations that are honored in
all states of nature.

Bank \(i\) lends \(L_i(=kD_i)\) and earns at the end of the year

\[
E\pi_i = p_1 [(1 + r_L)kD_i - (1 + r_D)D_i] + p_2 [kD_i - (1 + r_D)D_i],
\]

\[
= \left[ \bar{k} - \tilde{\delta}(D_0 + D_1) \right] D_i,
\]

where \(\bar{k} = k - 1 + p_1 k\bar{c}\) and \(\tilde{\delta} = p_1 k^2 c + b\). Sum of the two banks’ expected profits is

\[
E\pi = E\pi_0 + E\pi_1 = \left[ \bar{k} - \tilde{\delta}(D_0 + D_1) \right] (D_0 + D_1).
\]

From the area under the positively sloping deposit supply curve, we find the surplus that

\(^2\)Exogenously given schemes of deposit insurance can be introduced in our framework without
much complications. But in principle the insurer should be allowed to monitor the behaviors of
the banks, which is a non-trivial exercise and we do not pursue it here.
depositors expect to enjoy
\[ E[DS] = r_D (D_0 + D_1) - b \frac{(D_0 + D_1)^2}{2} = b \frac{(D_0 + D_1)^2}{2}. \] (4)

We have another group - entrepreneur-borrowers - whose expected surplus, based on
the negatively sloping loan demand curve, is
\[ E[BS] = p_1 \left[ \tilde{k} (D_0 + D_1) - c \frac{k^2 (D_0 + D_1)^2}{2} - r_L k (D_0 + D_1) \right], \]
\[ = p_1 k^2 c \frac{(D_0 + D_1)^2}{2}. \] (5)

Expected social welfare of this economy is then defined as \( SW = E\pi + E[DS] + E[BS] \). Adding equations (3), (4) and (5) we get
\[ E[SW] = \left[ \tilde{k} - \frac{\delta (D_0 + D_1)}{2} \right] (D_0 + D_1). \] (6)

The public bank pursues an objective function \( Z = \theta E[SW] + (1 - \theta) E\pi_0 \), where \( \theta \) is the degree of public ownership. Maximizing \( Z \) we derive the public bank’s deposit
reaction function as
\[ D_0 = \frac{\tilde{k}}{(2 - \theta) \delta} - \frac{D_1}{2 - \theta}. \]
Similarly, the private bank’s reaction function is
\[ D_1 = \frac{\tilde{k}}{2 \delta} - \frac{D_0}{2}. \]

The reaction curves of the two banks are shown in figure 1. Two thick curves, denoted
as \( RF_0 \) and \( RF_1 \) are drawn with the assumption that \( \theta \in (0, 1) \). The downward slopes
indicate that the deposits are strategic substitutes.
Figure 1: Deposit reaction functions: unlimited liability
If the private bank chooses zero deposit, the public bank will choose its monopoly deposit as \( \frac{k}{\delta(2-\theta)} \), and similarly, if the public bank chooses zero deposit the private bank’s manager will choose \( D_1 = \frac{k}{2\delta} \). Conversely, if the private bank chooses \( D_1 = \frac{k}{2\delta} \), the public bank will simply close down, and similarly, the public bank’s choice of \( D_0 = \frac{k}{\delta} \) will force the private bank to close down. Thus, the monopoly and entry-deterring levels of deposits of each bank can be defined in the usual way as quantity setting firms’ outputs are defined. The equilibrium deposits are given by point \( M \) comprising of \( D_0^* \) and \( D_1^* \), which we obtain as

\[
D_0 = \frac{k}{\delta(3-2\theta)}, \quad (7)
\]

\[
D_1 = \frac{k(1-\theta)}{\delta(3-2\theta)}, \quad (8)
\]

The aggregate deposit in this model is

\[
D_0 + D_1 = \frac{k(2-\theta)}{\delta(3-2\theta)}.
\]

It can be readily checked that both the public bank’s deposit and aggregate deposit are increasing in the degree of nationalisation. On the other hand, the private bank’s deposit is decreasing in the degree of nationalisation. In fact, when \( \theta = 0 \) and both banks were profit-maximizers we would have a pure (or equivalently private) duopoly. The public bank’s reaction curve would then swing inward and we have the Cournot deposits as \( D_0 = D_1 = \frac{k}{3\delta} \). This is given by point \( N \) in figure 1. Since point \( M \) lies South-East of point \( N \), it is clear that the mixed duopoly generates greater deposit for the public bank but lower deposit for the private bank than the private duopoly.

At the other extreme, when the public bank is fully public, \( \theta = 1 \), the private bank’s deposit is reduced to zero (point \( O \) in figure 1). This is the case where the public bank crowds out the private bank, a phenomenon commonly occurring mixed oligopoly
models.\textsuperscript{3}

In this case, when debt obligations are honored in all states of nature the standard results of mixed duopoly models continue to hold, in which the public bank acts more aggressively (in raising deposits) and the private bank acts more defensively. In other words, the standard profit-shifting motive of public ownership continues to hold even if there are some states of nature forcing defaults. It can also be checked that if the government was to choose optimal privatisation, it would choose $\theta = 1$. Total deposit under optimal (zero) privatisation would be $D_0 = \frac{k}{3\delta}$ and the expected profit of the public bank will be zero. That is, its expected loss in the bad state would be equal to its expected profit in the good state. The private bank is driven out of business.

We summarise the above results from the no limited liability case in the following proposition.

\textbf{Proposition 1} When debt obligations are always honoured by banks, the public bank acts more aggressively than the private bank, i.e. $D_0 > D_1$. Moreover the public bank’s deposit and aggregate deposit will increase with the degree of nationalisation while the private bank’s deposit will fall. That is, $\frac{\partial D_0}{\partial \theta} > 0$, $\frac{\partial D}{\partial \theta} > 0$ and $\frac{\partial D_1}{\partial \theta} < 0$.

4 Bank competition: Limited liability case

Now we assume that the banks are insufficiently capitalised, so that in the bad state they cannot meet their full obligations. This will be the case if $k = 1$ or $k$ is slightly greater than 1, or the bank is not able to recover the full amount of the principal from the entrepreneurs. For simplicity we will assume that the full amount of the principal is recovered, but $k$ is not large enough to cover the interest charges payable to the depositors in the equilibrium. Due to limited liability, depositors get back what is available with

\textsuperscript{3}However, in standard mixed oligopoly models for this to happen constant marginal cost is required. In our banking model, neither the marginal return curve nor the marginal cost curve is constant. But on both dimensions they depend on aggregate deposits. This is a crucial feature of banks, as opposed to standard firm.
the bank, \( kD \), when \( kD < (1 + r_D)D \). Since \( r_D \) increases with the total deposit \( D \), it is quite possible that the depositors may not lose at all if the total deposit is not large relative to \( k \). That is, if \( k > (1 + r_D) \) then even in the bad state the depositors are safe. For our story to hold, we need to focus on the case where \( k < (1 + r_D) \).

Under limited liability banks’ expected profit is given only by the good state.

\[
E_i = p_1 [(1 + r_L)kD_i - (1 + r_D)D_i],
\]

\[
= p_1 [k^* - \delta(D_0 + D_1)] D_i,
\]

where \( k^* = k - 1 + k\bar{c} \) and \( \delta = k^2 c + b \).

From the above one gets the aggregate expected profits as

\[
E_i = E_{i0} + E_{i1} = p_1 [k^* - \delta(D_0 + D_1)] (D_0 + D_1).
\]

The depositors now stand to lose in the bad state. Their surplus is

\[
E[DS] = p_1 (1 + r_D)(D_0 + D_1) + p_2 k(D_0 + D_1) - D - b \frac{(D_0 + D_1)^2}{2},
\]

\[
= \frac{1}{2}p_1 b(D_0 + D_1) + p_2 (k - 1) - p_2 b \frac{(D_0 + D_1)}{2} (D_0 + D_1).
\]

Note that with the restriction \( kD < (1 + bD)D \) or \( k - 1 < b(D_0 + D_1) \), (so that limited liability applies) \( E[DS] \) is now strictly less than \( b \frac{(D_0 + D_1)^2}{2} \), which was the expected depositor surplus under the ‘no limited liability’ case.

The expected borrower surplus remains unchanged from equation (5). Adding the expressions in (5), (10), and (11) we derive the expected social welfare as

\[
E[SW] = p_1 \left[ k^* - \delta \frac{(D_0 + D_1)}{2} \right] (D_0 + D_1) + p_1 b \frac{(D_0 + D_1)^2}{2} + p_2 (D_0 + D_1)(k - 1) - b \frac{(D_0 + D_1)^2}{2}.
\]
Following the same procedure as before we get the following two reaction functions

\[ D_0 = \frac{p_1 k^* + \theta p_2 (k-1)}{p_1 (2-\theta) + \theta p_2 b} - \frac{[p_1 \delta + \theta p_2 b]}{p_1 (2-\theta) + \theta p_2 b} D_1, \quad (13) \]

\[ D_1 = \frac{k^* - D_0}{2 \delta}. \quad (14) \]

Solving the above system of equations we get the following Cournot-Nash deposits

\[ D_0 = \frac{p_1 k^* \delta + \theta p_2 \{(k-1)2\delta - bk^*\}}{\delta [p_1 \delta (3-2\theta) + \theta p_2 b]}, \quad (15) \]

\[ D_1 = \frac{p_1 k^* (1-\theta) + \theta p_2 \{bk^* - (k-1)\delta\}}{\delta [p_1 \delta (3-2\theta) + \theta p_2 b]}, \quad (16) \]

These result in the aggregate deposit

\[ D_0 + D_1 = \frac{p_1 k^* (2-\theta) + \theta p_2 (k-1)}{p_1 \delta (3-2\theta) + \theta p_2 b}. \]

At this point we can make the following observations. Neither \( D_0 \) nor \( D_1 \) are unconditionally monotonic in \( \theta \). In particular, it is striking that even if \( \theta = 1 \), the private bank’s deposit does not become zero. It remains strictly positive. More importantly, the aggregate deposit under the mixed duopoly (with the public bank being fully public) may not necessarily be greater than that under pure duopoly. If both banks were privately owned (i.e. \( \theta = 0 \)), aggregate deposit is

\[ D(\theta = 0) = \frac{2k^*}{3 \delta}. \]

With full public ownership in the public bank, aggregate deposit is

\[ D(\theta = 1) = \frac{p_1 k^* + p_2 (k-1)}{p_1 \delta + p_2 b}. \]

By comparing these expressions and individual deposits of the banks we arrive at the following result.

**Proposition 2** If \( p_2 > \frac{6k^*}{k(3k^* - 2k^* - 3k^*)} \), aggregate deposit under full public ownership of
the public bank will be less than the aggregate deposit under pure duopoly. That is, 
\[ D(\theta = 1) < D(\theta = 0); \text{ moreover, } D_0(\theta = 1) < D_0(\theta = 0) = D_1(\theta = 0) < D_1(\theta = 1). \] 
On the other hand, if 
\[ p_2 < \frac{\delta k^*}{k(3\delta + 2k^*c)}, \] 
we have 
\[ D(\theta = 0) < D(\theta = 1), \text{ and } D_1(\theta = 1) < D_0(\theta = 0) = D_1(\theta = 0) < D_0(\theta = 1). \]

One implication of proposition 2 is that when the likelihood of the bad state is above a critical level, the depositors’ loss matters a lot and the publicly owned bank collects deposit very conservatively. However, this defensive act encourages the private bank to increase its deposit, because the two banks’ deposits are strategic substitutes. However, the aggregate deposit falls short of the fully private or pure duopoly level and thus the depositors’ loss is restricted to an optimal level. On the other hand, if the bad state is less likely then the government social welfare maximisation objective encourages its bank to mobilise greater deposits and the market is expanded well beyond the pure duopoly level. In the latter case, we have a similar outcome as if there was no uncertainty. But in the former case, public ownership leads to a contraction in the market, quite in contrast to standard mixed duopoly models.

We may relate this to the finding of Boyd and De Nicolo (2005) where greater competition reduces default risk by lowering the cost of borrowing (as the loan rate falls) and also by making more low-risk-low-profit projects viable. But our model suggests that this finding may not hold unconditionally. Though we considered only exogenous risk, it can be argued that in the presence of a public bank, results can go either way. If the default risk is significant, then the public bank’s conservative approach will lead to a contraction in total loans causing interest rate to rise. In the view of Boyd and De Nicolo then entrepreneurs will shift to more risky projects. If we allow endogenous default probability, we will be able to see how the public bank reacts and how the government’s choice of public ownership is affected.
5 Conclusion

This paper explores partial nationalisation and credit risk in the framework of a ‘mixed oligopoly’. We show that if there is no limited liability or if there is complete deposit insurance, then in the event of loan default the banks loses money, but depositors remain safe. But the banks’ loss is equal to depositors’ gain. Therefore, in terms of social welfare this loss does not matter. The public bank acts aggressively and mobilises greater deposits leading to an expansion of aggregate deposits, though in the process the private bank’s operation is squeezed. This is very much in line with the predictions of standard mixed duopoly models. But if the banks are protected by limited liability and deposits are only partially insured, then depositors are exposed to risk, and they cannot pass on this risk to anybody else. Therefore, in the social welfare calculations the expected loss of the depositors matters. The public bank takes into account this loss, and changes its behaviour in the following way. If the default risk is significant, it will mobilise less deposits resulting in a contraction of aggregate deposits, though the private bank will be able to profit from this defensive act. Essentially, the public bank will try to borrow less and lend less. This is a reversal of the standard mixed duopoly result. However, if the risk of default is small, the public bank returns to aggressive competition.
References


Appendix

Detailed derivation of proposition 1

It is straightforward to compare $D_0$ and $D_1$ from equations (7) and (8) and conclude that $D_0 > D_1$.

Next, we derive the following expressions:

\[
\frac{\partial D_0}{\partial \theta} = \frac{2}{(3-2\theta)^2} > 0.
\]

\[
\frac{\partial D_1}{\partial \theta} = -\frac{1}{(3-2\theta)^2} < 0.
\]

\[
\frac{\partial D}{\partial \theta} = \frac{1}{(3-2\theta)^2} > 0.
\]

Detailed derivation of proposition 2

We demonstrate the results graphically. Consider figure 2 where the deposit reaction functions of the two banks under limited liability are plotted under the assumption that the probability of default is above the critical level. While the equilibrium deposits are given by point $M$ as before, our interest is in comparing this with the case when $\theta = 1$. This is indicated by the shaded reaction function of the public bank $RF_0$ whereas the private bank’s reaction function $RF_1$ is unaffected. The equilibrium point $O$ under $\theta = 1$ lies North-West of point $M$. This indicates that full nationalisation leads to lower deposit collected by the public bank but higher deposit collected by the private bank. Figure 3 shows how it is the reverse when the probability of default is below the critical level. In this case, point $O$ lies South-East of point $M$ i.e. the public bank’s deposit level rises while the private bank’s deposit level falls which is similar to what was observed in the case of unlimited liability. In both these figures, as before, points $N$ denote the case of $\theta = 0$. 

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Figure 2: Deposit reaction functions: limited liability and high default risk
Figure 3: Deposit reaction functions: limited liability and low default risk