

The Jet Angular Profile and the Afterglow Light Curves

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Abstract. We investigate how the angular structure of GRB jets effects the afterglow light curves at different viewing angles, θ_v , from the jet symmetry axis. A numerical hydrodynamical modeling for the evolution of a relativistic collimated outflow, as it interacts with the surrounding medium, is carried out, and compared to two simple models that make opposite and extreme assumptions for the degree of lateral energy transfer. The Lorentz factor, Γ , and kinetic energy per unit solid angle, ϵ , are initially taken to be power laws of the angle θ from the jet axis. We find that the lateral velocity in the comoving frame, v'_θ , is typically much smaller than the sound speed, c_s , as long as $\Gamma \gg 1$, and the dynamics of relativistic structured jets may be reasonably described by a simple analytic model where ϵ is independent of time, as long as $\Gamma(\theta = 0) \gtrsim$ a few. We perform a qualitative comparison between the resulting light curves and afterglow observations. This constrains the jet structure, and poses problems for a ‘universal’ jet model, where all GRB jets are assumed to be intrinsically identical, and differ only by our viewing angle, θ_v .

1. Introduction

Most GRB jet models consider a uniform (or ‘top hat’) jet, where the Lorentz factor, Γ , and the energy per unit solid angle, ϵ , do not depend on the angle θ from the jet axis, within some finite well defined opening angle, θ_j , and drop sharply outside of θ_j . The possibility that GRB jets can display an angular structure, where ϵ and Γ are power laws in θ , was proposed by Mészáros, Rees & Wijers (1998). Recently, several different groups have analyzed afterglow observations within the frame work of the ‘top hat’ jet model, and have inferred a relatively narrow distribution both for the total energy output in gamma-rays (Frail et al. 2001) and in the initial kinetic energy of the relativistic outflow (Panaitescu & Kumar 2001; Piran et al. 2001). These results may alternatively be interpreted as GRB jets having a universal structure, which is intrinsically the same for all GRBs, and the observed differences between different GRBs are

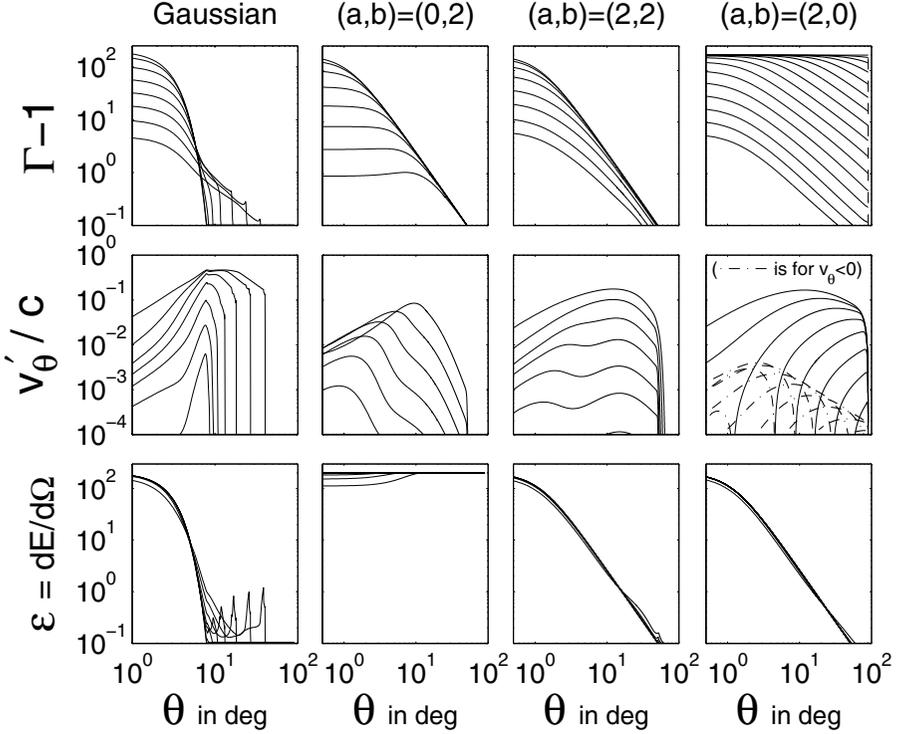


Figure 1. The hydrodynamic evolution of a structured jet, for $\Gamma_0 = 200$, $\epsilon_0 = (10^{53} \text{ erg})/(4\pi \text{ sr})$. We show $\Gamma - 1$, the lateral velocity in the local frame, $v'_\theta/c = \Gamma v_\theta/c$, and ϵ , for an initial Gaussian profile, and three different power law profiles where initially $\epsilon \propto \theta^{-a}$ and $\Gamma \propto \theta^{-b}$.

a result of different viewing angles, θ_v , w.r.t the jet symmetry axis (Lipunov, Postnov & Prokhorov 2001; Rossi, Lazzati & Rees 2002; Zhang & Mészáros 2002). Whereas in the ‘top hat’ jet interpretation, the jet break time, t_j , depends mainly on the initial opening angle of the jet, θ_j , in the universal ‘structured’ jet interpretation, t_j depends mainly on the viewing angle, θ_v , and the light curve is roughly similar to that for a ‘top hat’ jet with $\theta_j = \theta_v$ and $\epsilon = \epsilon(\theta_v)$.

While the evolution of ‘top hat’ jets and their light curves has been widely investigated, much less work has been done on ‘structured’ jets. Here we describe the main results of Kumar & Granot (2003, KG hereafter) and Granot & Kumar (2003, GK hereafter), and refer the reader to these works for more details.

2. The Jet Dynamics & Afterglow Light Curves

We begin with a numerical hydrodynamic calculation of the evolution of a relativistic jet (KG). The hydrodynamic equations are reduced to 1-D by assuming axial symmetry and integrating over the radial profile of the flow, thus consid-

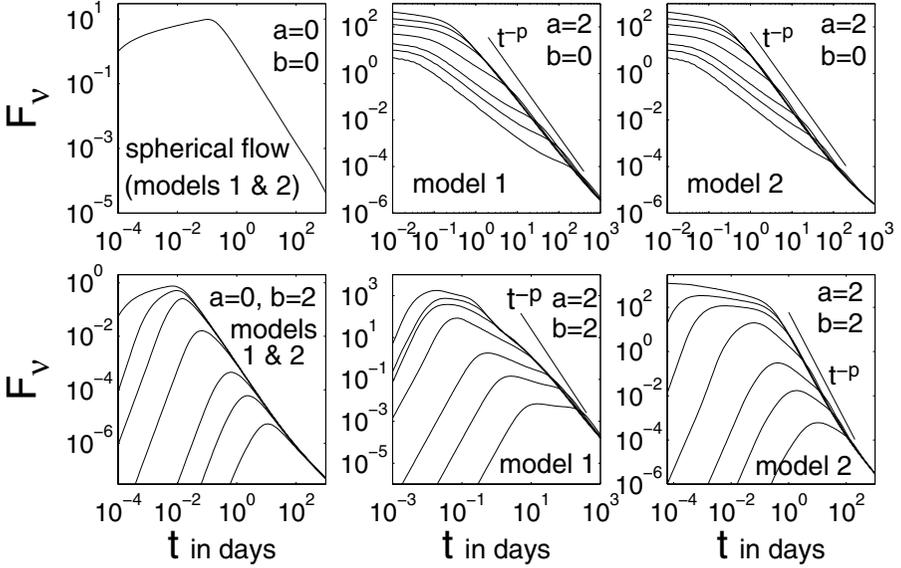


Figure 2. Light curves for structured jets (initially $\epsilon \propto \theta^{-a}$ and $\Gamma \propto \theta^{-b}$), for models 1 and 2, in the optical ($\nu = 5 \times 10^{14}$ Hz), for a jet core angle $\theta_c = 0.02$, viewing angles $\theta_{\text{obs}} = 0.01, 0.03, 0.05, 0.1, 0.2, 0.3, 0.5$, $p = 2.5$, $\epsilon_e = \epsilon_B = 0.1$, $n = 1 \text{ cm}^{-3}$, $\Gamma_0 = 10^3$, and ϵ_0 was chosen so that the total energy of the jet would be 10^{52} erg (GK). A power law of t^{-p} is added in some of the panels, for comparison.

erably reducing the computation time. We examined initial conditions where ϵ and Γ are power laws in θ , outside of a core angle, θ_c : $\epsilon(\theta, t_0) = \epsilon_0 \theta^{-a}$ and $\Gamma(\theta, t_0) = 1 + (\Gamma_0 - 1)\Theta^{-b}$, where $\Theta \equiv [1 + (\theta/\theta_c)^2]^{1/2}$, as well as a Gaussian profile: $[\Gamma(\theta, t_0) - 1]/(\Gamma_0 - 1) = \epsilon(\theta, t_0)/\epsilon_0 = \exp(-\theta^2/2\theta_c^2)$. For simplicity, we show results only for a uniform ambient medium. The hydrodynamic evolution is shown in Fig. 1. For power law jet profiles the lateral energy transfer is small, and as long as $\Gamma(\theta = 0) \gtrsim$ a few, $\epsilon(\theta, t) \approx \epsilon(\theta, t_0)$. For the Gaussian profile, which is the steepest, a shock forms in the lateral direction, but most of the energy still remains at $\theta \lesssim \theta_c$. The lateral velocity in the comoving frame is found to be $v'_\theta \sim c/\Gamma \ll c_s \approx c/\sqrt{3}$ for $\Gamma \gg 1$ (in KG we show analytically that $v'_\theta \sim c/(\Gamma\delta\theta)$ where $\delta\theta$ is the angle on which Γ or ϵ change significantly).

Now we examine two simple models, where either: (1) $\epsilon(\theta, t) = \epsilon(\theta, t_0)$, or (2) ϵ is averaged over the region to which a sound wave can propagate (this simulates the maximal lateral energy transfer that is consistent with causality). We assume initial power law profiles, as described above, and calculate the afterglow light curves for observers at different viewing angles, θ_v , assuming synchrotron emission (see Fig. 2). For $(a, b) = (0, 2)$ the light curve initially rises [before the deceleration time $t_{\text{dec}}(\theta_v)$], and there is no jet break, which is quite different from observations for most afterglows. For $(a, b) = (2, 2), (2, 0)$ we find a jet break at t_j when $\Gamma(\theta_v) \sim \theta_v^{-1}$. For $(a, b) = (2, 2)$ the value, α_1 , of

the temporal decay slope $\alpha \equiv d \log F_\nu / d \log t$ at $t < t_j$, increases with θ_v , while $\alpha_2 = \alpha(t > t_j)$ decreases with θ_v . This effect is more prominent in model 1, and appears to a lesser extent in model 2, but is prominent in the light curves from the numerical calculation of the jet dynamics (KG). This suggests that $\delta\alpha = \alpha_1 - \alpha_2$ should increase with t_j , which is not supported by observations. For $(a, b) = (2, 0)$, there is a flattening of the light curve just before the jet break, for $\theta_v \gtrsim 3\theta_c$. Again, this effect is larger in model 1, compared to model 2. The light curves for the simulated jet dynamics (KG) show a reasonably sharp jet break at $\theta_v \gtrsim (2-3)\theta_c$ and flattening of the light curve just before the jet break becomes strong at $\theta_v \gtrsim (5-7)\theta_c$. This leaves a factor of ~ 3 in θ_v/θ_c for which there is a sharp jet break not preceded by a flattening, as is typically observed in afterglow light curves. Since a larger range of θ_v ($\sim \theta_j$ for ‘top hat’ jets) is inferred ($\sim 2-20^\circ$), this poses a problem for a universal jet structure.

3. Conclusions

We have described a numerical scheme for calculating the dynamics of structured relativistic jets. The lateral velocity in the local frame is found to be smaller than the sound speed, except for very sharp jet angular profiles. This causes the energy per unit solid angle, ϵ , to remain close to its initial value in the first few days, suggesting that a simple model (model 1) where ϵ is constant in time, and each segment of the jet evolves as if it were part of a spherical flow, provides a good approximation for the jet dynamics and light curves for $t \lesssim$ a few days.

A universal jet profile, where all GRB jets are intrinsically identical, and only our viewing angle changes between different GRBs, has difficulty explaining all the afterglow light curves observed to date. This was demonstrated for $\epsilon(t_0) \propto \theta^{-2}$, which is needed to reproduce the recent results of a roughly constant energy in GRB jets. Furthermore, we find that the jet break for structured jets in an external density $\propto r^{-2}$ are much smoother compared to a uniform density, in agreement with the result of Kumar & Panaitescu (2000) for a ‘top hat’ jet. Finally, the light curves for a Gaussian initial jet profile are found to be similar to those for a ‘top hat’ jet, and (for $\theta_v \lesssim \theta_c$) produce sharp jet breaks, that are compatible with afterglow observations.

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