Scattering from long prisms: A comparison between ray tracing combined with diffraction on facets and SVM

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Abstract

A new model suitable for rapid computation of scattering on faceted dielectric objects such as ice crystals, which combines ray tracing with diffraction on flat facets, is tested against SVM. Phase functions and asymmetry parameters are compared for scattering from long hexagonal prisms at normal incidence.

1. Introduction

The importance of ice and mixed-phase clouds to the earth-atmosphere radiation balance and climate is well established. Yet, present understanding of cirrus with regard to scattering properties of ice crystals is weak, which is partly due to inadequate theoretical models. For realistic crystal shapes and sizes accurate models either do not exist, have not yet been adequately verified or are computationally very demanding, especially for larger size parameters. A modified Kirchhoff approximation (MKA) method has been introduced [1] to calculate far fields from classical geometric optics (GO) results, which encouraged the development of the improved GO model [2]. The latter is however computationally expensive. For moderate values of the size parameter the finite difference time domain (FDTD) method can be used [3] but it puts even more severe demands on computational resources. Thus, despite its limitations, geometric optics (GO) combined with projected-area diffraction [4] is still the most widely used model for moderate to large size parameters. Therefore, substantial improvement in the theory of scattering on ice crystals is important if phenomena such as radiative forcing by cirrus are to be understood and the sign and magnitude of cirrus cloud-climate feedback established.

2. Computational Method

We proposed a method using an approximation for the far field direction of the Poynting vector to modify ray tracing calculations of scattering on faceted dielectric bodies to take into account the diffraction of light refracted by and reflected from the facets [5]. For the modelling of scattering on faceted dielectric solids the GO code created by Macke et al. was used as a starting point [4] and modified to account for ray diffraction as a result of interaction with facet edges. Both refracted and reflected rays (including external ones) were subject to diffraction. In the first instance, two-dimensional geometry was used, allowing the modeling of scattering on long prisms (columns) at normal incidence. For small size parameters the model is tested against the Separation of Variables Method (SVM) [6,7].

3. Phase functions and asymmetry parameter

3.1 Definitions

The parameterisation of the scattering function in terms of the asymmetry parameter, g, is important for climate modelling since this parameter determines how much radiation is scattered in the forward and backward hemispheres in the space of scattering directions [8]. Because of symmetry, the scattering matrix for macroscopically isotropic and symmetric media is invariant with respect to the choice of the scattering plane and depends only on the angle between the incident and scattered beams, that is the scattering angle $\theta \in [0,\pi]$ [9]. The (1,1) element of the scattering matrix $p_{11}(\theta)$ is called the phase function and satisfies the normalisation condition

$$\frac{1}{4\pi}\int_{4\pi} d\mathbf{n}_{sca}\, p_{11}(\theta) = \frac{1}{2}\int_0^{\pi} d\theta \sin\theta\, p_{11}(\theta) = 1 \qquad (1)$$

where $\mathbf{n}_{sca}$ is a unit vector pointing in the direction of the scattered beam. The quantity

$$g = \frac{1}{2}\int_0^{\pi} d\theta \sin\theta\, p_{11}(\theta)\cos\theta \qquad (2)$$

is called the asymmetry parameter of the phase function. In the special case of light incidence perpendicular to the axis of long cylinders, scattering will take place almost exclusively in the equatorial plane, and the asymmetry parameter g defined in eq. (2) represents an average for a cylinder rotating around an axis parallel to the incident beam.

3.2. Calculated phase functions and asymmetry parameter

Phase functions and asymmetry parameters g were calculated for size parameter 50 and perpendicular incidence for a range of fixed orientations of the hexagonal cross section with respect to the incident beam (γ_{Euler}=0..30°, 1°steps) using SVM [6,7], classical GO [4] and GO with diffraction on facets [5]. The orientation γ_{Euler}=30° corresponds to perpendicular incidence on a facet of the hexagon, and the graphs g(γ_{Euler}) are mirror symmetric with respect to this orientation. Fig.1 shows phase functions for γ_{Euler}=30° and n=1.31, 1.5 and 1.7407. Fig. 2a, b and c show $g(\gamma_{Euler})$ for refractive indices n=1.31, 1.5 and 1.7407, respectively. As expected, the asymmetry parameter decreases with increasing refractive index. There are ranges of prism orientations where the g-values calculated using GO are lower than for SVM. For these regions classical GO predicts approach and occurrence of total internal reflection. The effect becomes more significant with increasing refractive index (total internal reflection occurs at γ_{Euler} = 17..24°, 3..15°, and 1..17° for n=1.31, 1.5 and 1.7407, respectively). In these regions the results are notably improved by including diffraction on facets, which allows some rays to emerge from the crystal where classical GO predicts total internal reflection (see also sketches on top of Fig.2). For n=1.5 and 1,7407 we notice also an improvement for γ_{Euler} = 18..30° when including diffraction on facets. This is mainly due to the widening of the halo peak (Fig.1), which reduces the asymmetry parameter. For n=1.31 the obtained reduction of asymmetry parameter is too large, which seems to be due to different shapes of the peaks obtained by SVM and GO with diffraction on facets (note that the term $\sin\theta\cos\theta$ in eq.(2) has a maximum at 45°).

4. Conclusions

We have compared phase functions and asymmetry parameters calculated with our new model which combines GO with diffraction on facets with classical GO and SVM. Because of the restriction to small size parameters due to computational limitations of SVM the comparison with SVM is a very strict test for the new model. One notable feature of the phase functions calculated using the new model is the broadening of the halo peak observed for lower size parameters (see Fig.1), an effect which GO alone does not predict as it is essentially size independent. Furthermore, some rays can escape the crystal where GO predicts total internal reflection. The latter effect leads to an improvement of the asymmetry parameter compared to classical GO. Concerning the effect of halo-widening on the asymmetry parameter, not only the peak width but also the shape and its symmetry seem to be important. It is expected that the phase function could be modelled more accurately by improving the approximation of exact diffraction theory implemented in the current version of the model.

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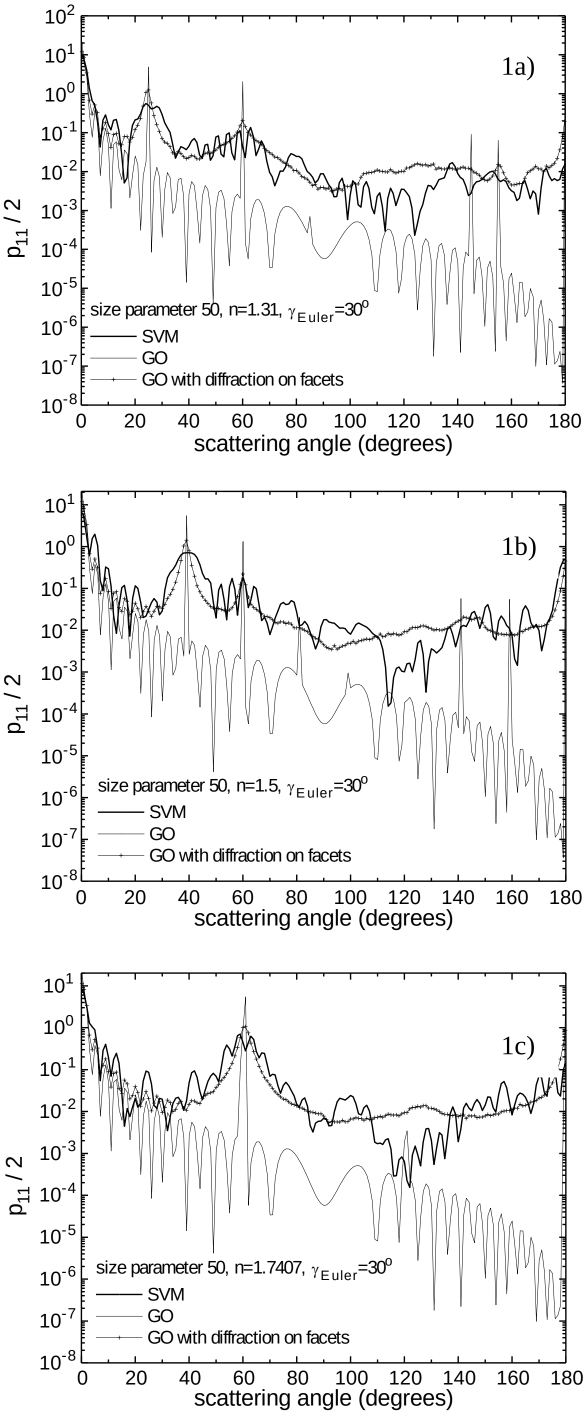

Fig. 1: Phase functions obtained by SVM, GO and GO combined with diffraction on facets for normal incidence, size parameter 50, γ_{Euler}=30°. The refractive index has a value of n=1.31 (a), 1.5 (b) and 1.7407 (c), respectively.

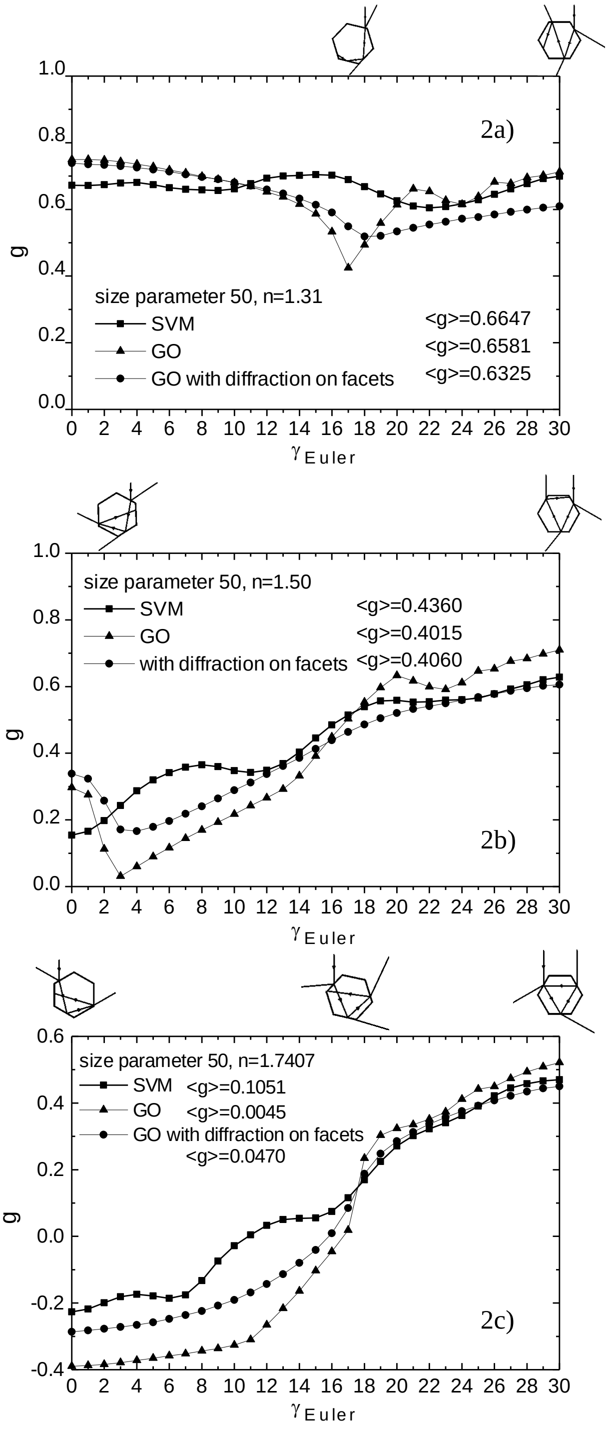

Fig. 2: Asymmetry parameter as a function of prism orientation angle γ_{Euler} obtained by SVM, GO and GO combined with diffraction on facets for normal incidence and size parameter 50. The refractive index has a value of n=1.31 (a), 1.5 (b) and 1.7407 (c), respectively.

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