Event-triggered control for robust exponential synchronization of inertial memristive neural networks under parameter disturbance[☆]

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Abstract

Synchronization of memristive neural networks (MNNs) by using network control scheme has been widely and deeply studied. However, these researches are usually restricted to traditional continuous-time control methods for synchronization of the first-order MNNs. In this paper, we study the robust exponential synchronization of inertial memristive neural networks (IMNNs) with time-varying delays and parameter disturbance via event-triggered control (ETC) scheme. First, the delayed IMNNs with parameter disturbance are changed into first-order MNNs with parameter disturbance by constructing proper variable substitutions. Next, a kind of state feedback controller is designed to the response IMNN with parameter disturbance. Based on feedback controller, some ETC methods are provided to largely decrease the update times of controller. Then, some sufficient conditions are provided to realize robust exponential synchronization of delayed IMNNs with parameter disturbance via ETC scheme.

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Moreover, the Zeno behavior will not happen in all ETC conditions shown in this paper. Finally, numerical simulations are given to verify the advantages of the obtained results such as anti-interference performance and good reliability.

Key words: Event-triggered control, inertial memristive neural networks, robust exponential synchronization, parameter disturbance.

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1. Introduction

Currently, complex networks are widely researched in the world [1, 2, 3, 4, 5]. In recent decades, neural networks and their dynamic behaviors have attracted increasing attention [6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. It is known that neural networks are usually applied in artificial intelligence field via adjusting synaptic weight. Using memristor to simulate synapse on account of the nonvolatile-memory characteristic of memristor, human brain can be emulated by constructing memristive neural networks (MNNs) model [16, 17, 18, 19, 20, 21, 22]. Up to now, synchronization which is an important dynamic behavior of complex systems has been applied to some potential areas [23, 24, 25], for instance, secure communication and image encryption. Therefore, some works on synchronization of MNNs have been studied [26, 27, 28, 29, 30, 31, 32, 33, 34, 35]. In [26], exponential synchronization of recurrent MNNs with time-varying delays was reseached by utilizing fuzzy theory and Lyapunov functional. Wang *et al.* studied exponential synchronization of coupled MNNs and proved that the coupled systems can be synchronized by a small fraction of controlled subsystems under a mild topology condition [27]. Using impulsive control method, dynamical and static multisynchronizations were realized to deal with coupled multistable neural networks in [29, 30]. Combining dynamical and static multisynchronizations, hybrid multisynchronization for delayed coupled MNNs was studied [31]. By using Halanary inequality, lay synchronization of coupled MNNs was investigated in [34].

It is worth noting that the major concern of these studies [26, 27, 28, 29, 30, 31, 32, 33, 34, 35] is the first-order differential MNNs. In recent years, inertial

MNNs (a type of second-order differential systems) and their dynamic behaviors were broadly researched [36, 37, 38, 39, 40, 41, 42, 43], because the inertial term of inertial MNNs (IMNNs) is regarded as an important tool to produce complicated chaos behavior. By utilizing matrix measure method, pinning synchronization of delayed IMNNs was addressed in [36]. Nonsmooth analysis and Lyapunov stability theories were used to deal with global stability of IMNNs with unbounded distributed delays [37]. It is found that synchronization of IMNNs can be achieved by using different control methods [39, 40, 41, 42, 43, 44]. For example, a kind of feedback control method was provided to achieve finite-time synchronization of delayed IMNNs in [39]. Alimi *et al.* realized fixed-time and finite-time synchronization of delayed inertial neural networks by using different state feedback controllers [40]. In [41], finite-time synchronization for delayed IMNNs was researched by using state feedback control method. Gong *et al.* achieved exponential synchronization of IMNNs and delayed coupled IMNNs via nonlinear control method in [42] and [43]. In [44], finite-time synchronization of delayed IMNNs was studied by using delay-dependent control method. These network control schemes designed in [39, 40, 41, 42, 43, 44], such as nonlinear control method and state feedback control method, have been broadly studied on account of high efficiency. However, these network control schemes are based on continuous-time feedback controllers, which means heavy computing burden, even congestion of communication channels will happen.

Event-triggered control (ETC) [45, 46, 47, 48, 49, 50, 51, 52] which is a kind of important sampling control scheme can effectively reduce computing cost and communication resources. Moreover, compared with time-triggered control method [53], ETC can ensure the performance of controlled system and distinctly decrease update times of controller. Hence, a growing number of researchers used ETC scheme to realize certain features of controlled systems [45, 46, 47, 48, 49, 50, 51, 52]. In [46], Zhou *et al.* investigated exponential synchronization problem of Markovian jump delayed complex networks by using a randomly occurring ETC method. For synchronizing coupled switched neural networks with communication delays, an ETC law which could effectively decrease the number of control updates was provided [47]. In [48], asymptotic stability of delayed MNNs was investigated via a discrete sampling ETC scheme for the first time. In [49], quasi-synchronization of delayed MNNs was achieved by using an impulsive ETC method. Furthermore, Zeno behavior did not appear in the controlled MNNs under ETC condition proposed in [49]. By introducing the discontinuous sign term and linear diffusive term, two event-based control schemes were proposed to realize the global synchronization for delayed MNNs [50]. Moreover, under the event-triggering conditions proposed in [50], Zeno behavior did not happen. However, these ETC methods are applied in the first-order systems [46, 47, 48, 49, 50, 51], which cannot be directly used in the second-order systems such as IMNNs due to requiring two types of errors. For synchronizing second-order IMNNs via ETC method, dynamic ETC method and static ETC method were provided to realize the asymptotic synchronization of IMNNs in [52]. As far as we know, there is little work on synchronization of second-order IMNNs under the influence of parameter disturbance via ETC scheme. What's more important is that there usually exist uncertain parameter disturbances in MNNs because of dependence on state for parameters of MNNs and some environmental disturbances. Thus, the uncertain parameter disturbances cannot be ignored on account of their unpredictable influence for MNNs [54, 55, 56, 57].

Inspired by the above discussion, this paper studies the robust synchronization of second-order IMNNs with time-varying delays under the influence of parameter disturbance via ETC scheme. The main contributions are summarized as follows.

1) A kind of state feedback controller is designed in this paper, which does not change value until the next event-triggered instant.

2) Several types of ETC methods based on state feedback controller are designed to deal with the robust synchronization problems and enhance anti-interference performance and reliability of second-order IMNNs.

3) For realizing robust exponential synchronization of delayed IMNNs with the influence of parameter disturbance, some sufficient conditions are provided

by using state feedback controller and ETC scheme.

4) Using ETC methods proposed in this paper, the computing burden and update times of feedback controller can be effectively decreased for the disturbed IMNNs. Moreover, the Zeno behavior does not happen in all ETC conditions.

The rest of the paper is organized as follows. In Section 2, the drive and response IMNNs with parameter disturbance are introduced and these IMNNs are changed into first-order MNNs with parameter disturbance by constructing proper variable substitutions. A kind of state feedback controller and some ETC methods are presented to achieve robust exponential synchronization of delayed IMNNs with parameter disturbance in Section 3. Section 4 provides numerical simulations to verify the validity of the obtained results. Finally, conclusions are given in Section 5.

2. Preliminaries

Notations: For a given vector $a = (a_1, a_2, \ldots, a_r)^T$, $\|a\|_1 = \sum\limits_{k=1}^{r} |a_k|$. For a given matrix $x = [x_{kh}]_{r\times r}$, $\|x\|_1 = \max\limits_{1\leq h\leq r} \sum\limits_{k=1}^{r} |x_{kh}|$. $\lambda_1 = \min\{\lambda(x)\}$ and $\lambda_2 = \max\{\lambda(x)\}$ represent the minimum and maximal eigenvalues of matrix x, respectively.

Consider a delayed IMNN as follows.

$$\begin{aligned}
&\tfrac{\mathrm{d}^2 x_k(t)}{\mathrm{d}t^2} = -v_k \tfrac{\mathrm{d}x_k(t)}{\mathrm{d}t} - o_k x_k(t) + \sum_{h=1}^{r} \alpha_{kh}(x_k(t)) \\
&\times f_h(x_h(t)) + \sum_{h=1}^{r} \beta_{kh}(x_k(t)) f_h(x_h(t-\tau_{kh}(t))) \\
&+I_k(t),\ k = 1, 2, \ldots, r,
\end{aligned} \tag{1}$$

where $\frac{\mathrm{d}^2 x_k(t)}{\mathrm{d}t^2}$ represents an inertial term, $x_k(t)$ denotes the state of the kth neuron, v_k and o_k are constants, time-varying delay $\tau_{kh}(t)$ satisfies $0 \leq \tau_{kh}(t) \leq \tau$, where τ is a positive constant, $f_h(\cdot)$ represents the activation function, $I_k(t)$ expresses the external input. $\alpha_{kh}(x_k(t))$ and $\beta_{kh}(x_k(t))$ represent memristive connection weights, which are given by

$$\alpha_{kh}(x_k(t)) = \begin{cases} \alpha_{1kh}, & \Psi_{k1} \\ \alpha_{2kh}, & \Psi_{k2}, \end{cases} \tag{2}$$

and

$$\beta_{kh}(x_k(t)) = \begin{cases} \beta_{1kh}, & \Psi_{k1}, \\ \beta_{2kh}, & \Psi_{k2}, \end{cases} \tag{3}$$

where α_{1kh}, α_{2kh}, β_{1kh} and β_{2kh} are constants, Ψ_{k1} and Ψ_{k2} represent $|x_k(t)| \leq \ell_k$ and $|x_k(t)| > \ell_k$, respectively, positive constant $\ell_k > 0$ denotes a switching jump.

Because of dependence on state for parameters of MNNs and some environmental disturbances, there usually exist uncertain bounded parameter disturbances in MNNs, actually. Hence, the more realistic delayed IMNN system can be written as follows.

$$\begin{aligned} &\tfrac{\mathrm{d}^2 x_k(t)}{\mathrm{d}t^2} = -\upsilon_k \tfrac{\mathrm{d}x_k(t)}{\mathrm{d}t} - o_k x_k(t) + \sum_{h=1}^{r} [\alpha_{kh}(x_k(t)) \\ &+\Delta\alpha_{kh}(t)]\, f_h(x_h(t)) + \sum_{h=1}^{r} [\beta_{kh}(x_k(t)) + \Delta\beta_{kh}(t)] \\ &\times f_h(x_h(t-\tau_{kh}(t))) + I_k(t),\ k = 1, 2, \ldots, r, \end{aligned} \tag{4}$$

where $\Delta\alpha_{kh}(t)$ and $\Delta\beta_{kh}(t)$ represent the uncertain parameters, and they are bounded as

$$|\Delta\alpha_{kh}(t)| \leq \varsigma_{kh}^{(1)}, \tag{5}$$

$$|\Delta\beta_{kh}(t)| \leq \varsigma_{kh}^{(2)}, \tag{6}$$

where $\varsigma_{kh}^{(1)}$ and $\varsigma_{kh}^{(2)}$ are positive constants.

Set $\hat{\alpha}_{kh} = \max\{|\alpha_{1kh}|, |\alpha_{2kh}|\}$, $\hat{\beta}_{kh} = \max\{|\beta_{1kh}|, |\beta_{2kh}|\}$, $\hat{\upsilon} = [\hat{\alpha}_{kh}]_{r\times r}$, $\hat{\Omega} = [\hat{\beta}_{kh}]_{r\times r}$. The initial conditions of the delayed IMNN (4) are considered as

$$\begin{cases} x_k(s) = \Upsilon_k(s), \\ \frac{\mathrm{d}x_k(s)}{\mathrm{d}s} = \Theta_k(s), \quad -\tau \leq s \leq 0. \end{cases} \tag{7}$$

Consider a constant γ_k and let $q_k(t) = \frac{\mathrm{d}x_k(t)}{\mathrm{d}t} + \gamma_k x_k(t)$, $k = 1, 2, \ldots, r$, then system (4) can be rewritten as

$$\begin{cases} \frac{\mathrm{d}x_k(t)}{\mathrm{d}t} = -\gamma_k x_k(t) + q_k(t), \\ \frac{\mathrm{d}q_k(t)}{\mathrm{d}t} = -(v_k - \gamma_k)q_k(t) - [o_k + \gamma_k(\gamma_k - v_k)] \\ \times x_k(t) + \sum\limits_{h=1}^{r} [\alpha_{kh}(x_k(t)) + \Delta\alpha_{kh}(t)]\, f_h(x_h(t)) \\ + \sum\limits_{h=1}^{r} [\beta_{kh}(x_k(t)) + \Delta\beta_{kh}(t)]\, f_h(x_h(t - \tau_{kh}(t))) \\ + I_k(t) \\ \triangleq -\tilde{v}_k q_k(t) - \tilde{o}_k x_k(t) + \sum\limits_{h=1}^{r} [\alpha_{kh}(x_k(t)) \\ + \Delta\alpha_{kh}(t)]\, f_h(x_h(t)) + \sum\limits_{h=1}^{r} [\beta_{kh}(x_k(t)) + \Delta\beta_{kh}(t)] \\ \times f_h(x_h(t - \tau_{kh}(t))) + I_k(t), \end{cases} \tag{8}$$

where $\tilde{v}_k = v_k - \gamma_k$, $\tilde{o}_k = o_k + \gamma_k(\gamma_k - v_k)$.

Then the initial conditions of the delayed IMNN (8) can be presented by

$$\begin{cases} x_k(s) = \Upsilon_k(s), \\ q_k(s) = \Theta_k(s) + \gamma_k \Upsilon(s), \quad -\tau \leq s \leq 0. \end{cases} \tag{9}$$

Let system (4) be the drive IMNN with parameter disturbance, then the response IMNN with uncertain bounded parameter disturbance can be written as

$$\begin{aligned} &\frac{\mathrm{d}^2 y_k(t)}{\mathrm{d}t^2} = -v_k \frac{\mathrm{d}y_k(t)}{\mathrm{d}t} - o_k y_k(t) + \sum_{h=1}^{r} [\alpha_{kh}(y_k(t)) \\ &+ \Delta\eta_{kh}(t)]\, f_h(y_h(t)) + \sum_{h=1}^{r} [\beta_{kh}(y_k(t)) + \Delta\mu_{kh}(t)] \\ &\times f_h(y_h(t - \tau_{kh}(t))) + I_k(t) + u_k(t),\ k = 1, 2, \ldots, r, \end{aligned} \tag{10}$$

where $u_k(t)$ is the controller, $\Delta\eta_{kh}(t)$ and $\Delta\mu_{kh}(t)$ denote the uncertain parameters, and they are bounded as

$$|\Delta\eta_{kh}(t)| \leq \rho_{kh}^{(1)}, \tag{11}$$

$$|\Delta\mu_{kh}(t)| \leq \rho_{kh}^{(2)}, \tag{12}$$

where $\rho_{kh}^{(1)}$ and $\rho_{kh}^{(2)}$ are positive constants.

Similarly, let $p_k(t) = \frac{\mathrm{d}y_k(t)}{\mathrm{d}t} + \gamma_k y_k(t),\ k = 1, 2, \ldots, r$. Then the response IMNN (10) can be rewritten as

$$\begin{cases} \frac{\mathrm{d}y_k(t)}{\mathrm{d}t} = -\gamma_k y_k(t) + p_k(t), \\ \frac{\mathrm{d}p_k(t)}{\mathrm{d}t} = -\tilde{\upsilon}_k p_k(t) - \tilde{o}_k y_k(t) + \sum\limits_{h=1}^{r} [\alpha_{kh}(y_k(t)) \\ +\Delta\eta_{kh}(t)]\, f_h(y_h(t)) + \sum\limits_{h=1}^{r} [\beta_{kh}(y_k(t)) + \Delta\mu_{kh}(t)] \\ \times f_h(y_h(t - \tau_{kh}(t))) + I_k(t) + u_k(t), \end{cases} \tag{13}$$

where $\tilde{\upsilon}_k = \upsilon_k - \gamma_k$ and $\tilde{o}_k = o_k + \gamma_k(\gamma_k - \upsilon_k)$.

Set errors $E_k(t) = y_k(t) - x_k(t)$ and $J_k(t) = p_k(t) - q_k(t)$. Then we can get errors as

$$\begin{cases} \frac{\mathrm{d}E_k(t)}{\mathrm{d}t} = -\gamma_k E_k(t) + J_k(t), \\ \frac{\mathrm{d}J_k(t)}{\mathrm{d}t} = -\tilde{\upsilon}_k J_k(t) - \tilde{o}_k E_k(t) + \sum\limits_{h=1}^{r} \alpha_{kh}(y_k(t)) \\ \times g_h(E_h(t)) + \sum\limits_{h=1}^{r} [\alpha_{kh}(y_k(t)) - \alpha_{kh}(x_k(t))]\, f_h(x_h(t)) \\ + \sum\limits_{h=1}^{r} [\Delta\eta_{kh}(t) f_h(y_h(t)) - \Delta\alpha_{kh}(t) f_h(x_h(t))] \\ + \sum\limits_{h=1}^{r} \beta_{kh}(y_k(t)) g_h(E_h(t - \tau_{kh}(t))) \\ + \sum\limits_{h=1}^{r} [\beta_{kh}(y_k(t)) - \beta_{kh}(x_k(t))]\, f_h(x_h(t - \tau_{kh}(t))) \\ + \sum\limits_{h=1}^{r} [\Delta\mu_{kh}(t) f_h(y_h(t - \tau_{kh}(t))) \\ -\Delta\beta_{kh}(t) f_h(x_h(t - \tau_{kh}(t)))] + u_k(t). \end{cases} \tag{14}$$

where $g_h(E_h(t)) = f_h(y_h(t)) - f_h(x_h(t))$. Moreover, the vector form of system (14) can be written as

$$\begin{cases} \frac{\mathrm{d}E(t)}{\mathrm{d}t} = -WE(t) + J(t), \\ \frac{\mathrm{d}J(t)}{\mathrm{d}t} = -\tilde{V} J(t) - \tilde{O} E(t) + \upsilon(y(t)) g(E(t)) \\ + [\upsilon(y(t)) - \upsilon(x(t))]\, f(x(t)) \\ +\Delta(\eta(t)) f(y(t)) - \Delta(\alpha(t)) f(x(t)) \\ +\Omega(y(t)) g(E(t - \tau(t))) \\ + [\Omega(y(t)) - \Omega(x(t))]\, f(x(t - \tau(t))) \\ + [\Delta(\mu(t)) f(y(t - \tau(t))) \\ -\Delta(\beta(t)) f(x(t - \tau(t)))] + u(t). \end{cases} \tag{15}$$

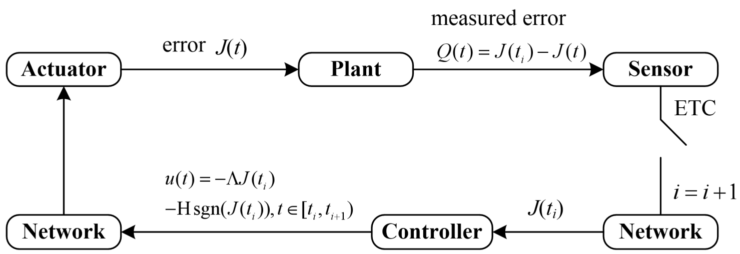

Figure 1: The mechanism of ETC scheme.

where $E(t) = (E_1(t), E_2(t), \ldots, E_r(t))^T$, $J(t) = (J_1(t),\, J_2(t), \ldots, J_r(t))^T$, $W = \mathrm{diag}\{\gamma_1, \gamma_2, \ldots, \gamma_\mathrm{r}\}$, $u(t) = (u_1(t), u_2(t), \ldots, u_r(t))^T$, $\tilde{V} = \mathrm{diag}\{\tilde{v}_1, \tilde{v}_2, \ldots, \tilde{v}_r\}$, $\tilde{O} = \mathrm{diag}\{\tilde{\mathrm{o}}_1, \tilde{\mathrm{o}}_2, \ldots, \tilde{\mathrm{o}}_\mathrm{r}\}$, $g(E(t)) = (g_1(E_1(t)), g_2(E_2(t)), \ldots, g_r(E_r(t)))^T$, $f(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \ldots, f_r(x_r(t)))^T$, $\upsilon(y(t)) = [\alpha_{kh}(y(t))]_{r\times r}$, $\upsilon(x(t)) = [\alpha_{kh}(x(t))]_{r\times r}$, $\Delta(\eta(t)) = [\Delta\eta_{kh}(t)]_{r\times r}$, $\Delta(\alpha(t)) = [\Delta\alpha_{kh}(t)]_{r\times r}$, $\Omega(y(t)) = [\beta_{kh}(y(t))]_{r\times r}$, $\Omega(x(t)) = [\beta_{kh}(x(t))]_{r\times r}$, $\Delta(\mu(t)) = [\Delta\mu_{kh}(t)]_{r\times r}$, $\Delta(\beta(t)) = [\Delta\beta_{kh}(t)]_{r\times r}$.

Set measured error as $Q(t) = J(t_i) - J(t)$, $\forall t \in [t_i, t_{i+1})$. t_i is an event-triggered instant, where $i = 1, 2, 3, \ldots$. For well activating ETC, set the first event-triggered instant $t_1 = 0$. The mechanism of ETC scheme is presented in Figure 1. The controller will be updated under a new triggering event when the measured error oversteps the threshold designed in ETC strategy in advance.

3. ETC for Robust Exponential Synchronization of IMNNs

A kind of state feedback controller is considered as

$$u(t) = -\Lambda J(t_i) - \mathrm{H}\mathrm{sgn}(J(t_i)), t \in [t_i, t_{i+1}), \tag{16}$$

where positive definite matrix $\Lambda = \mathrm{diag}(\Lambda_1, \Lambda_2, \ldots, \Lambda_r)^T$; $\mathrm{H} = \mathrm{diag}(\mathrm{H}_1, \mathrm{H}_2, \ldots, \mathrm{H}_r)^T$; sgn() is the sign function; t_i represents an event-triggered instant.

To get the main results, some necessary definition and assumptions will be presented.

Definition 1. The response IMNN (10) and the drive IMNN (4) with parameter disturbance are said to be robustly exponentially synchronized, if error system (15) is robustly exponentially stable, i.e., there exist positive constants Y and X such that

$$\|E(t)\|_1 \leq Y \sup_{-\tau \leq s \leq 0} \|E(s)\|_1 e^{-Xt} \tag{17}$$

for all $t \geq 0$, where $\sup\limits_{-\tau \leq s \leq 0} \|E(s)\|_1 \neq 0$.

Assumption 1. Function f_h satisfies Lipschitz and bounded conditions, i.e., $|f_h(a_1) - f_h(a_2)| \leq M_h |a_1 - a_2|$ and $|f_h(a_1)| \leq N_h$ for any $a_1,\ a_2 \in R$, $h = 1, 2, \ldots, r$, where M_h and N_h are positive constants.

Assumption 2. $\tau_{kh}(t)$ satisfies

$$\dot{\tau}_{kh}(t) \leq \theta < 1, \tag{18}$$

where $\theta > 0$ is a constant.

Next, we will present some theorems and corollaries about robust exponential synchronization via ETC method.

Theorem 1. IMNN systems (10) and (4) with parameter disturbance can be robustly exponentially synchronized under the controller (16), Assumptions 1 and 2 and the following ETC condition

$$\|Q(t)\|_1 \leq \varphi_1 \frac{(\vartheta \|J(t)\|_1 + \kappa)}{\lambda_2(\Lambda)}, \tag{19}$$

for $t \in [t_i, t_{i+1})$, if

$$\begin{aligned} &\lambda_1(W) > \delta + \max\left\{|\lambda(\tilde{O})|\right\} \\ &+\tfrac{M_{\max} e^{\delta\tau}}{1-\theta} \left\|\hat{\Omega}\right\|_1 + M_{\max} \|\hat{v}\|_1 \end{aligned} \tag{20}$$

$$\begin{cases} \mathrm{H}_k > \sigma_k, & \text{if } \mathrm{sgn}(J_k(t))\mathrm{sgn}(J_k(t_i)) > 0, \\ \mathrm{H}_k \leq -\sigma_k, & \text{otherwise}, \end{cases} \tag{21}$$

and

$$\begin{aligned} \sigma_k > &\sum_{h=1}^{r} \left[|\alpha_{1kh} - \alpha_{2kh}| + |\beta_{1kh} - \beta_{2kh}| \right. \\ &\left. +\rho_{kh}^{(1)} + \varsigma_{kh}^{(1)} + \rho_{kh}^{(2)} + \varsigma_{kh}^{(2)}\right] N_h, \end{aligned} \tag{22}$$

where $\varphi_1 \in (0, 1]$, $M_{\max} = \max\limits_{1 \leq h \leq r} \{M_h\}$, $\vartheta = -\delta - 1 + \lambda_1(\tilde{V}) + \lambda_1(\Lambda) > 0$, and $\kappa = \sum\limits_{k=1}^{r} \left\{\sigma_k - \sum\limits_{h=1}^{r} \left[|\alpha_{1kh} - \alpha_{2kh}| + |\beta_{1kh} - \beta_{2kh}| \ +\rho_{kh}^{(1)} + \varsigma_{kh}^{(1)} + \rho_{kh}^{(2)} + \varsigma_{kh}^{(2)}\right] N_h\right\}$.

Proof. Consider a Lyapunov functional as

$$\begin{aligned}F(t) &= e^{\delta t}\sum_{k=1}^{r}\left[|E_k(t)| + |J_k(t)|\right] \\ &+ \sum_{k=1}^{r}\sum_{h=1}^{r}\tfrac{\hat{\beta}_{kh}}{1-\theta}\int_{t-\tau_{kh}(t)}^{t}|g_h(E_h(s))|\, e^{\delta(s+\tau)}ds.\end{aligned} \tag{23}$$

For $t \in [t_i, t_{i+1})$, the upper right Dini-derivative of $F(t)$ can be given as

$$
\begin{aligned}
&D^+F(t) = \delta e^{\delta t}\left\{\sum_{k=1}^{r}\left[|E_k(t)| + |J_k(t)|\right]\right\} \\
&+e^{\delta t}[\mathrm{sgn}^T(E(t))\dot{E}(t) + \mathrm{sgn}^T(J(t))\dot{J}(t)] \\
&+\sum_{k=1}^{r}\sum_{h=1}^{r}\tfrac{\hat{\beta}_{kh}}{1-\theta}\left[|g_h(E_h(t))|\, e^{\delta(t+\tau)}\right. \\
&\left.-(1-\dot{\tau}_{kh}(t))\,|g_h(E_h(t-\tau_{kh}(t)))|\, e^{\delta(t-\tau_{kh}(t)+\tau)}\right] \\
&\leq \delta e^{\delta t}\left[\|E(t)\|_1 + \|J(t)\|_1\right] + e^{\delta t}\mathrm{sgn}^T(E(t)) \\
&\times\left[-WE(t) + J(t)\right] + e^{\delta t}\mathrm{sgn}^T(J(t)) \\
&\times\left\{-\tilde{V}J(t) - \tilde{O}E(t) + \upsilon(y(t))g(E(t))\right. \\
&+\left[\upsilon(y(t)) - \upsilon(x(t))\right]f(x(t)) + \Delta(\eta(t))f(y(t)) \\
&-\Delta(\alpha(t))f(x(t)) + \Omega(y(t))g(E(t-\tau(t))) \\
&+\left[\Omega(y(t)) - \Omega(x(t))\right]f(x(t-\tau(t))) \\
&+\left[\Delta(\mu(t))f(y(t-\tau(t))) - \Delta(\beta(t))f(x(t-\tau(t)))\right] \\
&\left.-\Lambda J(t_i) - \mathrm{Hsgn}(J(t_i))\right\} + \sum_{k=1}^{r}\sum_{h=1}^{r}e^{\delta t}\hat{\beta}_{kh} \\
&\times\left[\tfrac{1}{1-\theta}\,|g_h(E_h(t))|\, e^{\delta\tau} - |g_h(E_h(t-\tau_{kh}(t)))|\right] \\
&\leq \delta e^{\delta t}\left[\|E(t)\|_1 + \|J(t)\|_1\right] \\
&+e^{\delta t}\left[-\lambda_1(W)\|E(t)\|_1 + \|J(t)\|_1\right] \\
&+e^{\delta t}\left\{-\lambda_1(\tilde{V})\|J(t)\|_1 + \max\left\{|\lambda(\tilde{O})|\right\}\|E(t)\|_1\right. \\
&\left.+M_{\max}\|\hat{\upsilon}\|_1\|E(t)\|_1\right\} + e^{\delta t}\mathrm{sgn}^T(J(t)) \\
&\times\{\left[\upsilon(y(t)) - \upsilon(x(t))\right]f(x(t)) \\
&+\Delta(\eta(t))f(y(t)) - \Delta(\alpha(t))f(x(t)) \\
&+\left[\Omega(y(t)) - \Omega(x(t))\right]f(x(t-\tau(t))) \\
&+\Delta(\mu(t))f(y(t-\tau(t))) - \Delta(\beta(t))f(x(t-\tau(t))) \\
&-\Lambda J(t_i) - \mathrm{Hsgn}(J(t_i))\} \\
&+e^{\delta t}\mathrm{sgn}^T(J(t))\Omega(y(t))g(E(t-\tau(t))) \\
&-\sum_{k=1}^{r}\sum_{h=1}^{r}e^{\delta t}\hat{\beta}_{kh}\,|g_h(E_h(t-\tau_{kh}(t)))| \\
&+\tfrac{M_{\max}e^{\delta t}}{1-\theta}\left\|\hat{\Omega}\right\|_1\|E(t)\|_1 e^{\delta\tau}
\end{aligned}
\tag{24}
$$

Combining with $Q(t) = J(t_i) - J(t)$, we get

$$
\begin{aligned}
&-\mathrm{sgn}^T(J(t))\Lambda J(t_i) = -\mathrm{sgn}^T(J(t))\Lambda\left[J(t) + Q(t)\right] \\
&\leq -\lambda_1(\Lambda)\|J(t)\|_1 + \lambda_2(\Lambda)\|Q(t)\|_1.
\end{aligned}
\tag{25}
$$

According to Assumption 1 and the bounded conditions of uncertain parameters in (5)-(6) and (11)-(12), the following inequalities hold.

$$\begin{aligned}
&\mathrm{sgn}^T(J(t))\left\{[\upsilon(y(t))-\upsilon(x(t))]\,f(x(t))\right.\\
&+\Delta(\eta(t))f(y(t))-\Delta(\alpha(t))f(x(t))\\
&+[\Omega(y(t))-\Omega(x(t))]\,f(x(t-\tau(t)))\\
&+\Delta(\mu(t))f(y(t-\tau(t)))-\Delta(\beta(t))f(x(t-\tau(t)))\\
&\left.-\mathrm{H}\mathrm{sgn}(J(t_i))\right\}\\
&\leq \sum_{k=1}^{r}\sum_{h=1}^{r}\left[|\alpha_{1kh}-\alpha_{2kh}|+|\beta_{1kh}-\beta_{2kh}|\right.\\
&\left.+\rho_{kh}^{(1)}+\varsigma_{kh}^{(1)}+\rho_{kh}^{(2)}+\varsigma_{kh}^{(2)}\right]N_h\\
&-\sum_{k=1}^{r}\mathrm{sgn}(J_k(t))\mathrm{sgn}(J_k(t_i))\mathrm{H}_k\\
&\leq -\sum_{k=1}^{r}\left\{\sigma_k-\sum_{h=1}^{r}\left[|\alpha_{1kh}-\alpha_{2kh}|+|\beta_{1kh}-\beta_{2kh}|\right.\right.\\
&\left.\left.+\rho_{kh}^{(1)}+\varsigma_{kh}^{(1)}+\rho_{kh}^{(2)}+\varsigma_{kh}^{(2)}\right]N_h\right\}\\
&=-\kappa<0.
\end{aligned}\tag{26}$$

In addition, we can obtain

$$\begin{aligned}
&e^{\delta t}\mathrm{sgn}^T(J(t))\Omega(y(t))g(E(t-\tau(t)))\\
&-\sum_{k=1}^{r}\sum_{h=1}^{r}e^{\delta t}\hat{\beta}_{kh}\left|g_h(E_h(t-\tau_{kh}(t)))\right|\\
&=e^{\delta t}\sum_{k=1}^{r}\sum_{h=1}^{r}\left[\mathrm{sgn}(J_k(t))\beta_{kh}(y_k(t))g_h(E_h(t-\tau_{kh}(t)))\right.\\
&\left.-\hat{\beta}_{kh}\left|g_h(E_h(t-\tau_{kh}(t)))\right|\right]\\
&\leq 0.
\end{aligned}\tag{27}$$

Thus, it can be gained that

$$\begin{aligned}
&D^+F(t)\leq e^{\delta t}\left[\delta-\lambda_1(W)+\max\left\{|\lambda(\tilde{O})|\right\}\right.\\
&\left.+\tfrac{M_{\max}e^{\delta\tau}}{1-\theta}\left\|\hat{\Omega}\right\|_1+M_{\max}\|\hat{\upsilon}\|_1\right]\|E(t)\|_1\\
&+e^{\delta t}\left[\delta+1-\lambda_1(\tilde{V})-\lambda_1(\Lambda)\right]\|J(t)\|_1\\
&+e^{\delta t}\lambda_2(\Lambda)\|Q(t)\|_1-e^{\delta t}\kappa\\
&\leq e^{\delta t}(\varphi_1-1)\left[\vartheta\|J(t)\|_1+\kappa\right]\leq 0.
\end{aligned}\tag{28}$$

Therefore,

$$e^{\delta t}\|E(t)\|_1\leq F(t)\leq F(0),\tag{29}$$

and

$$
\begin{aligned}
F(0) = & \sum_{k=1}^{r} \left[|E_k(0)| + |J_k(0)| \right] \\
& + \sum_{k=1}^{r} \sum_{h=1}^{r} \frac{\hat{\beta}_{kh}}{1-\theta} \int_{-\tau_{kh}(0)}^{0} |g_h(E_h(s))| \, e^{\delta(s+\tau)} ds.
\end{aligned} \tag{30}
$$

If $\sup\limits_{-\tau \le s \le 0} \|E(s)\|_1 \neq 0$, there exists a positive constant P_1, such that

$$
\begin{aligned}
& \sum_{k=1}^{r} \left[|E_k(0)| + |J_k(0)| \right] \\
& + \sum_{k=1}^{r} \sum_{h=1}^{r} \frac{\hat{\beta}_{kh}}{1-\theta} \int_{-\tau_{kh}(0)}^{0} |g_h(E_h(s))| \, e^{\delta(s+\tau)} ds \\
& \le P_1 \sup_{-\tau \le s \le 0} \|E(s)\|_1 .
\end{aligned} \tag{31}
$$

Therefore, it can be acquired that

$$
\|E(t)\|_1 \le P_1 \sup_{-\tau \le s \le 0} \|E(s)\|_1 e^{-\delta t}. \tag{32}
$$

Thus, the IMNN (10) can achieve robust exponential synchronization and the IMNN (4) under the ETC condition (19).

Remark 1. Combining the state feedback controller (16) and ETC condition (19), the controller (16) just makes one update when measured error violates ETC condition (19). Therefore, the state feedback controller (16) does not be updated so long as measured error satisfies ETC condition (19). ETC can decrease the computational burden compared with the traditional continuous-time control [39, 40, 41, 42, 43, 44]. Therefore, it is very meaningful for ETC to realize synchronization of IMNNs.

Corollary 1. If inequalities (20)-(22) hold, IMNN systems (10) and (4) with parameter disturbance can be robustly exponentially synchronized under the controller (16), Assumptions 1 and 2 and the following ETC condition

$$
\|Q(t)\|_1 \le \frac{\varphi_1 \vartheta \|J(t)\|_1 + \varphi_2 \kappa}{\lambda_2(\Lambda)}, \tag{33}
$$

for $t \in [t_i, t_{i+1})$, and φ_1, ϑ and κ are given in Theorem 1, $\varphi_2 \in (0, 1]$.

Proof. Using (33) and the proof of Theorem 1, it can be gained that

$$\begin{aligned}
&D^{+}F(t) \leq e^{\delta t}\left[\delta - \lambda_1(W) + \max\left\{|\lambda(\tilde{O})|\right\}\right.\\
&\left.+\tfrac{M_{\max}e^{\delta\tau}}{1-\theta}\left\|\hat{\Omega}\right\|_1 + M_{\max}\|\hat{\upsilon}\|_1\right]\|E(t)\|_1\\
&+e^{\delta t}\left[\delta + 1 - \lambda_1(\tilde{V}) - \lambda_1(\Lambda)\right]\|J(t)\|_1\\
&+e^{\delta t}\lambda_2(\Lambda)\|Q(t)\|_1 - e^{\delta t}\kappa\\
&\leq e^{\delta t}(\varphi_1 - 1)\vartheta\|J(t)\|_1 + e^{\delta t}(\varphi_2 - 1)\kappa \leq 0.
\end{aligned} \tag{34}$$

The rest of proof is same as the proof of Theorem 1. Hence, the IMNN (10) can achieve robust exponential synchronization and the IMNN (4) under the ETC condition (33).

Corollary 2. If inequalities (20)-(22) hold, IMNN systems (10) and (4) with parameter disturbance can be robustly exponentially synchronized under the controller (16), Assumptions 1 and 2 and the following ETC condition

$$\|Q(t)\|_1 \leq \frac{\varphi_1\left(\vartheta\|J(t_i)\|_1 + \kappa\right)}{\lambda_2(\Lambda) + \varphi_1\vartheta}, \tag{35}$$

for $t \in [t_i, t_{i+1})$, and φ_1, ϑ and κ are given in Theorem 1.

Proof. Transforming the inequality (35), we can get

$$\begin{aligned}
&\lambda_2(\Lambda)\|Q(t)\|_1 \leq \varphi_1\left(\vartheta\|J(t_i)\|_1 + \kappa\right) - \varphi_1\vartheta\|Q(t)\|_1\\
&= \varphi_1\vartheta\left(\|J(t_i)\|_1 - \|Q(t)\|_1\right) + \varphi_1\kappa\\
&\leq \varphi_1\vartheta\left(\|J(t_i) - Q(t)\|_1\right) + \varphi_1\kappa = \varphi_1\vartheta\|J(t)\|_1 + \varphi_1\kappa,
\end{aligned} \tag{36}$$

for $t \in [t_i, t_{i+1})$.

Thus, inequality (19) can be acquired from inequality (35), that is to say, the conditions of Theorem 1 are satisfied. Therefore, the IMNN (10) can achieve robust exponential synchronization and the IMNN (4) under the ETC condition (35).

Remark 2. If the following condition holds,

$$\lim_{n\to\infty}\sum_{i=0}^{n}(t_{i+1} - t_i) = Q, \tag{37}$$

then Zeno behavior will happen in event-triggered system [58], where Q is a finite constant. Obviously, Zeno behavior is not expected in event-triggered system.

When execution time $\bar{t}_i = t_{i+1} - t_i$ is bigger than a positive constant, that is to say, event-triggered release times in finite time is finite, then event-triggered system will not exhibit Zeno behavior.

Theorem 2. If the ETC condition of Theorem 1 holds, then IMNN systems (10) and (4) with parameter disturbance can be robustly exponentially synchronized without Zeno behavior, and the execution time $\bar{t}_i = t_{i+1} - t_i$ satisfies the following condition

$$\begin{aligned} &\bar{t}_i > \tfrac{1}{\|\tilde{V}\|_1} \ln\left[\tfrac{\varphi_1 \|\tilde{V}\|_1 (\vartheta \|J(t)\|_1 + \kappa)}{\lambda_2(\Lambda)\mathrm{Z}} + 1\right] \\ &\geq \tfrac{1}{\|\tilde{V}\|_1} \ln\left[\tfrac{\varphi_1 \|\tilde{V}\|_1 \kappa}{\lambda_2(\Lambda)\mathrm{Z}} + 1\right], \end{aligned} \tag{38}$$

where $\mathrm{Z} = \left(\left\|\tilde{V}\right\|_1 + \|\Lambda\|_1 + \left\|\tilde{O}\right\|_1\right) F(0) + \|\mathrm{H}\|_1 + (2\|\hat{\upsilon}\|_1 + 2\left\|\hat{\Omega}\right\|_1 + \varsigma_{kh}^{(1)} + \varsigma_{kh}^{(2)}$ $+ \rho_{kh}^{(1)} + \rho_{kh}^{(2)})N_{\max}$, $N_{\max} = \max\limits_{1\leq h\leq r}\{N_h\}$.

Proof. When event is triggered for $t \in [t_i, t_{i+1})$, we can have

$$\|Q(t_{i+1})\|_1 > \varphi_1 \frac{(\vartheta\|J(t)\|_1 + \kappa)}{\lambda_2(\Lambda)}. \tag{39}$$

In addition,

$$\begin{aligned} &\tfrac{\mathrm{d}}{\mathrm{d}t}\|Q(t)\|_1 \leq \left\|\tfrac{\mathrm{d}}{\mathrm{d}t}Q(t)\right\|_1 = \left\|\dot{J}(t)\right\|_1 \\ &= \Big\| -\tilde{V}J(t) - \tilde{O}E(t) + \upsilon(y(t))f(y(t)) \\ &-\upsilon(x(t))f(x(t)) + \Delta(\eta(t))f(y(t)) - \Delta(\alpha(t))f(x(t)) \\ &+\Omega(y(t))f(y(t-\tau(t))) - \Omega(x(t))f(x(t-\tau(t))) \\ &+\Delta(\mu(t))f(y(t-\tau(t))) - \Delta(\beta(t))f(x(t-\tau(t))) \\ &-\Lambda J(t_i) - \mathrm{H}\mathrm{sgn}(J(t_i))\Big\|_1 \\ &\leq \left\|\tilde{V}\right\|_1 \|J(t)\|_1 + \left\|\tilde{O}\right\|_1 \|E(t)\|_1 \\ &+ \left(2\|\hat{\upsilon}\|_1 + 2\left\|\hat{\Omega}\right\|_1 + \varsigma_{kh}^{(1)} + \varsigma_{kh}^{(2)} + \rho_{kh}^{(1)} + \rho_{kh}^{(2)}\right) N_{\max} \\ &+\|\Lambda\|_1 \|J(t_i)\|_1 + \|\mathrm{H}\|_1 \\ &\leq \left\|\tilde{V}\right\|_1 \|Q(t)\|_1 + \left(\left\|\tilde{V}\right\|_1 + \|\Lambda\|_1\right) \|J(t_i)\|_1 \\ &+\left\|\tilde{O}\right\|_1 \|E(t)\|_1 + \|\mathrm{H}\|_1 \\ &+ \left(2\|\hat{\upsilon}\|_1 + 2\left\|\hat{\Omega}\right\|_1 + \varsigma_{kh}^{(1)} + \varsigma_{kh}^{(2)} + \rho_{kh}^{(1)} + \rho_{kh}^{(2)}\right) N_{\max}. \end{aligned} \tag{40}$$

Combining with the expression of $F(t)$ and $D^+F(t) \leq 0$, we can get

$$e^{\delta t}\|E(t)\|_1 \leq F(t) \leq F(0), \tag{41}$$

$$e^{\delta t}\|J(t)\|_1 \le F(t) \le F(0). \tag{42}$$

That is to say,

$$\|E(t)\|_1 \le e^{-\delta t}F(0) \le F(0), \tag{43}$$

and

$$\|J(t_i)\|_1 \le e^{-\delta t_i}F(0) \le F(0). \tag{44}$$

Then,

$$\begin{aligned}
&\tfrac{\mathrm{d}}{\mathrm{d}t}\|Q(t)\|_1 \le \left\|\tilde{V}\right\|_1\|Q(t)\|_1 + \left(\left\|\tilde{V}\right\|_1 + \|\Lambda\|_1\right)F(0)\\
&+\left\|\tilde{O}\right\|_1 F(0) + \|\mathrm{H}\|_1\\
&+\left(2\|\hat{\upsilon}\|_1 + 2\left\|\hat{\Omega}\right\|_1 + \varsigma_{kh}^{(1)} + \varsigma_{kh}^{(2)} + \rho_{kh}^{(1)} + \rho_{kh}^{(2)}\right)N_{\max}\\
&\le \left\|\tilde{V}\right\|_1\|Q(t)\|_1 + \mathrm{Z}.
\end{aligned} \tag{45}$$

Because $Q(t_i) = 0$, we can have

$$\|Q(t)\|_1 \le \frac{\mathrm{Z}}{\left\|\tilde{V}\right\|_1}\left[e^{\left\|\tilde{V}\right\|_1(t-t_i)} - 1\right], \tag{46}$$

for $t \in [t_i, t_{i+1})$. Therefore,

$$\begin{aligned}
&\varphi_1\tfrac{\left(\vartheta\|J(t)\|_1+\kappa\right)}{\lambda_2(\Lambda)} < \|Q(t_{i+1})\|_1\\
&\le \tfrac{\mathrm{Z}}{\left\|\tilde{V}\right\|_1}\left[e^{\left\|\tilde{V}\right\|_1(t_{i+1}-t_i)} - 1\right],
\end{aligned} \tag{47}$$

and

$$\begin{aligned}
&t_{i+1} - t_i > \tfrac{1}{\left\|\tilde{V}\right\|_1}\ln\left[\varphi_1\left\|\tilde{V}\right\|_1\tfrac{\left(\vartheta\|J(t)\|_1+\kappa\right)}{\lambda_2(\Lambda)\mathrm{Z}} + 1\right]\\
&\ge \tfrac{1}{\left\|\tilde{V}\right\|_1}\ln\left[\tfrac{\varphi_1\left\|\tilde{V}\right\|_1\kappa}{\lambda_2(\Lambda)\mathrm{Z}} + 1\right].
\end{aligned} \tag{48}$$

Therefore, under the conditions of Theorem 1, IMNNs systems (10) and (4) with parameter disturbance can be robustly exponentially synchronized without Zeno behavior.

Corollary 3. If the ETC condition of Corollary 1 holds, then IMNN systems (10) and (4) with parameter disturbance can be robustly exponentially synchronized without Zeno behavior, and the execution time $\bar{t}_i = t_{i+1} - t_i$ satisfies the following condition

$$\begin{aligned} &\bar{t}_i > \frac{1}{\|\tilde{V}\|_1} \ln \left[\frac{\|\tilde{V}\|_1 \left(\varphi_1 \vartheta \|J(t)\|_1 + \varphi_2 \kappa\right)}{\lambda_2(\Lambda) \mathrm{Z}} + 1 \right] \\ &\geq \frac{1}{\|\tilde{V}\|_1} \ln \left[\frac{\|\tilde{V}\|_1 \varphi_2 \kappa}{\lambda_2(\Lambda) \mathrm{Z}} + 1 \right], \end{aligned} \tag{49}$$

where Z and $N_{\max}$ are given in Theorem 2.

Corollary 4. If the ETC condition of Corollary 2 holds, then IMNN systems (10) and (4) with parameter disturbance can be robustly exponentially synchronized without Zeno behavior, and the execution time $\bar{t}_i = t_{i+1} - t_i$ satisfies the following condition

$$\begin{aligned} &\bar{t}_i > \frac{1}{\|\tilde{V}\|_1} \ln \left[\frac{\|\tilde{V}\|_1 \varphi_1 \left(\vartheta \|J(t_i)\|_1 + \kappa\right)}{(\lambda_2(\Lambda) + \varphi_1 \vartheta) \mathrm{Z}} + 1 \right] \\ &\geq \frac{1}{\|\tilde{V}\|_1} \ln \left[\frac{\|\tilde{V}\|_1 \varphi_1 \kappa}{(\lambda_2(\Lambda) + \varphi_1 \vartheta) \mathrm{Z}} + 1 \right], \end{aligned} \tag{50}$$

where Z and $N_{\max}$ are given in Theorem 2.

Set a dynamic variable $\chi_1(t)$ which satisfies the following condition

$$\dot{\chi}_1(t) = -\chi_1(t) + \varphi_1 \left(\vartheta \|J(t)\|_1 + \kappa\right) - \lambda_2(\Lambda) \|Q(t)\|_1, \tag{51}$$

where φ_1, ϑ and κ are given in Theorem 1. The initial value of equality (51) is $\chi_1(0)$ and satisfies $\chi_1(0) \geq 0$.

Then some ETC conditions which contain the dynamic variable $\chi_1(t)$ will be provided to achieve robust exponential synchronization between IMNNs systems (10) and (4) with parameter disturbance.

Theorem 3. If inequalities (20)-(22) hold, IMNN systems (10) and (4) with parameter disturbance can be robustly exponentially synchronized under the controller (16), Assumptions 1 and 2 and the following ETC condition

$$\|Q(t)\|_1 \leq \chi_1(t) + \varphi_1 \frac{\left(\vartheta \|J(t)\|_1 + \kappa\right)}{\lambda_2(\Lambda)}, \tag{52}$$

for $t \in [t_i, t_{i+1})$, and φ_1, ϑ and κ are given in Theorem 1.

Proof. From (51) and (52), it can be gained that

$$\begin{aligned} &\dot{\chi}_1(t) = -\chi_1(t) + \varphi_1 \left(\vartheta \|J(t)\|_1 + \kappa\right) - \lambda_2(\Lambda) \|Q(t)\|_1 \\ &\geq -\chi_1(t) + \lambda_2(\Lambda) \left(\|Q(t)\|_1 - \chi_1(t)\right) - \lambda_2(\Lambda) \|Q(t)\|_1 \\ &= -\left(1 + \lambda_2(\Lambda)\right) \chi_1(t). \end{aligned} \tag{53}$$

It can be obtained that the solution $\chi(t)$ satisfies $\chi(t) \geq 0$ for the equation $\dot{\chi}(t) = -(1+d)\chi(t)$ with $d > 0$ and $\chi(0) \geq 0$. Therefore, we can get that $\chi_1(t) \geq 0$ according to comparison lemma.

Set the following Lyapunov functional

$$
\begin{aligned}
&F_1(t) = F(t) + e^{\delta t}\chi_1(t) \\
&= e^{\delta t} \sum_{k=1}^{r} \left[|E_k(t)| + |J_k(t)| \right] \\
&+ \sum_{k=1}^{r} \sum_{h=1}^{r} \tfrac{\hat{\beta}_{kh}}{1-\theta} \int_{t-\tau_{kh}(t)}^{t} |g_h(E_h(s))| \, e^{\delta(s+\tau)} ds \\
&+ e^{\delta t}\chi_1(t),
\end{aligned} \tag{54}
$$

where $\delta \in (0,1)$. For $t \in [t_i, t_{i+1})$, the upper right Dini-derivative of $F_1(t)$ can be written as

$$
\begin{aligned}
&D^+F_1(t) = D^+F(t) + \delta e^{\delta t}\chi_1(t) + e^{\delta t}\dot{\chi}_1(t) \\
&\leq e^{\delta t} \left[\delta - \lambda_1(W) + \max\left\{ |\lambda(\tilde{O})| \right\} + \tfrac{M_{\max} e^{\delta\tau}}{1-\theta} \left\| \hat{\Omega} \right\|_1 \right. \\
&\left. + M_{\max} \|\hat{\upsilon}\|_1 \right] \|E(t)\|_1 + e^{\delta t} \left[\delta + 1 - \lambda_1(\tilde{V}) - \lambda_1(\Lambda) \right] \\
&\times \|J(t)\|_1 + e^{\delta t}\lambda_2(\Lambda)\|Q(t)\|_1 - e^{\delta t}\kappa \\
&+ \delta e^{\delta t}\chi_1(t) + e^{\delta t} \left[-\chi_1(t) + \varphi_1 \left(\vartheta \|J(t)\|_1 + \kappa \right) \right. \\
&\left. - \lambda_2(\Lambda) \|Q(t)\|_1 \right] \\
&\leq -e^{\delta t}\vartheta \|J(t)\|_1 - e^{\delta t}\kappa + \delta e^{\delta t}\chi_1(t) \\
&+ e^{\delta t} \left[-\chi_1(t) + \varphi_1 \left(\vartheta \|J(t)\|_1 + \kappa \right) \right] \\
&\leq e^{\delta t}(\varphi_1 - 1) \left(\vartheta \|J(t)\|_1 + \kappa \right) + (\delta - 1) e^{\delta t}\chi_1(t) \\
&\leq 0.
\end{aligned} \tag{55}
$$

Therefore,

$$
e^{\delta t} \|E(t)\|_1 \leq F_1(t) \leq F_1(0), \tag{56}
$$

and

$$
\begin{aligned}
F_1(0) = &\sum_{k=1}^{r} \left[|E_k(0)| + |J_k(0)| \right] \\
&+ \sum_{k=1}^{r} \sum_{h=1}^{r} \tfrac{\hat{\beta}_{kh}}{1-\theta} \int_{-\tau_{kh}(0)}^{0} |g_h(E_h(s))| \, e^{\delta(s+\tau)} ds \\
&+ \chi_1(0).
\end{aligned} \tag{57}
$$

If $\sup\limits_{-\tau \leq s \leq 0} \|E(s)\|_1 \neq 0$, there exists a positive constant P_2, such that

$$\begin{aligned}
&\sum_{k=1}^{r} [|E_k(0)| + |J_k(0)|] \\
&+ \sum_{k=1}^{r} \sum_{h=1}^{r} \tfrac{\hat{\beta}_{kh}}{1-\theta} \int_{-\tau_{kh}(0)}^{0} |g_h(E_h(s))| \, e^{\delta(s+\tau)} ds + \chi_1(0) \\
&\leq P_2 \sup_{-\tau \leq s \leq 0} \|E(s)\|_1 .
\end{aligned} \tag{58}$$

Therefore, it can be gained that

$$\|E(t)\|_1 \leq P_2 \sup_{-\tau \leq s \leq 0} \|E(s)\|_1 e^{-\delta t}, \tag{59}$$

where $\delta \in (0, 1)$.

Thus, the IMNN (10) can achieve robust exponential synchronization and the IMNN (4) under the ETC condition (52).

Set another dynamic variable $\chi_2(t)$ which satisfies

$$\begin{aligned}
\dot{\chi}_2(t) = -\chi_2(t) + \varphi_1 \vartheta \|J(t)\|_1 \\
+\varphi_2 \kappa - \lambda_2(\Lambda) \|Q(t)\|_1 ,
\end{aligned} \tag{60}$$

where φ_1, ϑ and κ are given in Theorem 1, $\varphi_2 \in (0, 1]$. The initial value of equality (60) is $\chi_2(0)$ and satisfies $\chi_2(0) \geq 0$.

Then new ETC conditions which contain $\chi_2(t)$ will be provided, such that IMNN systems (10) and (4) with parameter disturbance can be robustly exponentially synchronized.

Theorem 4. If inequalities (20)-(22) hold, IMNN systems (10) and (4) with parameter disturbance can be robustly exponentially synchronized under the controller (16), Assumptions 1 and 2 and the following ETC condition

$$\|Q(t)\|_1 \leq \chi_2(t) + \frac{\varphi_1 \vartheta \|J(t)\|_1 + \varphi_2 \kappa}{\lambda_2(\Lambda)}, \tag{61}$$

for $t \in [t_i, t_{i+1})$, and φ_1, ϑ and κ are given in Theorem 1, and $\varphi_2 \in (0, 1]$.

Proof. From (60) and (61), it can be gained that

$$\begin{aligned}\dot{\chi}_2(t) &= -\chi_2(t) + \varphi_1\vartheta\|J(t)\|_1 + \varphi_2\kappa \\ &\quad -\lambda_2(\Lambda)\|Q(t)\|_1 \\ &\geq -\chi_2(t) + \lambda_2(\Lambda)\left(\|Q(t)\|_1 - \chi_2(t)\right) \\ &\quad -\lambda_2(\Lambda)\|Q(t)\|_1 \\ &= -\left(1+\lambda_2(\Lambda)\right)\chi_2(t).\end{aligned} \tag{62}$$

Similarly, we can obtain that $\chi_2(t) \geq 0$ according to comparison lemma.

Set another Lyapunov functional

$$\begin{aligned}&F_2(t) = F(t) + e^{\delta t}\chi_2(t) \\ &= e^{\delta t}\sum_{k=1}^{r}\left[|E_k(t)| + |J_k(t)|\right] \\ &+ \sum_{k=1}^{r}\sum_{h=1}^{r}\tfrac{\hat{\beta}_{kh}}{1-\theta}\int_{t-\tau_{kh}(t)}^{t}|g_h(E_h(s))|\, e^{\delta(s+\tau)}ds + e^{\delta t}\chi_2(t),\end{aligned} \tag{63}$$

where $\delta \in (0,1)$. For $t \in [t_i, t_{i+1})$, the upper right Dini-derivative of $F_2(t)$ can be obtained as

$$\begin{aligned}&D^+F_2(t) = D^+F(t) + \delta e^{\delta t}\chi_2(t) + e^{\delta t}\dot{\chi}_2(t) \\ &\leq e^{\delta t}\left[\delta - \lambda_1(W) + \max\left\{|\lambda(\tilde{O})|\right\} + \tfrac{M_{\max}e^{\delta\tau}}{1-\theta}\left\|\hat{\Omega}\right\|_1\right. \\ &\left.+M_{\max}\|\hat{v}\|_1\right]\|E(t)\|_1 + e^{\delta t}\left[\delta + 1 - \lambda_1(\tilde{V}) - \lambda_1(\Lambda)\right] \\ &\times\|J(t)\|_1 + e^{\delta t}\lambda_2(\Lambda)\|Q(t)\|_1 - e^{\delta t}\kappa \\ &+\delta e^{\delta t}\chi_2(t) + e^{\delta t}\left[-\chi_2(t) + \varphi_1\vartheta\|J(t)\|_1\right. \\ &\left.+\varphi_2\kappa - \lambda_2(\Lambda)\|Q(t)\|_1\right] \\ &\leq -e^{\delta t}\vartheta\|J(t)\|_1 - e^{\delta t}\kappa + \delta e^{\delta t}\chi_2(t) \\ &+e^{\delta t}\left[-\chi_2(t) + \varphi_1\vartheta\|J(t)\|_1 + \varphi_2\kappa\right] \\ &\leq e^{\delta t}(\varphi_1 - 1)\vartheta\|J(t)\|_1 + e^{\delta t}(\varphi_2 - 1)\kappa \\ &+(\delta - 1)e^{\delta t}\chi_2(t) \\ &\leq 0.\end{aligned} \tag{64}$$

Therefore,

$$e^{\delta t}\|E(t)\|_1 \leq F_2(t) \leq F_2(0), \tag{65}$$

and

$$\begin{aligned}&F_2(0) = \sum_{k=1}^{r}\left[|E_k(0)| + |J_k(0)|\right] \\ &+ \sum_{k=1}^{r}\sum_{h=1}^{r}\tfrac{\hat{\beta}_{kh}}{1-\theta}\int_{-\tau_{kh}(0)}^{0}|g_h(E_h(s))|\, e^{\delta(s+\tau)}ds + \chi_2(0).\end{aligned} \tag{66}$$

If $\sup\limits_{-\tau \le s \le 0} \|E(s)\|_1 \neq 0$, there exists a positive constant P_3, such that

$$\begin{aligned}
&\sum_{k=1}^{r} \left[|E_k(0)| + |J_k(0)|\right] \\
&+ \sum_{k=1}^{r} \sum_{h=1}^{r} \tfrac{\hat{\beta}_{kh}}{1-\theta} \int_{-\tau_{kh}(0)}^{0} |g_h(E_h(s))| \, e^{\delta(s+\tau)} ds + \chi_2(0) \\
&\le P_3 \sup_{-\tau \le s \le 0} \|E(s)\|_1 .
\end{aligned} \tag{67}$$

Hence, we can gain

$$\|E(t)\|_1 \le P_3 \sup_{-\tau \le s \le 0} \|E(s)\|_1 e^{-\delta t}, \tag{68}$$

where $\delta \in (0,1)$.

Thus, the IMNN (10) can achieve robust exponential synchronization and the IMNN (4) under the ETC condition (61).

Subsequently, some theorems and corollaries will be presented to verify that IMNNs systems (10) and (4) with parameter disturbance can be robustly exponentially synchronized without Zeno behavior under the ETC conditions of Theorems 3 and 4.

Theorem 5. If the ETC condition of Theorem 3 holds, then IMNN systems (10) and (4) with parameter disturbance can be robustly exponentially synchronized without Zeno behavior, and the execution time $\bar{t}_i = t_{i+1} - t_i$ satisfies the following condition

$$\begin{aligned}
&\bar{t}_i > \tfrac{1}{\left\|\tilde{V}\right\|_1} \ln \left\{ \tfrac{\left\|\tilde{V}\right\|_1}{\mathrm{Z}} \left[\chi_1(t) + \varphi_1 \tfrac{\left(\vartheta \|J(t)\|_1 + \kappa\right)}{\lambda_2(\Lambda)} \right] + 1 \right\} \\
&\ge \tfrac{1}{\left\|\tilde{V}\right\|_1} \ln \left[\tfrac{\left\|\tilde{V}\right\|_1 \varphi_1 \kappa}{\lambda_2(\Lambda) \mathrm{Z}_1} + 1 \right],
\end{aligned} \tag{69}$$

where $\mathrm{Z}_1 = \left(\left\|\tilde{V}\right\|_1 + \|\Lambda\|_1 + \left\|\tilde{O}\right\|_1 \right) F_1(0) + \|\mathrm{H}\|_1 + (2\|\hat{\upsilon}\|_1 + 2\left\|\hat{\Omega}\right\|_1 + \varsigma_{kh}^{(1)} + \varsigma_{kh}^{(2)}$ $+ \rho_{kh}^{(1)} + \rho_{kh}^{(2)}) N_{\max}$, and $N_{\max}$ is given in Theorem 2.

Corollary 5. If the ETC condition of Theorem 4 holds, then IMNN systems (10) and (4) with parameter disturbance can be robustly exponentially synchronized without Zeno behavior, and the execution time $\bar{t}_i = t_{i+1} - t_i$ satisfies the following condition

$$\begin{aligned} \bar{t}_i &> \frac{1}{\left\|\tilde{V}\right\|_1} \ln\left\{ \frac{\left\|\tilde{V}\right\|_1}{\mathrm{Z}} \left[\chi_2(t) + \frac{\varphi_1\vartheta\left\|J(t)\right\|_1 + \varphi_2\kappa}{\lambda_2(\Lambda)}\right] + 1 \right\} \\ &\geq \frac{1}{\left\|\tilde{V}\right\|_1} \ln\left[\frac{\left\|\tilde{V}\right\|_1 \varphi_2\kappa}{\lambda_2(\Lambda)\mathrm{Z}_2} + 1 \right], \end{aligned} \tag{70}$$

where $\mathrm{Z}_2 = \left(\left\|\tilde{V}\right\|_1 + \left\|\Lambda\right\|_1 + \left\|\tilde{O}\right\|_1\right) F_2(0) + \left\|\mathrm{H}\right\|_1 + (2\left\|\hat{\upsilon}\right\|_1 + 2\left\|\hat{\Omega}\right\|_1 + \varsigma_{kh}^{(1)} + \varsigma_{kh}^{(2)}$ $+ \rho_{kh}^{(1)} + \rho_{kh}^{(2)})N_{\max}$, and $N_{\max}$ is given in Theorem 2.

Remark 3. Currently, there are increased researchers using ETC scheme to realize certain features of controlled systems [45, 46, 47, 48, 49, 50, 51, 52]. However, these ETC methods are presented in the first-order systems [46, 47, 48, 49, 50, 51], which cannot be directly utilized in the second-order IMNNs due to requiring two kinds of errors. On the other hand, there usually exist uncertain parameter disturbances in MNNs because of dependence on state for parameters of MNNs and some environmental disturbances. Thus, the uncertain parameter disturbances cannot be ignored on account of their unpredictable influence for MNNs [54, 55, 56, 57]. However, there is little work on ETC for achieving synchronization of second-order IMNNs under the influence of parameter disturbance. Therefore, this paper studies the robust synchronization of second-order delayed IMNNs under the influence of parameter disturbance via ETC scheme.

Remark 4. Compared with traditional control methods for achieving synchronization of MNNs [39, 40, 41, 42, 43, 44], such as nonlinear control method and state feedback control method, the ETC method proposed in this paper can decrease the computing burden and update times of feedback controller. Compared with these ETC methods applied in the first-order systems [46, 47, 48, 49, 50, 51], the ETC method proposed in this paper can deal with second-order systems. Compared with these results without the influence of parameter disturbance [45, 46, 47, 48, 49, 50, 51, 52], this exponential synchronization of delayed IMNNs under parameter disturbance via ETC has many advantages, such as anti-interference performance and good reliability.

Remark 5. In this paper, different types of ETC schemes are obtained in

Theorems 1, 3, 4 and Corollaries 1,2. The rule which produces different types of ETC schemes is to let the upper right Dini-derivative of Lyapunov functional be not more than 0. Take Theorem 1 and Corollary 1 for examples. To let $D^+F(t) \leq 0$ shown in inequalities (28) and (34), the ETC conditions (19) and (33) can be obtained. Thus, different types of ETC conditions can be produced according to the rule of letting the upper right Dini-derivative of Lyapunov functional be not more than 0.

4. Simulation

The section will present an example to verify the effectiveness of the obtained ETC.

Example. Consider a drive IMNN with parameter disturbance as

$$\begin{aligned}
&\tfrac{\mathrm{d}^2 x_k(t)}{\mathrm{d}t^2} = -v_k \tfrac{\mathrm{d}x_k(t)}{\mathrm{d}t} - o_k x_k(t) + \sum_{h=1}^{2} [\alpha_{kh}(x_k(t)) \\
&+\Delta\alpha_{kh}(t)]\, f_h(x_h(t)) + \sum_{h=1}^{2} [\beta_{kh}(x_k(t)) + \Delta\beta_{kh}(t)] \\
&\times f_h(x_h(t-\tau_{kh}(t))) + I_k(t),\ k = 1, 2,
\end{aligned} \tag{71}$$

where $v_1 = v_2 = 4.2$, $o_1 = o_2 = 2.2$, external input $I_1(t) = I_2(t) = 0$. $\tau_{kh}(t) = 0.05{+}0.05\sin(t)$, $k, h = 1, 2$, then we can choose $\tau = 0.1$ and $\theta = 0.05$. Uncertain parameters $\Delta\alpha_{11}(t) = 0.2\cos(t) + 0.1$, $\Delta\alpha_{12}(t) = \Delta\alpha_{21}(t) = 0$, $\Delta\alpha_{22}(t) = 0.16\cos(t) - 0.04$, $\Delta\beta_{11}(t) = \Delta\beta_{21}(t) = 0$, $\Delta\beta_{12}(t) = 0.2\sin(t)$, $\Delta\beta_{22}(t) = 0.14\cos(t) + 1.12$, then their bounded values can be chosen as $\varsigma_{11}^{(1)} = 0.3$, $\varsigma_{12}^{(1)} = \varsigma_{21}^{(1)} = 0$, $\varsigma_{22}^{(1)} = 0.2$, $\varsigma_{11}^{(2)} = \varsigma_{21}^{(2)} = 0$, $\varsigma_{12}^{(2)} = 0.2$, $\varsigma_{22}^{(2)} = 1.26$.

Memristive connection weights can be chosen as:

$$\alpha_{11}(x_1(t)) = \begin{cases} 0.38, & \Psi_{11}, \\ 0.25, & \Psi_{12}, \end{cases} \tag{72}$$

$$\alpha_{12}(x_1(t)) = \begin{cases} 0.16, & \Psi_{11}, \\ -0.26, & \Psi_{12}, \end{cases} \tag{73}$$

$$\alpha_{21}(x_2(t)) = \begin{cases} 0.39, & \Psi_{21}, \\ -0.24, & \Psi_{22}, \end{cases} \tag{74}$$

$$\alpha_{22}(x_2(t)) = \begin{cases} 0.06, & \Psi_{21}, \\ -0.35, & \Psi_{22}, \end{cases} \tag{75}$$

$$\beta_{11}(x_1(t)) = \begin{cases} 0.34, & \Psi_{11}, \\ -0.62, & \Psi_{12}, \end{cases} \tag{76}$$

$$\beta_{12}(x_1(t)) = \begin{cases} 0.13, & \Psi_{11}, \\ -0.46, & \Psi_{12}, \end{cases} \tag{77}$$

$$\beta_{21}(x_2(t)) = \begin{cases} -0.45, & \Psi_{21}, \\ 0.32, & \Psi_{22}, \end{cases} \tag{78}$$

$$\beta_{22}(x_2(t)) = \begin{cases} 0.26, & \Psi_{21}, \\ -0.34, & \Psi_{22}. \end{cases} \tag{79}$$

where switching jump $\ell_1 = \ell_2 = 1.5$.

Then, we can get that $\hat{\upsilon} = \begin{bmatrix} 0.38 & 0.26 \\ 0.39 & 0.35 \end{bmatrix}$, $\hat{\Omega} = \begin{bmatrix} 0.62 & 0.46 \\ 0.45 & 0.34 \end{bmatrix}$ and $\|\hat{\upsilon}\|_1$=0.77, $\left\|\hat{\Omega}\right\|_1 = 1.07$.

Let $\gamma_1 = \gamma_2 = 4$ and $q_k(t) = \frac{\mathrm{d}x_k(t)}{\mathrm{d}t} + 4x_k(t)$, $k = 1, 2$. Then $\lambda_1(W) = 4$ and system (71) can be rewritten as

$$\begin{cases} \frac{\mathrm{d}x_1(t)}{\mathrm{d}t} = -4x_1(t) + q_1(t), \\ \frac{\mathrm{d}x_2(t)}{\mathrm{d}t} = -4x_2(t) + q_2(t), \\ \frac{\mathrm{d}q_1(t)}{\mathrm{d}t} = -0.2q_1(t) - 1.4x_1(t) + \sum\limits_{h=1}^{2} [\alpha_{1h}(x_1(t)) \\ +\Delta\alpha_{1h}(t)]\, f_h(x_h(t)) + \sum\limits_{h=1}^{2} [\beta_{1h}(x_1(t)) \\ +\Delta\beta_{1h}(t)]\, f_h(x_h(t - \tau_{1h}(t))), \\ \frac{\mathrm{d}q_2(t)}{\mathrm{d}t} = -0.2q_2(t) - 1.4x_2(t) + \sum\limits_{h=1}^{2} [\alpha_{2h}(x_2(t)) \\ +\Delta\alpha_{2h}(t)]\, f_h(x_h(t)) + \sum\limits_{h=1}^{2} [\beta_{2h}(x_2(t)) \\ +\Delta\beta_{2h}(t)]\, f_h(x_h(t - \tau_{2h}(t))). \end{cases} \tag{80}$$

Then, we get the response IMNN as

$$\left\{\begin{array}{l}\frac{\mathrm{d}y_1(t)}{\mathrm{d}t}=-4y_1(t)+p_1(t),\\ \frac{\mathrm{d}y_2(t)}{\mathrm{d}t}=-4y_2(t)+p_2(t),\\ \frac{\mathrm{d}p_1(t)}{\mathrm{d}t}=-0.2p_1(t)-1.4y_1(t)+\sum\limits_{h=1}^{2}\left[\alpha_{1h}(y_1(t))\right.\\ \left.+\Delta\eta_{1h}(t)\right]f_h(y_h(t))+\sum\limits_{h=1}^{2}\left[\beta_{1h}(y_1(t))\right.\\ \left.+\Delta\mu_{1h}(t)\right]f_h(y_h(t-\tau_{1h}(t)))+u_1(t),\\ \frac{\mathrm{d}p_2(t)}{\mathrm{d}t}=-0.2p_2(t)-1.4y_2(t)+\sum\limits_{h=1}^{2}\left[\alpha_{2h}(y_2(t))\right.\\ \left.+\Delta\eta_{2h}(t)\right]f_h(y_h(t))+\sum\limits_{h=1}^{2}\left[\beta_{2h}(y_2(t))\right.\\ \left.+\Delta\mu_{2h}(t)\right]f_h(y_h(t-\tau_{2h}(t)))+u_2(t),\end{array}\right. \tag{81}$$

where uncertain parameters $\Delta\eta_{11}(t)=0.15+0.1\sin(t)$, $\Delta\eta_{12}(t)=\Delta\eta_{22}(t)=0$, $\Delta\eta_{21}(t)=0.15\sin(t)+0.13$, $\Delta\mu_{11}(t)=0.12\sin(t)-0.04$, $\Delta\mu_{12}(t)=\Delta\mu_{21}(t)=0$, $\Delta\mu_{22}(t)=0.04\cos(t)+0.22$. Thus, their bounded values can be chosen as $\rho_{11}^{(1)}=0.25$, $\rho_{12}^{(1)}=\rho_{22}^{(1)}=0$, $\rho_{21}^{(1)}=0.28$, $\rho_{11}^{(2)}=0.16$, $\rho_{12}^{(2)}=\rho_{21}^{(2)}=0$, $\rho_{22}^{(2)}=0.26$. Memristive connection weights are the same as the drive IMNN (80).

Setting $f_h(x)=\frac{|x+1|-|x-1|}{2}$, it can be gained that $M_h=1, N_h=1$, $h=1,2$, $M_{\max}=N_{\max}=1$. Combining with

$$\begin{array}{l}\sum\limits_{h=1}^{2}\left[|\alpha_{11h}-\alpha_{21h}|+|\beta_{11h}-\beta_{21h}|\right.\\ \left.+\rho_{1h}^{(1)}+\varsigma_{1h}^{(1)}+\rho_{1h}^{(2)}+\varsigma_{1h}^{(2)}\right]N_h=3.01\end{array} \tag{82}$$

and

$$\begin{array}{l}\sum\limits_{h=1}^{2}\left[|\alpha_{12h}-\alpha_{22h}|+|\beta_{12h}-\beta_{22h}|\right.\\ \left.+\rho_{2h}^{(1)}+\varsigma_{2h}^{(1)}+\rho_{2h}^{(2)}+\varsigma_{2h}^{(2)}\right]N_h=4.41,\end{array} \tag{83}$$

we can choose $\sigma_1=3.05$, $\sigma_2=4.43$. Then it can be obtained that $\kappa=\sum\limits_{k=1}^{2}\left\{\sigma_k-\sum\limits_{h=1}^{2}\left[|\alpha_{1kh}-\alpha_{2kh}|+|\beta_{1kh}-\beta_{2kh}|\ +\rho_{kh}^{(1)}+\varsigma_{kh}^{(1)}+\rho_{kh}^{(2)}+\varsigma_{kh}^{(2)}\right]N_h\right\}=0.06$, and we can set H as follows

$$\left\{\begin{array}{ll}\mathrm{H}_1=3.08, & \text{if } \operatorname{sgn}(J_1(t))\operatorname{sgn}(J_1(t_i))>0,\\ \mathrm{H}_1=-3.08, & \text{otherwise},\end{array}\right. \tag{84}$$

and

$$\begin{cases} \mathrm{H}_2 = 4.48, \quad \text{if } \operatorname{sgn}(J_2(t))\operatorname{sgn}(J_2(t_i)) > 0, \\ \mathrm{H}_2 = -4.48, \quad \text{otherwise}. \end{cases} \tag{85}$$

Considering $\delta = 0.05$, then it can be acquired that

$$\begin{aligned} &\delta + \max\left\{|\lambda(\tilde{O})|\right\} + \tfrac{M_{\max} e^{\delta\tau}}{1-\theta}\left\|\hat{\Omega}\right\|_1 + M_{\max}\|\hat{\upsilon}\|_1 \\ &= 0.05 + 1.4 + \tfrac{1\times e^{0.05\times 0.1}}{1-0.05} \times 1.07 + 1 \times 0.77 \\ &= 3.352 < \lambda_1(W). \end{aligned} \tag{86}$$

Choosing Λ= diag$\{1.0, 1.2\}$, we can get that $\vartheta = -\delta - 1 + \lambda_1(\tilde{V}) + \lambda_1(\Lambda)$ $= -0.05 - 1 + 0.2 + 1.0 = 0.15 > 0$.

Thus, the following ETC conditions can be gained.

1) ETC condition in Theorem 1:

$$\begin{aligned} &\|Q(t)\|_1 \leq \varphi_1 \tfrac{\left(0.15\|J(t)\|_1 + 0.06\right)}{1.2} \\ &= \varphi_1\left(0.125\|J(t)\|_1 + 0.05\right), \end{aligned} \tag{87}$$

2) ETC condition in Theorem 4:

$$\begin{aligned} &\|Q(t)\|_1 \leq \chi_2(t) + \tfrac{0.15\varphi_1\|J(t)\|_1 + 0.06\varphi_2}{1.2} \\ &= \chi_2(t) + 0.125\varphi_1\|J(t)\|_1 + 0.05\varphi_2, \end{aligned} \tag{88}$$

for $t \in [t_i, t_{i+1})$, $\varphi_1 \in (0,1]$, $\varphi_2 \in (0,1]$, where $\dot{\chi}_2(t) = -\chi_2(t) + 0.15\varphi_1\|J(t)\|_1 + 0.06\varphi_2 - 1.2\|Q(t)\|_1$, $\chi_2(0) = 0.14 \geq 0$.

It can be acquired from the conditions of Theorems 1 and 4 that IMNN systems (80) and (81) with parameter disturbance can be robustly exponentially synchronized under the controller (16), Assumptions 1 and 2 and the ETC conditions (87) and (88). When the initial conditions are $(x_1(s), x_2(s))^T = (1.05, 0.08)^T$, $\left(\frac{\mathrm{d}x_1(s)}{\mathrm{d}s}, \frac{\mathrm{d}x_2(s)}{\mathrm{d}s}\right)^T = (0.12, 0.09)^T$, $(y_1(s), y_2(s))^T = (0.17, 1.12)^T$, $\left(\frac{\mathrm{d}y_1(s)}{\mathrm{d}s}, \frac{\mathrm{d}y_2(s)}{\mathrm{d}s}\right)^T = (0.54, 0.16)^T$, we can get $(q_1(t), q_2(t))^T = (4.32, 0.41)^T$, $(p_1(t), p_2(t))^T = (1.22, 4.64)^T$. Considering $\varphi_1 = 0.5$, $\varphi_2 = 0.6$, the robust exponential synchronization of IMNN systems (80) and (81) with parameter disturbance under ETC condition (87) in Theorem 1 and ETC condition (88) in Theorem 4 are revealed in Figs. 2-14. Figs. 2 and 8 exhibit the synchronization errors $E_1(t)$ and $E_2(t)$ of IMNN systems (80) and (81) with parameter

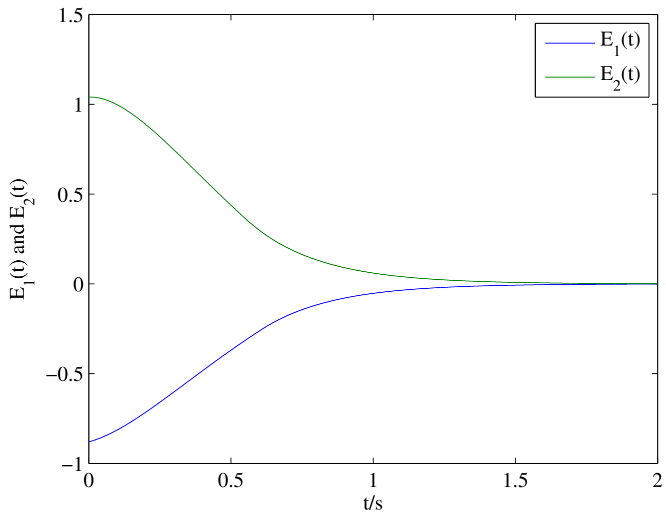

Figure 2: Robust exponential synchronization errors $E_1(t)$ and $E_2(t)$ of IMNN systems (80) and (81) with parameter disturbance under ETC condition (87) in Theorem 1 and $\varphi_1 = 0.5$.

disturbance under ETC conditions (87) and (88), respectively. It clearly shows the synchronization of IMNN systems (80) and (81) with parameter disturbance under ETC condition and verifies the effectiveness of the obtained results.

It is shown from Figs. 3 and 9 that errors $J_1(t)$ and $J_2(t)$ converge to zero under ETC conditions (87) and (88), respectively. Sample error $J_k(t_i)$ and measured error $Q_k(t)$ under ETC conditions (87) and (88) are shown in Figs. 4, 10 and Figs. 5, 11, respectively. It can be acquired from Figs. 4 and 10 that sample error $J_k(t_i)$ does not change if measured error $Q_k(t)$ does not breach the ETC conditions (87) and (88). It is shown in Figs. 6, 7 and Figs. 12, 13, when measured error $Q_k(t)$ breaches the ETC conditions (87) and (88), that is to say, $\|Q(t)\|_1$ oversteps the threshold $\varphi_1\,(0.125\|J(t)\|_1 + 0.05)$ of Theorem 1 and $\chi_2(t)+0.125\varphi_1\|J(t)\|_1+0.05\varphi_2$ of Theorem 4, the event is triggered. The finite number of event-triggered instants displayed in Figs. 7 and 13 reveals that the update times of feedback controller are effectively decreased. Furthermore, the subgraphs of Figs. 7 and 13 illustrate that the Zeno behavior will not happen in ETC conditions provided in this paper. Fig. 14 shows the trajectory of dynamic

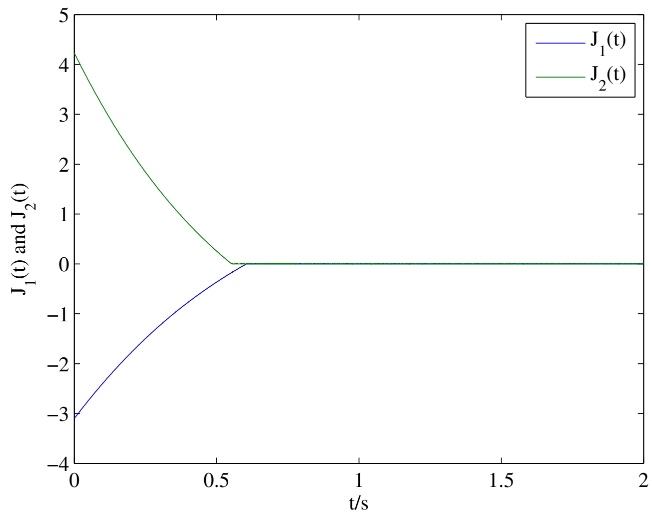

Figure 3: Synchronization errors $J_1(t)$ and $J_2(t)$ of IMNN systems (80) and (81) with parameter disturbance under ETC condition (87) in Theorem 1 and $\varphi_1 = 0.5$.

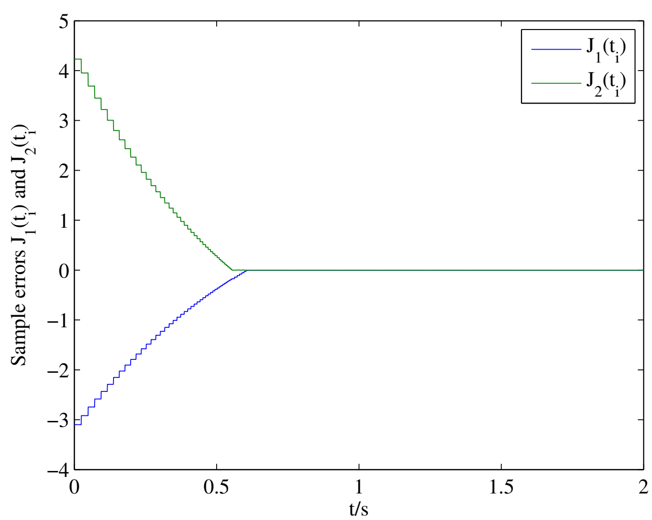

Figure 4: Sample errors $J_1(t_i)$ and $J_2(t_i)$ of IMNN systems (80) and (81) with parameter disturbance under ETC condition (87) in Theorem 1 and $\varphi_1 = 0.5$.

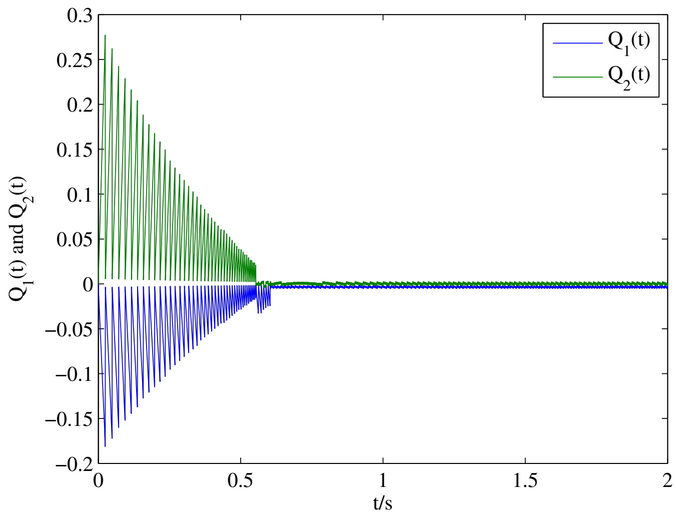

Figure 5: Measured errors $Q_1(t)$ and $Q_2(t)$ of IMNN systems (80) and (81) with parameter disturbance under ETC condition (87) in Theorem 1 and $\varphi_1 = 0.5$.

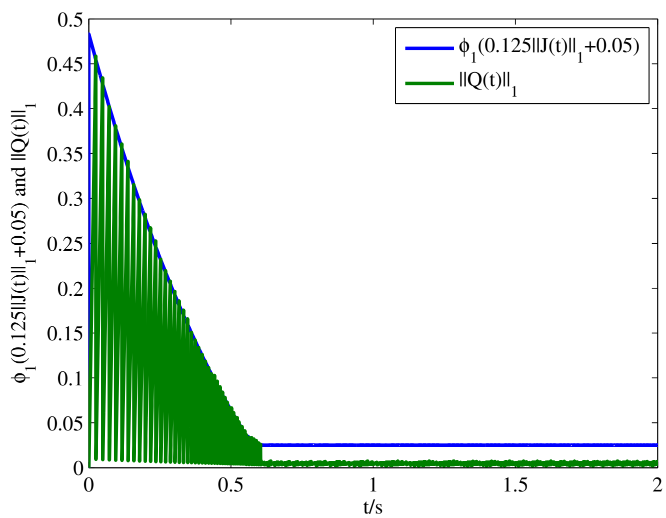

Figure 6: The relation between $\|Q(t)\|_1$ and the threshold $\varphi_1\left(0.125\|J(t)\|_1 + 0.05\right)$ of IMNN systems (80) and (81) with parameter disturbance under ETC condition (87) in Theorem 1 and $\varphi_1 = 0.5$.

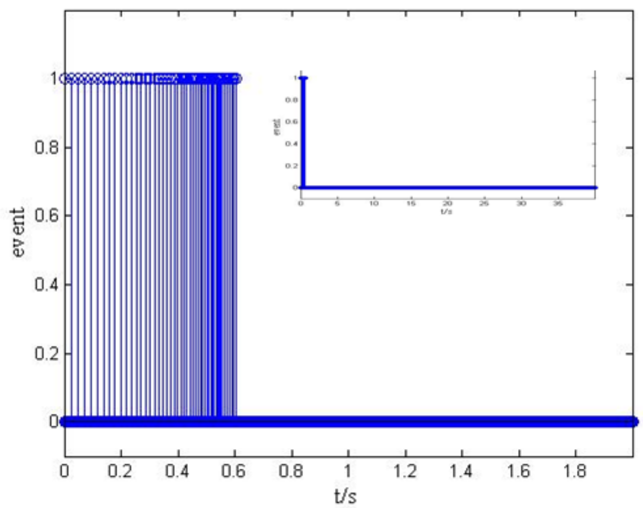

Figure 7: Event-triggered instants of IMNN systems (80) and (81) with parameter disturbance under ETC condition (87) in Theorem 1 and $\varphi_1 = 0.5$.

variable $\chi_2(t)$.

From Figs. 2-14, we can conclude that IMNN systems (80) and (81) with parameter disturbance can be robustly exponentially synchronized under ETC condition (87) in Theorem 1 and ETC condition (88) in Theorem 4. By utilizing ETC and a state feedback controller, the disturbed IMNNs can overcome the influence of parameter disturbance to achieve the exponential synchronization shown in Figs. 2 and 8, which means the disturbed IMNNs have anti-interference performance and good reliability. Moreover, the IMNNs can largely decrease the computing burden and update times of feedback controller via ETC scheme. The obtained results of this paper are effective.

5. Conclusion

This paper deals with the robust exponential synchronization problem of delayed IMNNs with parameter disturbance by utilizing ETC. A controller and some sufficient conditions based on ETC scheme are provided to realize robust

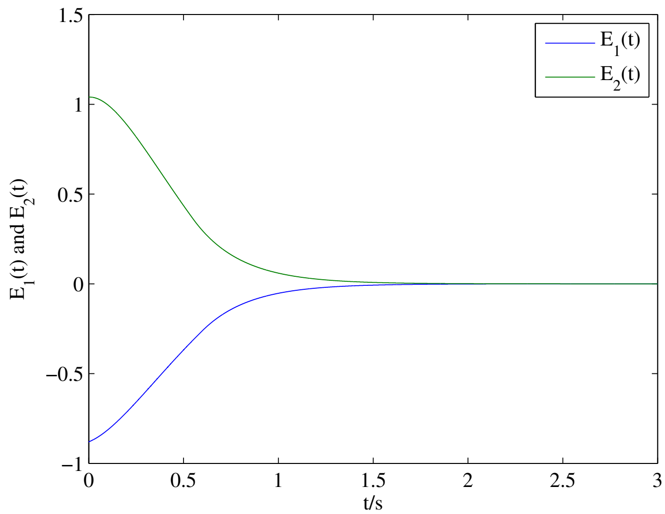

Figure 8: Robust exponential synchronization errors $E_1(t)$ and $E_2(t)$ of IMNN systems (80) and (81) with parameter disturbance under ETC condition (88) in Theorem 4 and $\varphi_1 = 0.5$, $\varphi_2 = 0.6$.

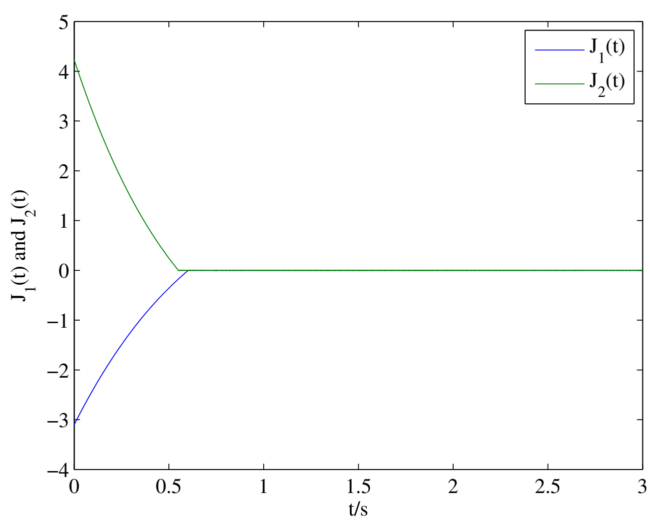

Figure 9: Synchronization errors $J_1(t)$ and $J_2(t)$ of IMNN systems (80) and (81) with parameter disturbance under ETC condition (88) in Theorem 4 and $\varphi_1 = 0.5$, $\varphi_2 = 0.6$.

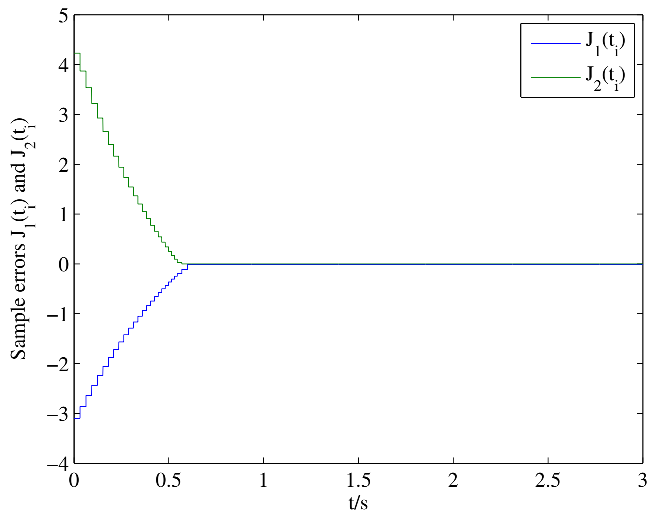

Figure 10: Sample errors $J_1(t_i)$ and $J_2(t_i)$ of IMNN systems (80) and (81) with parameter disturbance under ETC condition (88) in Theorem 4 and $\varphi_1 = 0.5$, $\varphi_2 = 0.6$.

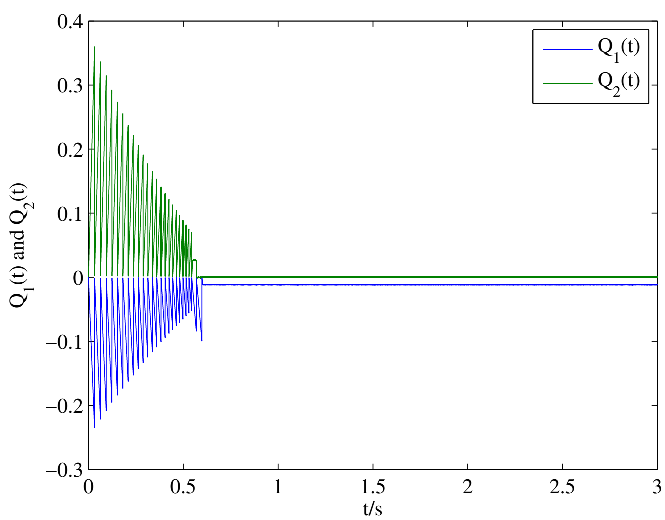

Figure 11: Measured errors $Q_1(t)$ and $Q_2(t)$ of IMNN systems (80) and (81) with parameter disturbance under ETC condition (88) in Theorem 4 and $\varphi_1 = 0.5$, $\varphi_2 = 0.6$.

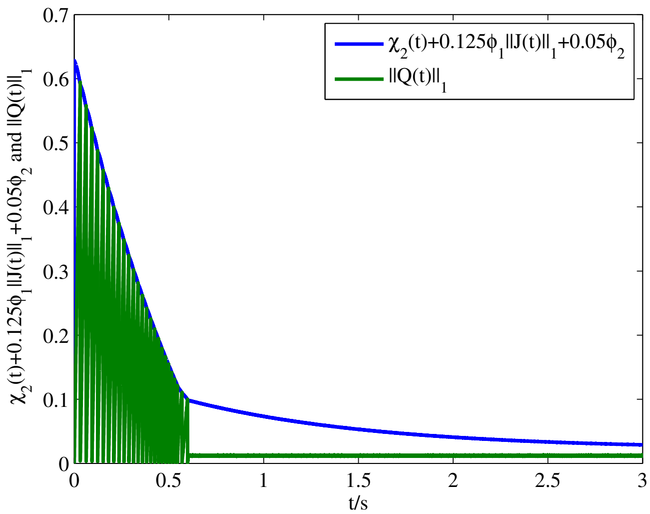

Figure 12: The relation between $\|Q(t)\|_1$ and the threshold $\chi_2(t) + 0.125\varphi_1 \|J(t)\|_1 + 0.05\varphi_2$ of IMNN systems (80) and (81) with parameter disturbance under ETC condition (88) in Theorem 4 and $\varphi_1 = 0.5$, $\varphi_2 = 0.6$.

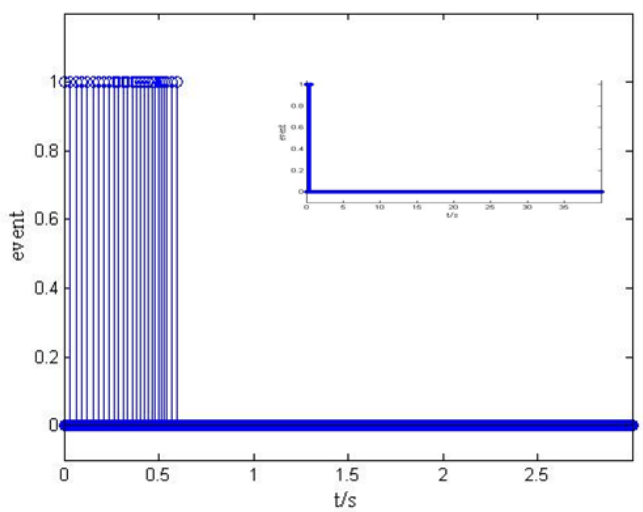

Figure 13: Event-triggered instants of IMNN systems (80) and (81) with parameter disturbance under ETC condition (88) in Theorem 4 and $\varphi_1 = 0.5$, $\varphi_2 = 0.6$.

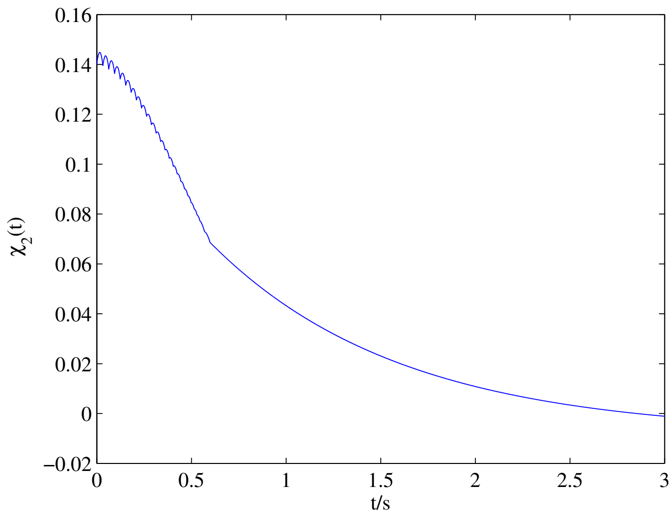

Figure 14: Dynamic variable $\chi_2(t)$ of ETC condition (88) in Theorem 4.

exponential synchronization of IMNNs under parameter disturbance. By using ETC scheme, the computing burden and update times of controller are effectively reduced. Compared with some existing results, this exponential synchronization of delayed IMNNs under parameter disturbance via ETC has many advantages, such as anti-interference performance and good reliability. Considering the excellent performance of ETC, it is very meaningful to research dynamical behaviors of different classes of MNNs via ETC scheme in the future.

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