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Lakatos and the Historical Approach to Philosophy of Mathematics

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At between 20,000 and 30,000 words, books in the Cambridge Elements series are an effort by Cambridge University Press to create a middle ground between the journal article and the full-length monograph. There are so far twenty Elements titles in philosophy of mathematics. This one, by Donald Gillies, is not principally a contribution to Lakatos scholarship, though it does gain some details from Gillies having studied for his doctorate under Lakatos. After a brief introduction, the second section neatly summarises Lakatos's *Proofs and Refutations* and presents it as the original and exemplary introduction of the historical method into philosophy of mathematics.

The third section explores the first part of Lakatos's legacy, namely contributions to philosophy of mathematics which appealed to historical case studies, published in the two decades after Lakatos's death. Gillies picks out works by Michael Crowe, Joseph Dauben, and papers by Caroline Dunsmore, Gillies himself and Giulio Giorello in the collection Gillies edited on revolutions in mathematics (1992). All five of these papers cite Lakatos favourably, so they do belong to his legacy. However, they all address the question of whether there are revolutions in mathematics in anything like the Kuhnian sense. As Gillies acknowledges, this sounds rather more like Kuhn's legacy than Lakatos's. Gillies's answer is that this question about revolutions in mathematics "should be considered as part of both Kuhn's and Lakatos' legacy." (p. 24). Perhaps these authors (and others) needed Lakatos to show the legitimacy of carrying the historical turn from the philosophy of science into the philosophy of mathematics. As Gillies observes, (pp. 2-3) historically minded philosophy of science predates Lakatos's work on mathematics by well over a century. Gillies mentions Whewell (1840), Mach (1883), Duhem (1904-5) and Popper (1963). Perhaps, without Lakatos, historical philosophy of science might have continued indefinitely without ever inspiring historical approaches to philosophy of mathematics. On the other hand, historically minded philosophy of mathematics does have long pedigrees in French and Russian. Brunschvicg (1912) initiated a tradition in France that continued with Cavaillès and Lautman in the inter-war years and survives to this day. Soviet philosophers sought to produce a Marxist philosophy of mathematics by giving a dialectical materialist reading of the history of mathematics. See Barabashev (1986) for a description of the Soviet scene, which Lakatos must have known something about. Even in English, there was at least one work in the two decades after Lakatos's death that puts the history of mathematics at the centre of its philosophical account, namely Kitcher (1983). Kitcher's book does not directly invoke Kuhn or Lakatos (though he did work with Kuhn while at Princeton). It is therefore not the case, from a global perspective, that Lakatos was the first to take a historical approach to philosophy of mathematics, nor do all subsequent works in that vein look back to him. That said, the French tradition in philosophy of mathematics has had no influence on English language philosophy and if the Soviet tradition has had any effect, it is through Lakatos. It is Lakatos, rather than Engels, Brunschvicg or Kitcher, whom philosophers usually cite in the opening paragraphs of books and articles in English that take a historical approach to the philosophy of mathematics.

Perhaps, then, Gillies is correct to suggest that we (English speakers) owe to Lakatos the very idea of historical approaches to the philosophy of mathematics. The Elements format imposes a tight word limit, which required Gillies to make a selection. Nevertheless, the selection he made (in this, third section) gives the erroneous impression that Lakatos's chief contribution was to give philosophers permission to ask Kuhnian questions about mathematics. Kuhn supplies the organising concept for these five papers (namely, revolution). This awkward point might have been avoided, had Gillies chosen to discuss two works that he mentions only to set them aside: Hallett (1979) and Koetsier (1991). These are, in different ways, sustained (book length) attempts to develop Lakatos's philosophy of mathematics and owe little or nothing to Kuhn. Also not mentioned is Glas (1995) (which admittedly seeks to unite Lakatos with Kuhn but at least is not preoccupied with revolutions).

The fourth section reviews some books in philosophy of mathematics from 1995 to the present day which Gillies claims for Lakatos's legacy. He discusses work by Paolo Mancosu, Niccolò Guicciardini, David Corfield, Ladislav Kvasz, Emily Grosholz and Carlo Cellucci. Unlike the previous section, there is no overarching theme. The extent of the intellectual debt to Lakatos various considerably across these works. Mancosu and Guicciardini explore the interplay between mathematics and philosophy in early modern Europe. Mancosu explains how Torricelli's 'horn of plenty' (a solid of rotation with infinite surface area but finite volume) caused trouble for Aristotelianism, and Guicciardini discusses the relationship between Newton's philosophical convictions and his mathematical practice. One might wonder whether these works are philosophy done with history, or simply history of mathematics that includes some history of philosophy. The history of mathematics inevitably involves some philosophy because there has always been, since Pythagoras at the latest, interplay between mathematics and philosophy. Writing up the history of mathematics therefore entails some history of philosophy. We don't see, in Mancosu and Guicciardini, a philosophical model being tested in a case study. We do see Aristotelianism and Newtonianism being severely tested, but Mancosu and Guicciardini write this up as historians. Other work reviewed in this section, most notably by Corfield and Cellucci, does engage directly with Lakatos's philosophy. All four of the remaining books discuss Lakatos and cite him as a source. Gillies highlights Corfield's observation that contemporary mathematics does not seem to divide into distinct research programmes in the way that an application of Lakatos's philosophy of science (the Method of Scientific Research Programmes) would require. Mathematicians with quite different hard-core commitments and positive heuristics can sometimes find their way to the same solution of the same problem. Cellucci describes his approach as a 'heuristic' philosophy of mathematics in a clear nod to Lakatos.

In the final section, Gillies makes the case that the historical approach to the philosophy of mathematics is a progressive research programme in Lakatos's sense. His argument is that the historical approach has identified at least three problems which are of independent interest: the relationship between philosophy of mathematics and the sociology of mathematicians; the status of logic; and the role of axiomatics in mathematics. The first and the third of these seem to me to be on firm ground. Ahistorical philosophy of mathematics either removes the mathematician from the picture altogether, or at most leaves an abstract, attenuated model of the knowing subject. For example, Brouwer's intuitionism rooted mathematics in the subject's experience of the division of time into past and future, but this experience is common to all subjects at all times and places (or so Brouwer seems to have assumed). Hence, although it mentions temporality, it is not in any real sense a historical approach to philosophy. For ahistorical philosophy, the question of the relevance of the sociology of knowledge does not arise. Turning to the third problem, this question about axiomatics is about practice, about axiomatisation as a process and a goal. In other words, it is about heuristics. It may be possible to ask this question in an ahistorical spirit, but it is hard to see how it might have arisen without the turn of philosophical attention to history and practice. The second question, about the status of logic, is less of a clear win for the historical approach because the role of logic as a foundation of mathematics has been complicated by Gödel's results and the emergence of category theory. These would have happened with or without historical approaches to the philosophy of mathematics.

Throughout this book, Gillies writes of *the* historical approach to the philosophy of mathematics, and in the final section he refers to *the* historical research programme. This invites obvious questions. The negative heuristic of this research programme is obviously not to treat mathematics as a body of eternal truths. But what is the hard core? What is the positive heuristic? Examination of the examples here reveals that there are at least two historical approaches. One is the use of historical case studies for testing general philosophical theories. This is what we find in Popperian philosophy of science and in Lakatos's Methodology of Scientific Research Programmes (amongst others). The other is to address philosophical questions about mathematics by telling the history of mathematics over a long stretch of history. This is what we find in Kitcher, possibly in Kvasz and arguably in *Proofs and Refutations*. This latter approach is descended from grand historico-philosophical narratives as written by Hegel, Marx and perhaps Vico. Such narratives face critical questions (about the order of events and the comprehensiveness of the account) that the case-study approach does not. One might further distinguish historical accounts over shorter timescales that are not tests of a philosophical model but rather seek to understand how mathematics changed through the 1660s, say, or the 1930s. The works mentioned by Mancosu, Guicciardini, and Corfield might fit in this category.

These cavils aside, Gillies succeeds in arguing for his principal theses, namely that Lakatos introduced historical approaches to the philosophy of mathematics (in the English-speaking world at least), and that such approaches are producing results that would not otherwise have been forthcoming. Even if we suppose that the turn to history and practice in philosophy of science would have spilled over into philosophy of mathematics without Lakatos, he ensured that historically informed philosophy of mathematics was not dominated by Kuhnian categories and questions. As Gillies notes (p. 17), Lakatos did not found a school. This, on the evidence here presented, is a good thing.[1]

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[1] I am grateful to Donald Gillies for comments on an earlier draft of this review, which helped me to improve it considerably.

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