Consumption, Wealth and Indebtedness in the Context of Uncertainty: The Consumption Function meets Portfolio Theory

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Abstract: The objective of this paper is to provide a sound theoretical framework for the empirical analysis of consumer indebtedness, by integrating Portfolio theory with the Life-Cycle hypothesis (LCH) model of consumption. Modern versions of this LCH theory almost always assume that utility is additive over time, but in this study the multiplicative 'Cobb-Douglas' function is used. The new synthesis also explains the stochastic properties of consumption more fully and clearly than previous studies, in particular the uncertainty arising from rates of return on risky assets. The new theory will also help to improve the explanation of the 'surprise' changes in consumption because these sources of risk are incorporated explicitly into the analysis.

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Introduction[1]

The plan of the discussion within the paper is, first, to outline the crucial elements of Modern Portfolio theory and the Capital Asset Pricing Model. The crucial theoretical contribution of this theory is to categorise all assets as falling into one of only two categories, 'risk free', or 'risky'. Second, the deterministic Life-Cycle theory is used to derive consumption clearly from income and real wealth, but initially in the context of certainty. Third, a synthesis is set out to show unequivocally how individual or household expenditure depends on the risks associated with these two types of asset. The sources of uncertainty are then discussed in the framework of the stochastic consumption function derived. Finally, the theory is applied to consumer indebtedness[i] and aggregate consumption.

Portfolio Theory and the Capital Asset Pricing Model[ii]

Uncertainty in the standard capital asset pricing model (CAPM) relates to the rate of return on each risky asset during each 'period'; investors are taken to be concerned about the expected values and variances (and covariances) of the rates of return on risky assets. By assumption, because differing investors have similar expectations and knowledge of the stochastic properties of the risky assets and borrowings (or holdings of the risk free asset), there is a 'capital market line' in the expected portfolio rate of return and standard deviation space. This is the same line for all investors in the market, that is

$$e_t = r_t + \alpha_t \sigma_t, \qquad [1]$$

where e_t is the expected rate of return earned by the investor in period t;

r_t is the (non-stochastic) rate of return on the risk free asset in period t;

α_t is $(R_t - r_t)/\sigma_t^m$;

R_t is the expected rate of return on the market portfolio of risky assets in period t;

σ_t^m is the standard deviation on the market portfolio of risky assets in period t;

σ_t is the standard deviation on the investor's (whole) portfolio in period t.

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Investors have varying preferences with respect to return and risk, because they choose portfolios on differing points of the capital market line. The power and relevance of the CAPM, is that, according to the model each investor's asset holdings can be taken to consist of only two elements: a share of the market portfolio of risky assets as well as borrowing or lending of the risk free security. This result requires only that investors are risk averse to varying degrees.

To illustrate this theory in a form suitable for application, assume the investor wishes to maximise a 'certainty equivalent' return,

$$E_t = e_t - h_t^2\sigma_t^2, \qquad [2]$$

subject to equation [1][iii]. Ideally, the analysis requires a utility function to be consistent with earning the risk free rate of return, in the absence of risk, or in conditions of certainty, when $\sigma = 0$. The study also requires that the first derivative to be negative with respect to sigma for risk aversion, and the second derivative to be minus for a maximum. Three possibilities, therefore, can be considered: $(1/\sigma)^h$, where 'h' would be some constant; $(1 - h\sigma)$; and $(1 - h^2\sigma^2)$. The first possibility would be undefined in the absence of risk, and has a positive second order partial derivative. The second one has a second order derivative of zero, which is not negative. The third, which is adopted in the analysis, is consistent with conditions of certainty as well as having first and second order partial derivatives that are minus.

Substituting for e_t : maximise $r_r + \alpha_t\sigma_t - h_t^2\sigma_t^2$.
The first order condition is: $\alpha_t - 2h_t^2\sigma_t = 0$,
so $\sigma_t = \alpha_t/(2h_t^2)$. [3]

The investor chooses a level of risk which depends directly on the slope of the capital market line, and inversely with the 'risk aversion' parameter, h. If the individual is almost risk neutral, and h approaches zero, the chosen risk level is very high. If the person is very risk averse, and h is high, then the chosen σ is low, and the rate of return earned will tend towards the risk free rate. Changes in the parameter h have the effect of changing the shares of the risk free asset and the market portfolio of risky assets in the investor's overall portfolio. Extreme risk aversion would mean mostly holding the risk free asset. Approaching risk neutrality would mean extensive borrowing of the risk free asset to fund excessive holding of the market selection of risky assets, that is, excessive in the context of the overall portfolio, from a lender's point of view.

One way of trying to extend the results of this study would be to incorporate extensions of the standard CAPM. For example, for various interest rates on risk free borrowing and lending, or different borrowing rates for diverse investors, but then there can be differences in the constituents of their portfolios of risky assets and the simplicity of the later analysis would be lost.

The Life-Cycle Theory of Consumption[iv]

The most frequently used hypothesis to explain individual or household spending is the Life-Cycle model (Guariglia, 2001), which involves the maximization of a utility function subject to a lifetime budget constraint. In the absence of any borrowing or lending, the consumer accounting identity for each period t, where $t = 1$ to n, holds:

$$Y_t + (1 + E_t)Q_{t-1}A_{t-1} = P_tC_t + Q_tA_t, \quad [4]$$

where C_t denotes household consumption for each period over the lifespan, P_t represents consumer good prices during the interval, Y_t equals non-asset income per year over the work span of the household, plus any welfare payments, which act as automatic stabilizers, A_t stands for asset quantities and Q_t corresponds to asset prices at the end of each phase with E_t equal to the asset rate of return for the period, including appropriate capital gains, which defines the discount factor as:

$$F_t = F_{t-1}(1 + E_t), \quad [5]$$

with $F_0 = 1$. This allows the accounting identity [4] for each period to be divided by its corresponding discount factor.

By summation and collapsing these equations into one intertemporal budget constraint has the effect of permitting borrowing and lending over the whole lifetime, giving:

$$Q_0 + A_0 + \sum_{t=1}^{n} \frac{Y_t}{F_t} = \sum_{t=1}^{n} \frac{P_t C_t}{F_t} + \frac{Q_n A_n}{F_n}. \qquad [6]$$

In words: initial assets plus discounted income equals discounted consumption and discounted final assets[v].

Typically, investors are also consumers. The life cycle theory assumes that every person has a utility function. This is underpinned by the principle of diminishing marginal utility of income, which implies that consumers choose a relatively stable, habit-forming life style. This leads to saving in periods of plenty to add to the stock of wealth in order to maintain a similar level of consumption from period to period, even in retirement. This lifetime utility function of consumption along with the accumulation of final real assets can be expressed as follows:

$$U = U\left[C_1, C_2, \dots \dots, C_t \dots., C_n, \frac{Q_n A_n}{P_n}\right], \qquad [9]$$

where U is of any functional form, where ordinal utility is only unique within the range of monotonic transformations. Such generality, however, prevents the derivation of any explicit consumption solution from the constrained maximisation problem. The analysis, therefore, makes the more specific assumption of a Cobb-Douglas form, where it is convenient to denote final real assets, $\frac{Q_n A_n}{P_n}$, as W_n:

$$U = a\left(C_1^{b_1}, C_2^{b_2}, \dots \dots, C_t^{b_t}, \dots \dots., C_n^{b_n}, W_n^{f}\right). \qquad [8]$$

The coefficients b and f sum to one, as is usual in a Cobb-Douglas formulation, because a monotonic transformation can be chosen to ensure it.

As the survey article by Browning and Lusardi (1996) makes clear, most other work in this field uses utility functions based on the CEQ models, which are additive over time. A false assumption seems to have influenced the literature, that a simple multiplicative form such as is used here, cannot be analytically solved for current consumption. In their words:

“If we wish to analyse many period non-CEQ models then we must have re-course either to approximations,… to the CARA utility form,… or to simulations.” (P.1807)

With this element in mind, the next step is to denote the Lagrange multiplier on the budget constraint as Z, so that the first-order conditions for the maximum can be derived, with respect to each period's consumption along with real assets at the end of the lifespan, which are:

$$\frac{b_t U}{C_t} = \frac{Z P_t}{F_t}, \quad \text{for } t = 1 \text{ to } n, \qquad [9]$$

and

$$\frac{f\,U}{W_n} = \frac{Z\,P_n}{F_n}, \qquad [10]$$

re-arranging:

$$\frac{P_t\,C_t}{F_t} = (U/Z)b_t, \qquad [11]$$

and

$$\frac{P_n W_n}{F_n} = (U/Z)f. \qquad [12]$$

If these equations are summed, the total equals (U/Z) as the b and f amount to one. This sum, however, is also discounted consumption with final assets, which by the intertemporal budget constraint is also equal to present value of (non-asset) income with initial assets:

$$(U/Z) = Q_o A_0 + \sum_{t=1}^{n} \frac{Y_t}{F_t}. \qquad [13]$$

In particular, the analysis finds that the explicit expression for C_1, by substitution for (U/Z) from [13] in equation [11], is:

$$C_1 = gF_1 Q_0 A_0 + gY_1 + gF_1 \left(\sum_{t=2}^{n} \frac{Y_t}{F_t} \right), \qquad [14]$$

where $g = b_1/P_1$.

Equation [14] shows how current consumption depends specifically on returns from real initial assets, current non-asset and benefit earnings, as well as future non-asset and welfare income, appropriately discounted. Differing consumers/investors will have varying resources from these three elements, which may vary with age along with other demographic and economic factors.

Uncertainty is not considered at this stage. The analysis in this section assumes that prospective borrowers are only constrained by their lifetime budget constraint, not short term liquidity constraints. The absence of uncertainty means that actual, expected and 'certainty equivalent' rates of return are all the same. The notation, E_t, relates to the 'certainty equivalent' rate of return, as in the context of Portfolio theory, for relevance to the subsequent synthesis.

Portfolio Theory, Life Cycle Consumption and Sources of Uncertainty in Consumption

The analysis now incorporates the CAPM results, expressed as equations [1], [2], and [3] into the Life cycle theory, exemplified by the consumption function [14], to bring together all the channels of uncertainty into one. Thus, the Life cycle theory is re-examined, incorporating the uncertain rates of return into the utility function and the budget constraint, in the form of 'certainty equivalent' rates of return, E_t. This entails utilising the CAPM assumption that investors' preferences depend on both the expected rates of return on risky assets and on the standard deviations (for each time period), σ. The results will differ from those of earlier 'synthesisers', such as Fama (1970), because the functional form assumptions are more specific here.

The overall market value of assets for an investor, where Q_t and A_t, are defined earlier, becomes

$$Q_t^1 A_t^1 + Q_t^2 A_t^2, \qquad [15]$$

where superscript "1" denotes the risk free asset and "2" denotes the market portfolio of all risky assets at the end of each period t.

There are also rates of return r_t and R_t on each of these types of asset, which depend on the interest, dividends or rental yield, and the movement in prices. The return on the risk free asset, almost by definition, is non-stochastic. An interesting special case is where the risk free asset is a deposit, or short term government bond, in which case $Q_t^1 = 1$ for all t, and the return is 'the' interest rate.[vi] The rate of return on the market portfolio of risky assets, however, is stochastic, and has an expected value, which is the weighted sum of the component rates. The standard deviation depends on the covariance of these elements. The 'certainty equivalent' rate of return is defined from these in equation [2].

Moreover, the Cobb-Douglas assumption, embraced previously, was consistent with respect to final wealth, with the requirements that the first order partial derivative is positive, and the second one is negative. In the case of risk, or the standard deviation of the rate of return, the requests are that the first order be negative (more risk leading to less satisfaction, or lower preference), and that the second one be negative as well, as discussed in the context of the CAPM earlier.

The utility function is still given by [8], but in principle the budget constraint could be used to substitute for real final wealth, $\mathrm{W_n}$, and then the 'certainty equivalent' rates of return, E_t, would appear directly in the utility function. The problem here is specified as maximising a Lagrange function formed from expression [8], subject to the overall budget constraint equation [6], where the rates of return in the discount factors (F) are taken to be the 'certainty equivalent' values, incorporating the capital market lines for each period from 1 to n, equation [1]. This Lagrange is optimised with respect to consumption (C), final wealth (W) and the standard deviations, σ, after substitution for $\mathrm{E_t}$ using equation [2].

Re-calling the Lagrange multiplier on the budget constraint was denoted previously by Z, the conditions for a maximum are still expressions [11] and [12], together with the first order terms for σ. Denoting the whole Lagrange equation as L, and using the function of a function rule, initially for $\partial \mathrm{L}/\partial \sigma_\mathrm{n}$, because it occurs in only the one $\mathrm{F_n}$, then

$$\partial \mathrm{L}/\partial \sigma_\mathrm{n} = (\partial \mathrm{L}/\partial \mathrm{F_n})(\partial \mathrm{F_n}/\partial \mathrm{E_n})(\partial \mathrm{E_n}/\partial \sigma_\mathrm{n}) = 0. \qquad [16]$$

$\partial \mathrm{F_n}/\partial \mathrm{E_n}$ is $\mathrm{F_{n-1}}$, from [5], and not zero. $\partial \mathrm{L}/\partial \mathrm{F_n}$ can be obtained from differentiation of the budget constraint, [6], because $\mathrm{F_n}$ does not appear in the utility function. The result has $\mathrm{F_n^2}$ squared as a denominator, and only some terms from the budget constraint, and therefore is not zero. $\partial \mathrm{L}/\partial \sigma_\mathrm{n} = 0$, however, is exactly how equation [3] is derived, in the context of the CAPM. Expression [3] therefore still holds, at this stage for σ_n in this new framework.

In fact, equation [3] holds for all σ_t, because the study can work down from $n-1$ to 1, taking account of one extra F_t each time. $\partial L/\partial\sigma_t$ includes a $\partial E/\partial\sigma_t$ term for each relevant F, which implies a zero for an optimum. This is because $\partial F/\partial E_t$ is still not zero, and nor is $\partial L/\partial F$ for all the relevant Fs. Equation [3], hence, serves to determine the choices of risk in each period. The expected rates of return e_t can be found by substitution into [1], the capital market line. This means that expression [14] can now be written as

$$C_1 = g(1+E_1)(Q_0A_0) + gY_1 + g(1+E_1)\left(\sum_{t-2}^{n}\frac{Y_t}{F_t}\right), \qquad [17]$$

where $g = b_1/P_1$ and $(1+E_1)$is defined in [2] with F in [5]. Expression [17] shows how the determination of the consumption of an individual household depends on prior real wealth,[vii] current non-asset income, and future discounted non-asset earnings, which is necessarily uncertain. In addition, it illustrates how consumption depends on rates of return,E_t, and discount factors, F_t, which are also uncertain, if the person or household has any share of the market portfolio of risky assets.

From Theory to Practice

There are essentially two sources of uncertainty in the consumption function. There is insecurity in expected future non-asset income, Y_t. This has been analysed by Caballero (1990) as well as Guariglia and Rossi (2002)[viii]. There is also uncertainty, however, in the discount factors (or rates of return), which are specific to each consumer or investor. The source is the uncertainty in the rates of return on the market portfolio of risky assets. This component is present in the original formulation of the Life-Cycle hypothesis, but only implicitly. Definitions of rates of return were left vague. This aspect of the analysis is advanced with the embodiment of the CAPM into the consumption function.

In fact, the factor $(1+E_t)$, which comes from equation [2], brings risk/uncertainty into the 'picture' from the unanticipated innovations (or shocks) that arise from financial markets in the form of capital gains/losses, or interest and dividends from the holding of a "menu of assets" (Deaton, 1992). The risk varies in a predictable way from one investor to another; it is higher for those individuals who have relatively high weightings of the market portfolio of risky assets in their selection; it is low for those persons who have relatively high weightings of the risk free asset. This variation in risk between investors explains why individuals can face higher risks than the aggregate market does, partly because other people choose less risk, and choose higher weightings of the risk free asset in their portfolios. Those households who borrow the risk free asset will have higher expected discount rates than those who hold the risk free asset, or lend it. This is because the expected rate of return on the market portfolio of risky assets is higher than the (non-stochastic) rate of return on the risk free asset, and has a higher contribution to their portfolios. The corollary is that the variance, or standard deviation, will be larger too, because the risky market portfolio has a greater weight for them.

To apply this theory, some categorisation of individual or household assets into 'market portfolio of risky assets' and 'risk free assets' must be made. Company shares, or equity, and real estate, housing or property, would presumably be considered 'risky'. Government bonds, particularly ones with a short maturity, and perhaps index linked bonds which help to remove inflation risk, might be 'risk free'. Some other 'assets' raise more difficulty; for example pension entitlement.

Consumer Indebtedness

Consumer indebtedness can be analysed in the context of the synthesis expressed as equation [17]. The risk aversion parameters, h_t^2, may vary predictably over lifetimes. Older people will have a larger marginal propensity to spend out of wealth because of a prior accumulation of income and property assets to fund an endowment for their retirement period.

An ageing population could mean that the number of consumers wanting to hold risky assets, such as equities for property income growth, may fall relative to safe holdings with a secure income-earning potential, depending on the degree of risk aversion of older households.

By contrast, young individuals plan to consume the increase in wealth over a longer horizon, and therefore will spend a smaller fraction of the increased income from assets in any year whilst borrowing on expected labour income (Betti and et al., 2003). Obviously, the individual consumption function, equation [17], depends on the membership and age of the household, the expected retirement period as well as the presence or absence of social security payments. Briefly, the variables highlighted by this theory are real current and future non-asset income along with the accumulation of wealth in the form of assets from saving, which can be converted into uncertain ‘property’ earnings, or the accumulation of debts and need to pay interest.

Future discounted income, however, is not only dependant on the rate of return, but also is a function of the make-up of the market portfolio of risky assets, such as real estate or property, which not only generate additional income to a household for consumption in the form of mortgage equity, but also determine the degree of liquidity constraint faced by an individual household, as well as the level of ‘indebtedness’ from borrowing.

The difficulty is that, in reality the prospective lenders have differing expectations of future non-asset income from prospective borrowers, which should only be constrained by the life-time budget. Short-term liquidity restrictions can then constrain long-term future expectations.

These short-run expectations of non-asset income inside the economy, in certain circumstances, will override those of the individual or household concerned. Then consumption may depend on the status of individuals within the economy, for example whether or not they are employed. Similarly, it may be easier for certain groups of households to borrow risk free assets if a share of the market portfolio of risky assets is already owned, for example as a house, which is dependent on age and status of individuals.

The Aggregate Consumption Function and Automatic Stabilisers

This discussion relates equation [17] to the contemporary literature. It may be that the prevailing belief within this field of study is fallacious; that unequivocal solutions can only come from utility functions that are quadratic in consumption and additive over time periods. Equation [17] and its derivation demonstrates that explicit solutions can be obtained by employing a Cobb-Douglas utility function, which shows that the presumptions of the economic profession in this regard are unfounded.

The implications for the aggregate consumption function in an economy follow from the summation of equation [17] over all the individual households in the economy. The additive structure of [17] means that aggregate consumption is related to aggregate wealth, aggregate current non-asset income and discounted future non-asset income, although the weighting required involves some complications, because the summation cannot be taken inside the terms.

It may be helpful to categorise households by a number of states at the economy level (Deaton, 1992). The consumption of one category may be determined by liquidity constraints perceived in the short-run, meaning that for a percentage of households, S, their consumption is ruled by Keynes' (1936) marginal propensity to consume out of current disposable income, because they have no assets, and prospective lenders take too poor a view of their income expectations to lend. They can only spend out of their current income, which may be replaced or supplemented by government benefits.

The other segment of the population, $(1 - \mathrm{S})$ will be able to consume from property as well as current non-asset income, and can borrow on discounted future expected income without constraint, although affected by uncertain rates of return. This means that the real cause of changes in consumption is unexpected changes in the rate of return on risky assets. An unexpectedly low return will reduce prior asset value in future periods, and may affect adversely future risk aversion parameters, h, so increasing the 'certainty equivalent' discount rates E and reducing future discounted non-asset income.

In fact, it is in this wealthier category of households that the element of uncertainty is concentrated, because current income is not generally as uncertain as the market portfolio of risky assets. Changes in consumption can be explained from [17] as:

$$\Delta \mathrm{C} = \Delta\left(\mathrm{g}\, Y_1 + g(1+E_1)(Q_0 A_0) + g(1+\ E_1)\sum_{t=2}^{n}\frac{Y_t}{F_t}\right). \qquad [18]$$

The source of uncertainty in changes in consumption can be captured by relating the “surprise” element of the modern theory (Hall, 1978) to the variations in the pattern of consumption from the present trend arising over and above current non-asset income, which can be denoted by ε.

This can also be expressed in the Campbell and Mankiw (1989) form as

$$\Delta C = s(g_1 Y_1) + (1 - S)\varepsilon. \qquad [19]$$

On the one hand, the analysis that underlies expression [19] is reinforced by the empirical findings of

Betti and et al. (2003) that high income status bestowed on a certain per cent of households, embodied in ε can borrow and accumulate liabilities/assets early in their working life, although they could face uncertain ‘surprises’ in their undertakings to maintain a smooth pattern of consumption. According to the analysis here, the uncertainty and the element of surprise could well arise from the uncertain rates of return embodied in [18].

On the other hand, in the other category, low income groups face limited uncertainty because they are credit rationed, and so, consume out of current labour and benefit income to sustain a constant level of consumption. This is reinforced by the function of automatic stabilizers such as the income-support system, or the welfare state, with unemployment benefits and credits, in conjunction with the progressive tax instruments of local and national governments, to insure against shortfalls in current labour income (Muellbauer, 1994). The end-result for this hypothesis is that the built-in fiscal flexibility does not dampen the element of uncertainty and risk associated with asset-generating incomes that arise from the portfolio of assets and indebtedness held to finance the consumption path.

This discussion implies that liquidity constraints together with the welfare system acting as automatic stabilizers lead to certain consumption configurations for certain groups of households in the model. The expectations of future discounted income add a degree of uncertainty because of the varying rates of return. The insecurity of the asset markets is transmitted into property income along with values. In other words, illiquid assets such as equities, housing and land appreciate and depreciate according to the health and uncertainty of the economy as well as depending on the rates of return encapsulated in expression [18].

Summary

The objective of the analysis in this paper is to integrate Modern Portfolio theory with the Life-Cycle hypothesis of consumption. The latter theory can show how consumption is maintained over a life time by varying proportions of income and wealth. The former theory, in particular the Capital Asset Pricing Model, shows how an investor should acquire wealth (or loans) as a holding of a share of the market portfolio of risky assets, together with risk free assets or liabilities. The synthesis of the two theories enriches the explanation of the consumption function, and integrates a risk dimension into the analysis to account also for asset accumulation, or indebtedness *via* borrowing.

This paper therefore constructs a theoretical framework in which consumption, the holding of assets, and indebtedness can be analysed under conditions of uncertainty. Individual households can face greater uncertainty than the personal or household sector in an economy, so issues requiring an individual focus such as consumer indebtedness, may benefit more from this approach than issues such as aggregate consumption, although they should benefit too.

Endnotes

[i] See the analysis by Betti and et al. (2003).

[ii] See Elton and et al. (2003) for an overview of portfolio theory. A recent study in the empirical aspect of this field comes from a discussion paper by Tofallis (2008).

[iii] The notion of certainty equivalence solution is also adopted by the empirical investigation by Lyhagen (2001).

[iv] The theory was developed by Modigliani and Brumberg in a series of articles based on Fisher's model of consumption. The Nobel Prize lecture, which outlines the bulk of the work, is in Modigliani (1986). For a straightforward discussion of the Life-Cycle model see European Commission (2004). For a summary of the other 'stylized parables' in the field of modelling consumption, see Muellbauer (1994).

[v] The intermediate asset terms all cancel out, and an important ingredient is to define the discount factors correctly, so that this happens.

[vi] At times of low (or absence) inflation, the risk free asset can be identified with government securities, bank and building society deposits, which do not change in monetary terms (Mill, 1848). The return here is entirely attributed to income, and not capital gain or change in the value of wealth.

[vii] Another issue concerns the utility function. Given the definition of $Q_n A_n$, and f, the analysis has an apparent anomaly that:

$$W_n^f = \left(\frac{Q_n^1 A_n^1}{P_n}\right)^{f^1} \left(\frac{Q_n^2 A_n^2}{P_n}\right)^{f^2}.$$

The overall asset value, however, equals the sum of the two asset values, not a product. The solution to this anomaly is set out in Theil (1973). There is a reliable way of calculating weights (in this case the 'f's') from the elements of a sum, so that a product relationship holds to a very high degree of accuracy and He shows that way.

[viii] They introduce uncertainty into their analysis by presuming a stochastic process for employment income based on the AR (1) procedure with drift.

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