Disinvestment and Bank Competition

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Abstract

We address the questions of disinvestment (partial privatization) and entry in the context of quantity competition between a partially privatized public bank and a private bank. We find that social welfare improves with entry only when the private banks are more efficient than the public bank. We also determine socially optimum degree of disinvestment and entry.

Keywords: banking, partial privatization, entry.

JEL classification: G21, L13, L33.
1 Introduction

Partial government ownership of banks is a common phenomena all over the world. In a study covering 92 countries, La Porta, Lopez-de-Silanes, and Shleifer (2000) found that “in an average country, 42 percent of equity of the 10 largest banks was still owned by the government in 1995”. This is especially true for transition economies where nationalized banks are gradually being privatized. To give an example, in India, the government has expressed its intention to bring down its holding in public sector banks to 33 per cent through gradual privatization (or ‘disinvestment’) and the process is underway. Moreover, although new banks have been entering the Indian banking industry, entry is strictly monitored and controlled by the government. Such a situation indicates the presence of not too many banks in the industry which results in imperfect competition between them. As a result, it is reasonable to expect elements of strategic behaviour among the competing banks.

Surprisingly, there is a paucity of models analyzing strategic interactions between partially nationalized firms and private competitors. The growing body of mixed oligopoly models\(^1\) considers either fully privatized or fully nationalized firms. Therefore, the issues of partial privatization (henceforth disinvestment) and entry deregulation have not been given due recognition.

The central concern of this paper is to fill this lacuna, with reference to the banking industry.

Closely related to our work are three papers, viz. Purroy and Salas (2000), Matsumura (1998) and Fershtman (1990). Purroy and Salas (2000) study competition between a savings institution\(^2\) and a profit maximizing private

\(^1\)Bos (1991) contains an exhaustive discussion of the issues.

\(^2\)A savings institution is akin to a workers’ cooperative and it exhibits ‘expense pref-
bank. They show that the savings institution outperforms the private bank in terms of deposit collection and profit. The private bank can partly restore the asymmetry by offering managerial incentives (as in Fershtman and Judd, 1987). While the savings institution bears some resemblance to a public sector bank, their model is not useful to address the question of disinvestment. The question of entry is also ignored. We address both these issues in this paper.

Matsumura (1998) on the other hand, though ignoring the issue of entry, directly deals with the question of optimal disinvestment. He shows that mixed ownership is an optimal situation compared to full nationalization or full privatization. However his result depends on the specification that the government is consumer surplus oriented and the objective of the partially privatized firm is to maximize a weighted average of its profit and the government’s utility. In contrast, we consider the government’s objective to be profit oriented since it went for the process of disinvestment with profitability of the public bank in mind and still we obtain partial privatization of the public bank as a solution. Moreover, we show that social optimum requires deregulation of entry accompanied by partial privatization when entry brings

cence behaviour', i.e. utility maximization where utility here is the sum of profit and a positive weight on workers’ wage-bill.

3This is not a surprising result and is in line with the well known Cournot intuition and the results obtained by Fershtman and Judd (1987). The Cournot intuition says that if a firm could commit to a strategy that enables it to produce at a higher level than what the Cournot-Nash equilibrium predicts, e.g. by offering managerial incentives as in Fershtman and Judd (1987), its profit will be higher.

4The government’s utility is taken as the sum of social welfare and a non-negative weight on consumer surplus.

5When the government’s objective is profit oriented, mixed ownership is no longer optimal and the government goes for either full nationalization or full privatization.
with it efficiency into the industry.

Matsumara’s approach to modelling partially privatized firms is more conventional and is in line with other mixed oligopoly models, such as De Fraja and Delbono (1989), Sen and Saha (1992), Pal and White (1998), White (2001) and Nishimori and Ogawa (2002). De Fraja and Delbono (1989) show that with one social welfare maximizing firm and \( n \) profit maximizing firms, when the number of firms is sufficiently large, the optimal strategy of the social welfare maximizing firm is to act as a profit maximizer.\(^6\)

Fershtman (1990) suggested an alternative approach to modelling a partially privatized firm. Instead of specifying an objective function, he proposed a special type of reaction function that would suitably describe the behaviour of a partially privatized firm.\(^7\) Though the objective of the firm remains an open question, his reaction function approach is extremely convenient for characterizing mixed oligopoly competition. Fershtman shows that the partially privatized firm earns higher profit than if it were privatized when both firms are equally efficient. When the private firm is more efficient, nationalization of the partially privatized firm reduces social welfare.\(^8\)

To address both the issues of disinvestment and entry, we adopt the reaction function suggested by Fershtman (1990) and endogenously determine the degree of disinvestment.\(^9\) Our premise is that disinvestment of the pub-

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6This suggests the possibility of privatization but partial privatization is not considered.

7The reaction function is a weighted average of the reaction functions of a social welfare maximizing firm and a profit maximizing firm. It is shown that the partly nationalized firm earns higher profit.

8Further, Fershtman finds that the public characteristic of an incumbent partially privatized firm serves to deter entry when all firms are equally efficient. However, there will be entry if the incumbent is sufficiently privatized and consequently, social welfare could increase.

9The degree of disinvestment is exogenous in Fershtman (1990).
lic bank is used as a strategy to improve its profit performance. Hence a reservation level of the public bank’s profit is set as part of the government’s objective, which otherwise maximizes social welfare.\textsuperscript{10} While meeting reservation profit, ownership needs to be divested to bring profit orientation in the public bank’s behaviour. Accordingly optimal disinvestment is determined. When the number of potential entrants is fixed to begin with, while greater entry results in higher disinvestment, there is a limit on the number of private banks that can be allowed in the industry for a given reservation profit. However, entry deregulation is a suboptimal strategy from the social welfare point of view, when all banks are equally efficient.\textsuperscript{11} When private banks are more efficient than the public bank, their entry leads to more disinvestment and also raises social welfare. Thereby, the model can determine both socially optimal disinvestment and entry. Finally, we study the case of product differentiation with price competition.

The rest of the paper is organized as follows. Section 2 introduces the model framework. Section 3 discusses the implications for disinvestment when the potential number of entrants is exogenously fixed. Section 4 discusses the optimality of entry deregulation policy with disinvestment. Section 5 presents the case of price competition. Section 6 concludes.

\section{The Model Framework}

We assume that one public bank (represented by subindex 0) and \(n(\geq 1)\) private banks (represented by subindex \(i\), \(\forall i = 1\) to \(n\)) can engage in a

\textsuperscript{10}This makes the government profit oriented unlike that in Matsumura (1998).

\textsuperscript{11}This is contrary to Fershtman (1990) where entry increases social welfare if the partially privatized firm is sufficiently privatized, even when both firms are equally efficient.
Cournot-type quantity competition. A typical bank $i$ mobilizes deposits $D_i$ by offering an interest rate $r$ to depositors by the rule

$$r = bD, \quad b > 0,$$

where $D = \sum_0^n D_i$ is the total supply of deposits coming from depositors. All banks face a constant rate of return $R$ on each unit of investment made out of these deposits. It is not very difficult to think of fixed $R$ when money markets and loan markets are competitive in which case all banks earn similar rates of return on their investments.

A private bank’s objective is to choose $D_i$ so as to maximize profit

$$\Pi_i = (R - r)D_i$$

Given others’ deposits, the private bank’s deposits are

$$D_i = \frac{R - bD_{-i}}{2b},$$

which is its reaction function (say, $RF_i$), where $D_{-i} = \sum_{j \neq i} D_j$

In India, before the deregulation of interest rates in the nineties, banks were not free to choose interest rates as they were fixed by the regulator. Even after reforms and deregulation, some interest rates such as that offered on deposits with low maturities continue to be fixed by the regulator. In such situations, one can expect banks to compete in terms of deposit collection by setting targets for deposit mobilization or through setting up of branches.

Albeit all our results are valid even if $R$ is generalized to be inversely dependent on $D$ (e.g. when the deposits are loaned out, higher $D$ would increase the total loans which would reduce the interest rate in the market for loans and hence lower the rate of return on investments). We can even incorporate a reserve requirement of the central bank (known as Cash Reserve Ratio in India) whereby banks are required to keep a fraction, say $\gamma$ of their reserves as cash with the central bank and the rest, i.e. $(1 - \gamma)D$ is free to be invested. In this case the return on unit investment is $(1 - \gamma)R + \gamma r_0$ (where $r_0$ is the return paid on cash reserves) which can be redefined as $R$ without affecting the analysis.
The public bank is partly owned by the government and is partly private. The decision of how much private participation to allow (i.e. disinvestment) rests with the government. However the choice of deposits of the public bank rests with the managers of the bank. The bank’s strategy is a mix of social welfare maximizing and profit maximizing strategies depending on the degree of government vis-a-vis private ownership. Next, we separate the objective of the government from the strategy of the public bank. The government’s objective is to maximize social welfare subject to a reservation level of the public bank’s profit and this determines the level of disinvestment that the government goes for.

Social welfare is given by the sum of depositor surplus (DS) and bank profits. Therefore

\[ SW = DS + \sum_{i=0}^{n} \Pi_i \]

where \( DS = rD - \int_0^D bDz = rD - \frac{bD^2}{2} \)

Therefore

\[ SW = (R - \frac{bD}{2})D \]

To maximize social welfare, a pure public (i.e. fully nationalized) bank would choose \( D_i \) as

\[ D_i = \frac{R - bD_{-i}}{b} \]

which is its reaction function (say, \( RF_i \)).

However, the mixed ownership of the public bank places it in between the above two extreme situations. Since the bank cannot ignore the interests of its private shareholders, it cannot be a pure social welfare maximizer. On
the other hand, by virtue of being a shareholder of the bank, the government can indirectly influence its activities. Hence the public bank cannot be a pure profit maximizer. The approaches in the literature towards modelling the mixed ownership nature of a public firm is divided into two. One way is to model the mixed ownership through the public firm’s objective function. Matsumura (1998) follows this approach by incorporating both social welfare and profit in the partially privatized public firm’s objective function. The other way is to allow for the mixed ownership to be manifested directly in the reaction function. Without worrying about the objective of the firm or the government, Fershtman (1990) follows the second approach by considering the reaction function of the partly nationalized firm as a weighted average of the reaction functions of a pure social welfare maximizing firm and a profit maximizing firm. The *raison d’etre* of such a reaction function is the assumption that “the conflict between the two interest groups is resolved through a compromise” (Fershtman, 1990). We follow Fershtman’s suggestion of incorporating the conflict in strategies in the reaction function itself, while also explicitly introducing an objective function of the government.

Therefore, the public bank’s reaction function is a weighted average of $\tilde{RF}$ and RF, where RF is the reaction function from profit maximization

$$RF_0^* = \theta \tilde{RF}_0 + (1 - \theta)RF_0,$$

where $\theta \in [0, 1]$

$\theta$ is the degree of nationalization or government control which is positively linked to the proportion of shares the government holds and $(1 - \theta)$ is the degree of disinvestment. $\theta$ is unity if the bank is fully nationalized and zero if the bank is fully privatized.

Fershtman (1990) however does not discuss the objective of the public firm
or the government. While a variety of objective functions could be consistent with the above reaction function, we too desist from discussing the objective function of the public bank.\textsuperscript{14} Instead, we introduce an objective function of the government which serves to determine how much private participation to be allowed in the public bank. Therefore, we separate the objective of the government from the strategy of the public bank. The government’s objective is

\begin{equation}
\text{Maximize } SW \\
\text{Subject to } \pi_0 \geq \bar{\pi}_0,
\end{equation}

where $\pi_0$ is a reservation profit.\textsuperscript{15}

In other words, although the government is a social welfare maximizer, being a shareholder it would want the public bank to earn a minimum level of profit, also serving as a profit commitment to facilitate entry.\textsuperscript{16} The reservation profit can also be interpreted as being a participation constraint of the private partner which induces him to buy stakes in the public bank. This objective function that we assign to the government is similar to that in Bos (1986). The choice of $\bar{\pi}_0$ is a political decision and comes from the ‘profit orientation’ of the government. We consider a two-stage full information game where in the first stage, the government chooses $\theta$ and $n$. Given $\theta$ and $n$, in the second stage, the public bank and the private banks compete for

\textsuperscript{14}Bos and Peters (1989) as cited in Fershtman (1990) contains a detailed discussion on the objective function of a partly nationalized firm.

\textsuperscript{15}Since disinvestment is done to impart profit behaviour to the public bank, it could be expected that the government will impose some reservation level of profit even after partially abdicating control which can be represented as a minimum reservation profit.

\textsuperscript{16}$\bar{\pi}_0$ can be seen as a commitment to some reservation level of profit which signals that the government will not drive out all potential entrants from the industry by maximizing social welfare, thereby driving profit to zero.
deposits.

As a benchmark case, let us first determine optimal disinvestment under monopoly of the public bank. Note that before disinvestment, the public bank makes zero profit, whereas its social welfare is at the maximum. Now suppose that government stakes are reduced from unity to $\theta$. Then the deposits produced is given as

$$D_0 = \frac{(1 + \theta)R}{2b} = D$$

This gives social welfare and profit as

$$SW^M = \frac{R^2}{8b} (3 - \theta)(1 + \theta)$$

$$\pi^M_0 = \frac{(1 - \theta^2)R^2}{4b}$$

It is straightforward to check that $SW$ is increasing in $\theta$ while $\pi_0$ is decreasing in $\theta$. Hence the optimal $\theta$ is determined by the intersection of $\pi^M_0$ and $\bar{\pi}_0$

$$\theta^* = \sqrt{1 - \frac{4b\bar{\pi}_0}{R^2}}$$

\textsuperscript{17}The maximum social welfare is $\frac{R^2}{8}$. 

\textsuperscript{17}The maximum social welfare is $\frac{R^2}{8}$. 

10
Figure 1: Bank Competition

\[
\frac{\bar{n}}{\bar{n} + 2} \quad \theta^* \quad 1
\]

\[
\pi_0(\bar{n})
\]

\[
\pi_0
\]

\[
\pi_M
\]

\[
SW
\]

\[
SW^M
\]
The graphical solution is shown in figure 1. Intersection of $\pi_0^M$ and $\bar{\pi}_0$ gives the equilibrium $\theta$.

Notice that, disinvestment rises with ‘profit orientation’ and falls with the rate of return earned on deposits

$$\frac{\partial \theta^*}{\partial \bar{\pi}_0} < 0$$

$$\frac{\partial \theta^*}{\partial R} > 0$$

3 Bank Competition

Now we consider the case where the disinvestment authority decides only on the extent of disinvestment, taking the existence of a private sector as given. In other words, the number of private banks is exogenously determined. It may be imagined that the questions of entry are largely determined by the government’s overall policy of liberalization, which may not be sector-specific, though disinvestment may be handled by a more specialized body, which is indeed the case in reality. Hence we assume that $n$ private banks compete for deposits with the public bank\(^{18}\) in a Cournot game where all banks have complete information about the game. We solve the game by backward induction. First, for given $\theta$ and $n$, the equilibrium deposits are determined. Next, equilibrium $\theta$ is determined. Finally, we obtain an upper limit on $n$ beyond which the profit constraint of the public bank becomes untenable.

In the presence of $n$ private banks, reaction function of the public bank

\(^{18}\)In our setup, coexistence of private banks with a pure public bank is not possible because simple social welfare maximization by a fully nationalized bank would drive out all other banks in the absence of any capacity constraint. Hence, $n$ banks cannot exist beforehand. The public bank has to be disinvested in order to create room for entry.
is

\[ D_0 = \frac{(1 + \theta)(R - bD_{-i})}{2b} \]

Private banks simply maximize profit. Therefore, the private banks’ reaction functions are

\[ D_i = \frac{R - bD_{-i}}{2b}, \forall i = 1, ..., n \]

Assuming that all the \( n \) private banks are identical, the reaction functions become

\[ D_0 = \frac{(1 + \theta)(R - bD_1)}{2b} \quad (1) \]
\[ D_1 = \frac{R - bD_0}{(n + 1)b}, \]

where \( D_1 \) refers to a representative private bank’s deposits.

Solving the two reaction functions for \( D_0 \) and \( D_1 \)

\[ D_0 = \frac{(1 + \theta)R}{[(2 + n(1 - \theta))]b} \]
\[ D_1 = \frac{(1 - \theta)R}{[(2 + n(1 - \theta))]b} \]
\[ D = D_0 + nD_1 = \frac{[(1 + \theta) + n(1 - \theta)]R}{[(2 + n(1 - \theta))]b} \]

This results in social welfare and profit as

\[ SW = \frac{R^2[3 + n(1 - \theta) - \theta][(1 + \theta) + n(1 - \theta)]}{2b[2 + n(1 - \theta)]^2} \]
\[ \pi_0 = \frac{(1 - \theta^2)R^2}{[2 + n(1 - \theta)]^{2b}} \quad (2) \]
\[ \pi_1 = \frac{(1 - \theta^2)R^2}{[2 + n(1 - \theta)]^{2b}} \]

Note that profit of the public bank is higher which is because of the deviation from Cournot-Nash equilibrium as a result of mixed ownership.\footnote{This finding is consistent with Fershtman (1990).}
Also note that social welfare decreases with disinvestment as before, but the public bank’s profit now rises in \( \theta \) up to a point and then declines. To be more precise, its profit is a concave function of \( \theta \) with a maximum at \( \theta = \frac{n}{n+2} \). We obtain this by differentiating its profit twice with respect to \( \theta \) as

\[
\frac{\partial \pi_0}{\partial \theta} = 0 \Rightarrow \theta = \frac{n}{n+2}
\]

\[
\frac{\partial^2 \pi_0}{\partial \theta^2} = \frac{-4R^2[n(n+2)(1-\theta) + 2]}{b[2+n(1-\theta)]^4} < 0
\]

The government’s objective remains the same as discussed in the previous section. Once again, the profit constraint determines the level of disinvestment that the government goes for.\(^{21}\) The graphical solution is given in figure 1. Intersection of \( \pi_0 \) and \( \bar{\pi}_0 \) gives the equilibrium \( \theta \).

Mathematically, the solution is given by

\[
\theta^* = \frac{n(n+2) + \sqrt{k[k - 4(n+1)]}}{n^2 + k},
\]

where, \( k = \frac{R^2}{b\bar{\pi}_0} \)

Note that we choose the higher root since higher \( \theta \) is preferred to lower \( \theta \) because of higher social welfare.

**Proposition 1** For a given level of reservation profit \( \bar{\pi}_0 \) \((0 < \bar{\pi}_0 < \pi_0^M)\), a larger scale of entry is associated with a higher degree of disinvestment.

**Proof**: As \( n \) increases, for any given \( \theta \) the \( \pi_0 \) curve shifts down (see figure 1), since from equation (2), \( \frac{\partial \pi_0}{\partial n} < 0 \).

\(^{20}\)Note that if \( n=1 \), profit of the public bank is maximized at \( \theta = \frac{1}{3} \) (see figure 1), which is the same as obtained by Fershtman (1990).

\(^{21}\)Note that this does not mean that the reservation profit alone always determines the level of disinvestment. While this is true in the case of exogenous entry, in the case of endogenously determined scale of entry which we analyse later, the level of disinvestment is determined both by the reservation profit and the chosen scale of entry.
The $\pi_0$ curve being concave, as it shifts down, its relevant intersection point with $\bar{\pi}_0$ moves to the left, leading to a lower $\theta^*$, i.e. higher disinvestment.

**Proposition 2** For any given level of reservation profit $\bar{\pi}_0$ ($0 < \bar{\pi}_0 < \pi_0^M$), the maximum number of entrants and the corresponding degree of disinvestment are as follows

\[ n^* = \frac{R^2}{4b\bar{\pi}_0} - 1 \quad (4) \]

\[ \theta^* = \frac{n^*(n^* + 2)}{n^{*2} + k^2}, \text{ where } k = \frac{R^2}{b\bar{\pi}_0} \quad (5) \]

*Proof:* As the $\pi_0$ curve shifts down with rising $n$, beyond a particular value of $n$, the $\pi_0$ curve falls below the reservation level, $\bar{\pi}_0$, where the profit constraint becomes untenable (see figure 1). Hence, the value of $n$ for which the $\pi_0$ is tangent to the $\bar{\pi}_0$ line gives the maximum number of entrants to be allowed by the government. This value for $n$ can be obtained when equation (3) has equal roots. In other words, $k[k - 4(n + 1)] = 0$. Notice that, higher reservation profit reduces this maximum scale of entry and increases the corresponding level of disinvestment. It can be easily seen from equation (4)

\[ \frac{\partial n^*}{\partial \bar{\pi}_0} < 0 \]

Graphically in figure 1, as the $\bar{\pi}_0$ line shifts up, its tangency with the $\pi_0$ curve is reached much earlier for a lower $n$. Hence the maximum scale of entry is less.

From equation (5)

\[ \frac{\partial \theta^*}{\partial n^*} > 0 \]

Hence, as the maximum number of entrants falls, the corresponding $\theta$ falls, i.e. there is higher disinvestment.
Proposition 3  A rise in reservation profit $\bar{\pi}_0$ ($0 < \bar{\pi}_0 < \pi_0^M$) raises profit of the private banks.

Proof: A rise in $\bar{\pi}_0$ reduces $\theta$ and $n$ which in turn increases $\pi_1$. This can be seen from equation (2)

$$\frac{\partial \pi_1}{\partial \theta} < 0$$
$$\frac{\partial \pi_1}{\partial n} < 0$$

Therefore, higher profit of the public bank does not cut into the profit of the private banks which is unlike the Cournot result. This happens because the chosen $\theta$ is on the falling part of the $\pi_0$ curve. Hence, as $\bar{\pi}_0$ rises, $\theta$ falls. As the public bank moves away from social maximizing behaviour, its aggressiveness lessens and this helps the private banks to increase their profit. Moreover, as $\bar{\pi}_0$ rises, the maximum number of entrants falls and thus, because of less competition, each private bank is now able to make more profit.

Higher rate of return earned on deposits raises the maximum scale of entry and increases the corresponding disinvestment. From equation (4)

$$\frac{\partial n^*}{\partial R} > 0$$

From equation (5)

$$\frac{\partial \theta^*}{\partial n^*} > 0$$

Hence, as the maximum number of entrants falls, the corresponding $\theta$ falls, i.e. there is higher disinvestment.
4 Welfare Consequences of Entry

Entry has a complementary relation with disinvestment. However, entry per se may not be desirable from the point of view of social welfare unless it brings with it enhanced efficiency into the industry. In this section, we study the impact of entry on social welfare.

Proposition 4 Entry reduces social welfare when all banks are equally efficient.

Proof: Suppose, to the contrary of our claim, social welfare rises with entry which is possible only if $D$ rises. This would mean either of the following three cases happen, viz. $D_0$ falls or remains unchanged or rises. When $D$ rises, $r$ must be rising because of the positively sloped deposit supply curve. Hence profit of the public bank, $(R - r)D_0$ gets squeezed since $R$ is fixed. But $\pi_0$ has to satisfy the reservation profit of $\bar{\pi}_0$ which rules out $D_0$ falling or remaining unchanged. The only other case left is that of $D_0$ rising. From equation (1) we know that when $n$ rises and subsequently $\theta$ falls, $D_0$ cannot rise. Hence all the three cases are ruled out. Which means with entry, $D$ cannot rise. In fact, $D$ would fall. Since $\frac{\partial SW}{\partial D} > 0$, social welfare falls. Hence entry reduces social welfare.

Hence, there is no reason to deregulate entry at all. Monopoly of the public bank seems to be the optimal situation from the welfare point of view. The real culprit behind this result is the reservation profit of the government. Explicit profit constraint of the incumbent works like an entry accommodation strategy by scaling down its deposits, so much so that total industry deposits also declines. The reason is clear. As the incumbent must maintain a fixed level of profit and the entrants also make profit in equilibrium, industry deposits must contract and the social welfare will fall. Therefore, if the
disinvestment authority had the power to regulate entry, it should allow none to enter, and disinvest appropriately. This result also suggests that to enable entry, the public bank has to take a hit in terms of profit. Hence, political pressure to retain profitability (manifested in the reservation profit) even after disinvestment can actually be counterproductive. The moral of this exercise is that entry in this environment is useless unless it brings some efficiency gains. This finding is consistent with other models that have dealt with privatization or disinvestment as well as some with empirical experiences.

We now consider the case where private banks earn a higher rate of return on their investments than does the jointly owned bank, i.e. \( R_1 > R_0 \). This is justifiable in many ways. One readily available argument can be found in India’s long-standing policy of ‘priority sector lending’ (statutory lending to relatively low-return sectors such as agriculture, small industries etc.), which is mainly applied to public sector banks.\(^{22}\)

Now the profit of bank \( i \) becomes

\[
\Pi_i = (R_i - r)D_i
\]

Substituting in the expression for \( SW \)

\[
SW = R_0D_0 + nR_1D_1 - \frac{bD^2}{2}
\]

The equilibrium deposits now are

\[
D_0 = \frac{(1 + \theta)[n(R_0 - R_1) + R_0]}{b[n(1 - \theta) + 2]}
\]

\[
D_1 = \frac{2R_1 - (1 + \theta)R_0}{b[n(1 - \theta) + 2]}
\]

\[
D = \frac{n(1 - \theta)R_1 + (1 + \theta)R_0}{b[n(1 - \theta) + 2]}
\]

\(^{22}\)Other justifications for this assumption could be better fund-management practices of private banks, inefficiency of the public sector, poor debt recovery by public banks due to political interference etc.
Note that there is an upper bound on \( R_1 \), since too high \( R_0 \) could drive \( D_0 \) to zero.\(^{23}\)

Social welfare and \( \pi_0 \) are given as

\[
SW = \frac{A + B + C}{2b[n(1 - \theta) + 2]^2},
\]

where, \( A = R_0^2(1 + \theta)[(3 - \theta)(1 + 2n) + 2n^2(1 - \theta)] \)

\( B = nR_1^2[8 + n(1 - \theta)(3 + \theta)] \)

\( C = -2nR_0R_1(1 + \theta)[(5 - \theta) + 2n(1 - \theta)] \)

\[
\pi_0 = \frac{(1 - \theta^2)[R_1 + n(R_1 - R_2)]^2}{b[n(1 - \theta) + 2]^2}
\]

\( \pi_0 \) has the same shape as before. Social welfare is positively sloping in \( \theta \) as before provided \( R_1 \) is not too high, since too high \( R_1 \) has an adverse effect on \( \pi_0 \).\(^{24}\)

The objective of the government remains as before. The profit constraint determines the level of disinvestment. The graphical solution is the same as in figure 1. Intersection of \( \pi_0 \) and \( \bar{\pi}_0 \) gives the equilibrium \( \theta \).

Mathematically, the solution is given by,

\[
\theta^* = \frac{n(n + 2) + \sqrt{k[k - 4(n + 1)]}}{n^2 + k}, \quad \text{where, } k = \frac{[R_0 + n(R_0 - R_1)]^2}{b\bar{\pi}_0}
\]

As before we choose the higher root since higher \( \theta \) is preferred to lower \( \theta \) because of higher social welfare.

\(^{23}\) \( R_1 - R_0 < \frac{R_0}{n} \)

\(^{24}\) \( R_1 - R_0 < \frac{(1 - \theta)}{(3 - \theta) + n(1 - \theta)} \frac{R_0}{n} \)
Proposition 5 Entry increases social welfare when the potential entrants are more efficient than the incumbent, provided the efficiency of the potential entrants is within an upper bound.

Proof: We maximize social welfare simultaneously choosing \( n \) and \( \theta \), subject to the profit constraint. The Lagrangian for this problem is,

\[
L = SW + \lambda(\pi_0 - \bar{\pi}_0)
\]

The optimal \( n \) and \( \theta \) are given by the first order conditions

\[
\frac{\partial SW}{\partial \theta} = \frac{\partial \pi_0}{\partial \theta}, \quad \frac{\partial SW}{\partial n} = \frac{\partial \pi_0}{\partial n}
\]

and,

\[
\pi_0 = \bar{\pi}_0
\]

Solving, we get optimal \( n \) as

\[
n^* = \frac{R_0(1 + \theta^*)^2 - 4R_1\theta^*}{(1 - \theta^*)^2(R_1 - R_0)},
\]

where \( \theta^* \) is given from equation (3). It can be easily shown that \( n^* \) is positive\(^{25} \) which means that a positive value of the number of potential entrants exists for which social welfare is maximum. However, \( R_1 \) has an upper bound as defined earlier, so as to make social welfare an increasing function of \( \theta \).

Therefore, we see that for a given level of reservation profit of the public bank, social welfare declines with entry when all banks are equally efficient. This is because the government’s reservation profit reduces the public bank’s deposits and consequently the industry deposits. However, social welfare

\(^{25}\)The other two values of \( n^* \) obtained are \( \frac{R_\theta}{R_1 - R_0} \) and \( \frac{-2}{1 - \theta^*} \). However, at the first value of \( n^* \), \( \pi_0 = 0 \). Therefore, the reservation profit is not attained and so this value is ruled out. The second value of \( n^* \) is negative and hence ruled out.
improves with entry if the rate of return on investments of the private banks is higher (but not too high). Here again, the profit constraint leads to a decline in the industry deposits, but the loss in the depositor surplus is now compensated by substantial profit gains of the private banks.

While the presence of the government with its social welfare maximizing objective is essential to our story, it is worthwhile to mention that the exit of the government does not unambiguously improve or reduce social welfare. It can be easily shown that there is a range of values of $R_1$ for the presence of only private banks in the industry gives rise to a higher welfare as compared to the mixed oligopoly case. If $R_1$ is too low, the relative inefficiency of the private banks leads to a lower social welfare than compared with the mixed oligopoly case. On the other hand, if $R_1$ is too high, it drives depositor surplus down so that the social welfare is lower than compared with the mixed oligopoly case.

5 Price Competition

In this section we provide an insight into choice of disinvestment in the presence of product differentiation and price competition. This is relevant on both empirical and theoretical grounds. With financial deregulation, banks are expected to engage in product differentiation and interest rate competition. This is being observed in India and many other emerging economies. Theoretically also, the implication of price competition for disinvestment needs to be understood. Due to public ownership in a mixed oligopoly price competition becomes much more intense, but it does not always improve social welfare mainly because of the retaliatory feedbacks from rival prod-

\footnote{In India, banks are now free to choose interest rates on deposits with higher maturities.}
ucts. To be more precise, assume two substitute products, one produced by a partially public firm and the other by a private firm. With an increase in the extent of public ownership in the first market, the social welfare in the second market will surely fall, and in turn the price retaliation of the second market will arrest and even may reverse the growth of the social welfare in the first market. Thus, there is a need for optimal extent of public ownership, or privatization.

We consider a similar setup as before with one public bank and only one private bank, but the type of deposit account each offers is different from the other's. Each chooses its own interest rate keeping in mind the competing response of its rival.

Suppose the deposit supply functions are

\[ D_0 = A + r_0 - \gamma r_1 \]
\[ D_1 = A + r_1 - \gamma r_0 \]

where \(0 < \gamma < 1\).

The private bank’s objective is to choose \(r_1\) so as to maximize profit, \(\Pi_1 = (R - r_1)D_1\), according to its reaction function \(RF_1\):

\[ r_1 = \frac{R - A + \gamma r_0}{2}. \]

As before, the private partner of the public firm is interested in its share

\(^{27}\)We abstract from the question of entry.

\(^{28}\)However, in practice, banks are seen to provide a wide range of deposits but they tend to specialize on different types of deposits to reduce competition. For example, in India, many newly permitted private banks offer overdraft facilities to savings deposit holders, while a public bank does not offer such benefits. However, this does not construe a vertical product differentiation because the private bank also requires a minimum balance which is much larger than that required by a public bank. Here, however, for model simplicity we restrict to the case where each bank offers only one type of deposit instead of a basket.
of the profit, and the government in social welfare (profits plus depositors’ surplus). However, in the present case, social welfare can have two components, arising from the two markets. But for analytical simplicity we assume that the government is mainly concerned about the social welfare of the market in which the public bank operates, $SW_0 = \Pi_0 + \frac{D_0^2}{2}$, given the profitability constraint $\pi_0 \geq \bar{\pi}_0$.

As was earlier explained, the mixed ownership of the public bank leads to its reaction function being a weighted average of $\tilde{RF}$ and $RF$, where $RF$ is the reaction function from profit maximization

$$RF^*_0 = \theta \tilde{RF}_0 + (1 - \theta)RF_0$$

$$= \frac{(1 + \theta)R + (1 - \theta)(\gamma r_1 - A)}{2}$$

As in the quantity competition case, $\theta$ $(0; \theta; 1)$ is the degree of nationalization or government control which is positively linked to the proportion of shares the government holds and $(1 - \theta)$ is the degree of disinvestment.³⁰

To solve the game by backward induction, we first determine the equilibrium interest rates for a given $\theta$. Solving the two reaction functions for the interest rates, we get

$$r_0 = \frac{2 + \gamma(1 - \theta)][R - A] + 2\theta(R + A)}{4 - (1 - \theta)^2 \gamma^2}$$

$$r_1 = \frac{2 + \gamma][R - A] + \gamma \theta(R + A)}{4 - (1 - \theta)^2 \gamma^2}$$

²⁹It can be hypothesised that the government’s objectives may vary depending on the level of operation. While at the level of the bank, the government representative on the bank’s board is instructed to look after the welfare effects in his market alone, the disinvestment authority may have a broader concern in terms of aggregate welfares. For simplicity, we assume that the government’s objective remains the same at both levels, and the insights we derive, as we show later, can be applied to the general case also.

³⁰Note that $\tilde{RF} : r_0 = R$ and $RF_0 : r_0 = \frac{R - A + \gamma r_1}{2}$
and equilibrium deposits as

\[ D_0 = \frac{[2 + \gamma](1 + \theta)[A + R(1 - \gamma)]}{4 - (1 - \theta)\gamma^2} \]  \hspace{1cm} (10)

\[ D_1 = \frac{[2 + \gamma(1 - \theta)][A + R(1 - \gamma)]}{4 - (1 - \theta)\gamma^2} \]  \hspace{1cm} (11)

Comparing the above expressions, we observe that the public bank offers a higher interest rate and mobilizes more deposits than the private bank (i.e. \( D_0 > D_1, r_0 > r_1 \)). This is because of the social welfare objective of the government which takes into account depositors’ benefit. By putting \( \theta = 0 \) we can verify that \( D_0 = D_1 \) and \( r_0 = r_1 \). This point is similar to the quantity competition case.

Next the equilibrium profits are,

\[ \pi_0 = \frac{(2 + \gamma)^2[A + R(1 - \gamma)]^2(1 - \theta)(1 + \theta)}{2[4 - (1 - \theta)\gamma^2]^2} \]  \hspace{1cm} (12)

\[ \pi_1 = \frac{[2 + \gamma(1 - \theta)]^2[A + R(1 - \gamma)]^2}{2[4 - (1 - \theta)\gamma^2]^2} \]

In contrast to the quantity competition case, the public bank does not always make higher profit. It does so only if the government’s share in ownership is below a critical level - i.e. \( \theta < \frac{\gamma(2 + \gamma)}{\gamma(2 + \gamma) + 2} \). When \( \theta \) is sufficiently high (above the critical level), the price reaction curve of the public firm shifts out so much that the Bertrand-Nash equilibrium moves closer to the so-called Stackelberg equilibrium with profit maximizing firms. To elaborate more, if both bank 0 and 1, were (fully) privately owned, and bank 0 was a price leader, then the resulting prices (which are the Stackelberg prices) would be similar to that in the simultaneous move game that we are considering. That \( \theta \) causing a shift in the reaction function of the public bank is equivalent to assigning a leadership role in the context of pure profit maximization. Then by the standard result in industrial organization, price leader makes smaller profit than the follower (Dowrick, 1986). With large \( \theta \) this effect sets in.
On the other hand, with smaller $\theta$, the public bank is closer to the so-called Cournot-Nash equilibrium, and by the observation of Fershtman (1990) we know that the partially public firm makes more profit.

Now we arrive at the first stage of the game to determine optimal disinvestment. With the help of equations (*)-(*), one can determine social welfares in both the markets:

$$SW_0 = \frac{(2 + \gamma)^2[A + R(1 - \gamma)]^2(3 - \theta)(1 + \theta)}{2[4 - (1 - \theta)\gamma^2]^2}$$

$$SW_1 = \frac{[2 + \gamma(1 - \theta)]^2[A + R(1 - \gamma)]^2}{2[4 - (1 - \theta)\gamma^2]^2}$$

The government’s objective is to maximize $SW_0$ with respect to $\theta$ subject to $\pi_0 \geq \bar{\pi}_0$. Since it is a one-variable optimization problem, the solution must be given by either the constraint or the objective function alone. Suppose $\hat{\theta}$ sets $\pi_0(\theta)$ as given in equation (9?) equal to $\bar{\pi}_0$.

It can be checked that $SW_0$ is a concave function of $\theta$ with a maximum at $\theta = 1 - \gamma^2$, whereas $SW_1$ is falling all through in $\theta$. Profits of both banks fall with $\theta$.

The reason for obtaining a peak in the social welfare function ($SW_0$) is quite clear. When $\theta$ is low (say close to zero), private participation in the public bank is substantial. Consequently, price is high and the depositor

$$\frac{\partial SW_0}{\partial \theta} = \frac{4(2 + \gamma)^2[A + R(1 - \gamma)]^2(1 - \theta - \gamma^2)}{[4 - (1 - \theta)\gamma^2]^3} \Rightarrow \theta = 1 - \gamma^2$$

$$\frac{\partial^2 SW_0}{\partial \theta^2} = -\frac{4(2 + \gamma)^2[A + R(1 - \gamma)]^2[4 + 2\gamma^2(1 - \theta) - 3\gamma^4]}{[4 - (1 - \theta)\gamma^2]^4} < 0$$

$$\frac{\partial SW_1}{\partial \theta} = -\frac{6\gamma(2 + \gamma)[A + R(1 - \gamma)]^2[2 + \gamma(1 - \theta)]^2 \partial \pi_0}{[4 - (1 - \theta)\gamma^2]^3 \partial \theta} = -\frac{2(2 + \gamma)^2[A + R(1 - \gamma)]^2[4\theta + (1 - \theta)\gamma^2]}{[4 - (1 - \theta)\gamma^2]^3} < 0$$

$$\frac{\partial \pi_1}{\partial \theta} = -\frac{4\gamma(2 + \gamma)[A + R(1 - \gamma)]^2[2 + \gamma(1 - \theta)]}{[4 - (1 - \theta)\gamma^2]^3} < 0$$

25
surplus is moderate. As $\theta$ increases, profit falls, depositor surplus increases. But at low $\theta$, the increase in the depositor surplus dominates the fall in profit. Thus the social welfare increases. This continues up to $\theta = 1 - \gamma^2$. Beyond this, the increase in the depositor surplus begins to abate and gets outweighed by the fall in profit, which leads to an overall decline in social welfare.

This discussion helps us to conclude that the optimal $\theta$ is given by $\text{minimum}[\tilde{\theta}, 1 - \gamma^2]$. The graphical solution is given in figure (2).
Figure 2: Price Competition
To summarize, we arrive at the following proposition.

**Proposition 6**  (a) Deposits and interest rate of the public bank are higher than that of the private bank. However, profit of the public bank is lower (greater) than that of the private bank, if $\theta > (\leq) \frac{\gamma(2+\gamma)}{(2+\gamma)+2}.$

(b) Optimal disinvestment is given by $\theta = \min[\tilde{\theta}, 1 - \gamma^2]$

It is worthwhile to note that when $\gamma$ is relatively small, the optimal disinvestment is likely to be given by $\tilde{\theta}$ (rather than $1 - \gamma^2$). The outcome is qualitatively similar to the homogeneous product and quantity competition case as discussed in section 3. But if $\gamma$ is sufficiently high (i.e. the products being closer substitutes), the social welfare function reaches its peak much earlier, largely because of the feedback effect of the rival’s interest rate, which is a strategic complement. Consequently, a greater degree of disinvestment is chosen which gives rise to profit of the public bank in excess of the reservation level.\(^{32}\)

Thus we see that the profitability constraint is not the all important determinant for disinvestment. Social welfare considerations are also important, particularly when the products are closer substitutes. It can also be argued that in the framework of product differentiation and price competition, the degree of disinvestment is likely to be higher and more so if the disinvestment authority were concerned with the welfare in the second market as well.\(^{33}\)

Social welfare in the second market always falls in $\theta$, hence when it is included in the government’s objective function, disinvestment will be even

\(^{32}\)Check that if $\gamma = 0$ in equation (*), $SW_0$ would be an increasing function, and $SW_2$ would be unaffected by $\theta$. Therefore, $\gamma > 0$ plays a crucial role in determining the shape of the social welfare function, and consequently optimal disinvestment.

\(^{33}\)This is evident from the fact that $SW_1$ is a declining function of $\theta$. So if an optimal $\theta \geq 0$ exists, it must be less than $(1 - \gamma^2)$. 

28
more. The earlier the rise of the social welfare function is arrested, greater is the disinvestment. In the quantity competition case, the social welfare function was always rising in $\theta$. Hence the optimal disinvestment would be less. In other words, price competition would lead to greater disinvestment.

6 Conclusion

This paper provides a theoretical view of the complementarity of disinvestment and entry deregulation in the banking industry. We study a ’mixed oligopoly’ involving a partly disinvested public bank and $n$ private banks competing for deposits. We show that when entry is exogenously given, while a larger scale of entry is associated with a higher degree of disinvestment, there is an upper bound on the scale of entry and consequently the degree of disinvestment that can be allowed by the government. However, we find that entry deregulation along with disinvestment is the best policy for the government from the point of view of social welfare only when private banks are more efficient than the public bank. Finally, we study the case of price competition.

We do not discuss the economic reasons behind the initial decision to disinvest which could be a political decision taken by the government. The government might want to disassociate itself from business and facilitate the entry of market forces in the industry. Our point of inquiry is the strategic role of disinvestment and entry deregulation once the initial decision to disinvest has been taken by the government. Moreover, we do not study the competition for loans which is the other function of a bank. Asymmetric information can be introduced and the deposit supply function can be generalized. Future research could consist of extending our model in the above
directions.
References


