

A study of transmission properties in tapered graded-index fibre

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ABSTRACT

We investigated the transmission properties of tapered fibres both experimentally with simulations in order to select the appropriate fibre for the high-resolution spectrograph EXOhSPEC. The work focuses on testing commercial produced graded-index (GI) and custom made graded-index (GI) multimode fibres and compares these with a 10 μm step-index (SI) conventional fibre. The performance of a spliced taper package (SP) is also investigated. All tests are performed with coherent light at 635 nm. The main focus of the investigation was to determine the factors that optimised the light transmission by matching the numerical aperture to the fibre and minimising misalignment. We characterised how the transmission through the fibre depends on the effective numerical aperture (NA_{eff}) or input beam numerical aperture. The results suggest that under optimal coupling conditions, the transmission of a Custom GI taper is better than the 10 μm SI fibre and the commercial Thorlabs taper over the range of NA_{eff} from 0.02 to 0.25. In addition, we characterised a spliced taper package (SP) which contained a tapered fibre spliced to untapered fibre. Additionally a comparison of transmission via two coupling methods was investigated in the work. The results indicate splicing a large core GI fibre onto a tapered fibre is an appropriate technique for the fibre link of a fibre-fed spectrograph.

Keywords: adiabatic tapered fibre, graded-index taper, transmission, taper length, spliced fibre, numerical aperture

1. INTRODUCTION

Fibre optics are increasingly used as the light pipe in astronomical instruments. MEDUSA¹ opened the possibility of developing a sophisticated spectrograph by separating the optics from the telescope mount and applying the fibre as a light link. As a result, the stability of an instrument contained in a clean room away from the telescope was increased. We are developing a light link system from a telescope to the high-resolution spectrograph EXOhSPEC.² The link system contains two fibre optic cables a larger core multimode fibre (50 μm) for light gathering from the telescope and a smaller core a small core ($\sim 10 \mu\text{m}$) for the light input to the spectrograph. The two fibres are linked by ‘butt coupling’ i.e., placing two fibre tips very close to each other to transfer the light from one to another. The input of the spectrograph needs $\sim 10 \mu\text{m}$ fibre core to reach the desired resolution. However, light is lost when coupling from the larger core to a smaller core fibre. This motivated the study of a 5:1 taper ratio in 50 μm graded-index fibre, By tapering the 50 μm core is reduced to a 10 μm core and so the requirement for the of the fibre input to the spectrograph should be achieved. Previous studies^{3–5} have shown this approach is a good candidate for coupling the light from the telescope fibre into the spectrograph instead of using a conventional single mode or small-size multimode fibre.

Previous studies^{6–8} have explained the concept of using tapered fibre as a means of inputting light into the spectrograph instead of using conventional multimode or single-mode fibre. This is possible as^{7–9} the fibre

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numerical aperture (NA) can be changed from its manufactured NA by tapering adiabatically. Our previous work^{3,4} suggests that a short taper length in graded-index fibre provides the most promising candidate for a fibre link with less modal noise and higher transmission. Test performed on a taper of the same geometry and fibre type⁵ and our previous work has shown that graded-index taper suffers less modal noise compared to the similar core size conventional 10 μm step-index fibre. This motivated the design of the ‘spliced taper’ (SP) studied in this work. The untapered end of the custom graded-index taper was spliced into the conventional 50 μm graded-index fibre (Thorlabs: GIF50E). The packaging is 3D printed and tapering and splicing was performed on a Thorlabs Vytran GPX3400 glass processor.

In this paper, we investigate the transmission and how to optimise the coupling for various possible light link options for EXOhSPEC including both tapered and untapered fibre. Section 2 contains an explanation of NA of taper (Section 2.1) and mode propagation in tapered fibre (Section 2.2). Section 3 describes the experimental setup and results. We explain how to optimise the transmission by NA matching in Section 3.2. The results of transmission characterisation are given in Section 3.3. Lastly, we discuss and conclude the work in Sections 4 and 5, respectively.

2. GEOMETRY OF THE TAPERED FIBRE

Tapered fibre is fabricated by heating and pulling the glass in a precise and controlled manner. This causes the fibre geometry to change. When tapering down to a waist, the core and cladding diameters are both reduced by the same ratio. Work by Birks, et al⁹ presented the principle of tapered fibre and how the shape is changed. After the pull, the tapered fibre has three main parts (shown in Figure 1): untapered section, transition section and waist section. The tapered fibre has a certain transition (taper length) that can be controlled and selected. The waist is the section of the taper that has a reduced core and cladding diameter size. The waist dimensions define the final size of the tapered core/cladding. Ideally, a fibre tapered adiabatically has a low loss. In other words, energy is conserved. Factors for defining the ‘adiabatic’ tapered fibre depend on the transition angle, taper ratio and taper length.⁹

In this work, we used a down-transition injected light into the untapered end and characterised the output light from the waist of the end of the taper.

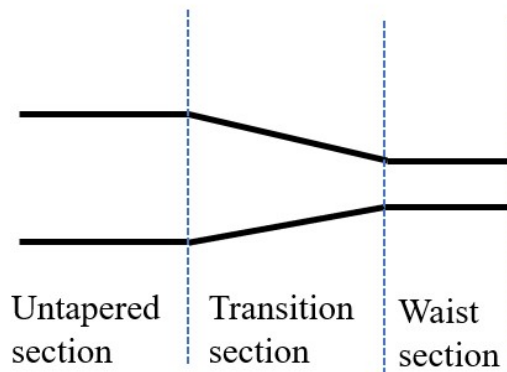


Figure 1. A drawing to illustrate a tapered fibre section. There are three main sections, which are untapered section, transition section and waist section.

2.1 Numerical aperture of the taper

We investigated how the transmission of the fibre depended on the effective numerical aperture (NA_{eff}) or the input beam NA. Numerical aperture or NA is defined from the angle of the beam propagating onto the fibre surface and is limited by the refractive index difference between core and cladding materials. This can also be written as $NA = n \sin \theta_{\text{max}}$, where $\sin \theta_{\text{max}}$ is half the maximum angle acceptance and n is the refractive index of

the glass. Typically, the difference in refractive index of core and cladding material defines the NA of the fibre optics as $NA = \sqrt{n_{\text{clad}}^2 - n_{\text{core}}^2}$.

With a tapered fibre, there is a certain NA of the input light that allows optimal transmission. This depends on the geometry of the taper and is different to the NA of the original untapered fibre. Figure 2 shows the drawing of tapered fibre with light entering from the untapered end to the tapered end. NA_i is manufactured or original NA of the fibre. R is the taper ratio. d_i and d_o are the diameter of the core of the untapered end and tapered end, respectively. If the fibre is tapered adiabatically, the energy is conserved by the etendue principle and if the core and cladding indices remain unchanged after the tapering process, the optimal input NA for maximum transmission along the taper ($NA_{\text{taperinput}}$) can be simplified and expressed as follows:

$$NA_{\text{taperinput}} = R NA_i = \frac{d_o}{d_i} NA_i. \quad (1)$$

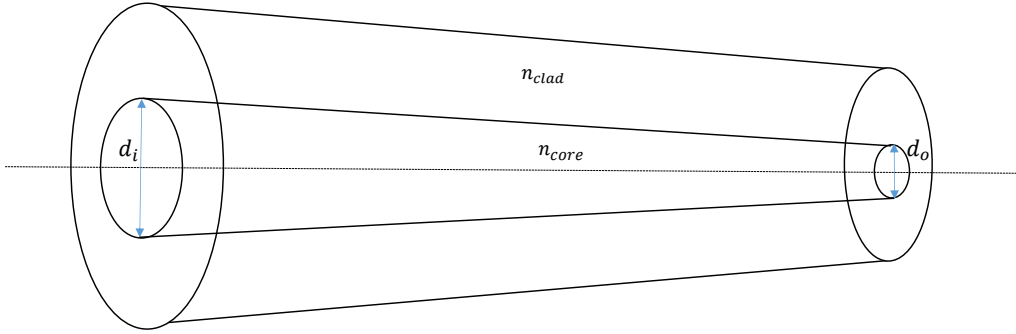


Figure 2. A diagram to illustrate a tapered fibre. This illustration shows the light is incident from the untapered end with diameter d_i toward to tapered end with diameter d_o . core and cladding refractive index are labelled as n_{core} and n_{clad} , respectively.

2.2 Mode propagation considerations in tapered fibre

In optical fibres, light propagation is treated as electromagnetic waves. Due to the complexity of waveguide calculations in multimode fibres, this is usually explored using numerical methods (e.g.^{10,11}) and modelling. The fibre optics shape is too complex to be treated as a 1D or 2D waveguide for accurate predictions. Hence, this work focuses on true-to-scale 3D models of tapered fibre and uses the COMSOL multiphysics code package.¹²

The NA of an input light to fibre optics can be represented by the fibre and taper mode characteristics. Ideally, the lowest order mode propagation is a property of a single mode fibre ($\sim 10 \mu\text{m}$ or less core-diameter size). Multimode fibre allows a large number non-fundamental modes to propagate ($> 10 \mu\text{m}$ core diameter). A previous study⁴ explained overall transmission calculated from the summation of transmission of all the modes that were excited ($0 \leq NA \leq NA_{\text{eff}}$). An individual mode i defines effective numerical aperture NA_i by $NA_i = \sqrt{n_{\text{core}}^2 - n_i^2}$. The overall transmission (T_{NA}) can be expressed as

$$T(NA) = \sum_0^{NA} t_i * C_i(NA) * \Delta NA_i, \quad (2)$$

where $C_i(NA)$ is coupling efficiency ($=1$ for no corrections for mode coupling efficiency) and t_i is transmission of each mode. Note that the calculations include only $t > 1\%$ which is why mode coupling for small NA_{eff} are significantly less and show the drop in transmission at $NA_{\text{eff}} < NA_{\text{taper}}$.

3. EXPERIMENTATION AND RESULTS

This section describes how the transmission along the various test tapers is affected by the effective numerical aperture of the incident light. Optimisation of the coupling is vital to ensure maximum transmission. The input beam numerical aperture or effective NA (NA_{eff}) needs to match with the manufacturer or taper NA of the fibre. We explain the setup along with sample specifications in Section 3.1 and conditions of the light coupling in Section 3.2.

3.1 The setup and the test fibre

The experimented setup (Figure 3) comprises off-the-shelf components and is designed to test how the variation of the beam input NA (NA_{eff}) affects transmission. A red laser diode 635 nm was selected as a coherent light source and the laser beam diameter was magnified by x3 with a pair of achromatic doublets to fill the entrance pupil of a microscope objective lens. An adjustable iris was placed before the collimated light entering the objectives for NA_{eff} adjustment. A 90:10 beam splitter splits the light into two paths. This enables the input light and output light to be measured at the same time. Input light is measured at P6(d) which is a proportion of light (10% side) after passing through an iris diaphragm with diameter d . A 10x Olympus objective is used to focus the light onto a test fibre. With this setup NA_{eff} can be varied from 0.02 to 0.21. A tip/tilt kinematic mount and 3-axis translation stage were used to optimise light coupling conditions i.e., the smallest number of modes or highest output power detected.

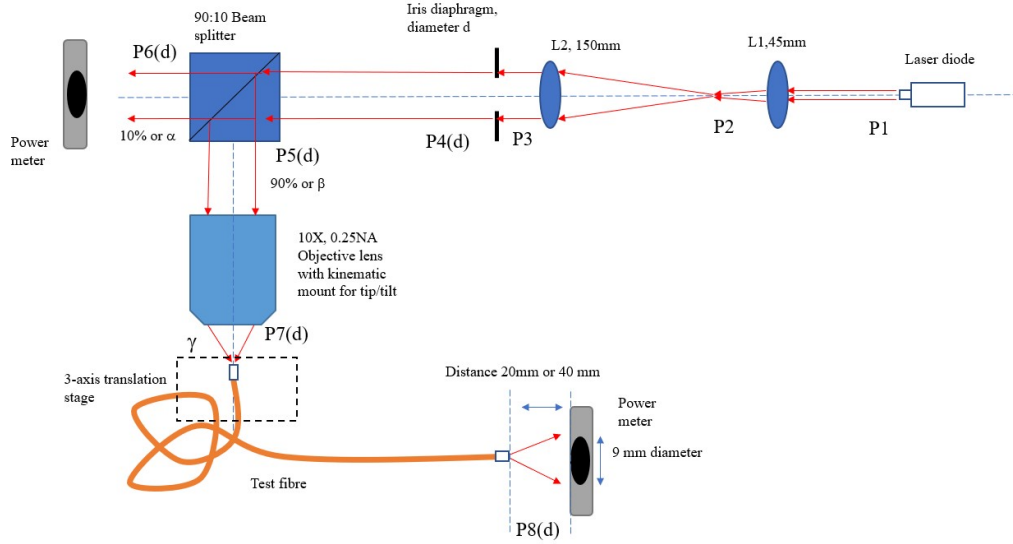


Figure 3. A drawing of the fibre transmission characterisation dependant on the NA_{eff} setup.

The ratio between output and input power is defined as ‘transmission’ in this work. The iris diaphragm is used to vary the collimated beam diameter before the beam is focused onto the fibre input end. We call this an ‘NA matching technique’. NA_{eff} can be varied by adjusting the iris diaphragm diameter and can be written as:

$$NA_{\text{eff}} = \frac{d_{\text{iris}}}{d_{\text{MOL}}} NA_{\text{MOL}}. \quad (3)$$

where d_{iris} and d_{MOL} are iris diameter and objective lens entrance pupil (9 mm for 10x Olympus lens), respectively. The NA_{MOL} is the numerical aperture of the objective lens (0.25 for 10x Olympus lens).

Table 1 shows optical fibres studied in this work. In order to determine the effect of the FC/PC connectors on the measured transmission through the fibre, untapered fibre of the same type as used in the taper samples was cleaved carefully and cleaned to ensure good quality end faces. The transmission through these bare fibres and then compared to the transmission of patch cables of the same fibre type with FC/PC connectors on the ends. Less than 50% difference in transmission was seen.

Table 1. The specification of tapered fibres. All tapers have a 5:1 taper ratio and 25 mm taper length. NA_{manu} is a manufacturer NA.[†] and defines for NA of untapered and tapered Bath custom fibre and * defines for NA of Thorlabs GI fibre.

Description	NA_{manu}	NA_{taper}
Thorlabs 50 μm GI	0.2	0.04
Thorlabs 62.5 μm GI	0.275	0.055
Bath custom 50 μm GI fibre	0.3	0.06
Spliced taper Package	$0.3^{\dagger}$ & 0.2^*	$0.06^{\dagger}$ & 0.2^*

3.1.1 Spliced tapered fibre

We designed a bespoke ‘spliced taper’ package (SP). This was fabricated using Thorlabs Vytran GPX3400 glass processor. The fused spliced part is between the untapered end of the Bath custom graded-index taper and the untapered end of 50 μm Thorlabs graded-index fibre. The fibre was packaged carefully inside a sturdy 3D-printed package. Figure 4 shows a drawing of spliced taper package. The input end is 50 μm Thorlabs graded-index fibre (GIF50E) and the output end is the tapered end of the custom graded-index taper. Epoxy covered the taper and spliced and was used for cladding light suppression in the transition region. Image of the splicing part between the untapered end of the custom GI taper and 50 μm graded-index Thorlabs fibre (GIF50E) is shown in Figure 5.

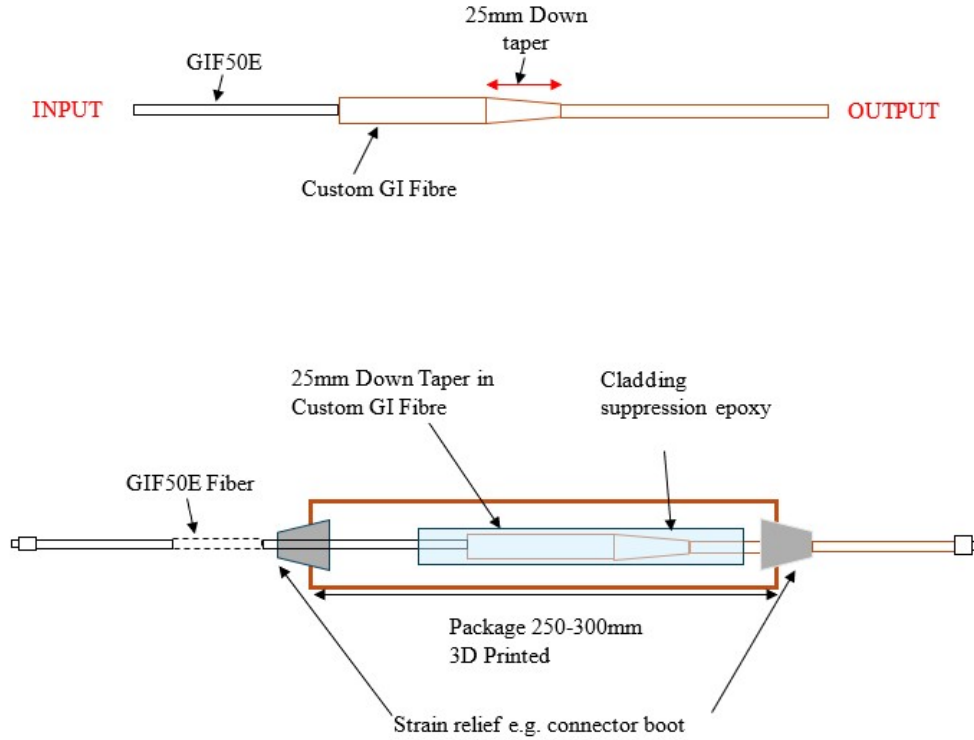


Figure 4. Drawing of the spliced taper packaging. The top image shows the fusing of untapered fibre ends between 50 μm GI Thorlabs and Bath Custom GI taper. The bottom image shows the splicing sections contained and secured with UV-cured epoxy to suppress the cladding light inside the 3D-printed box.

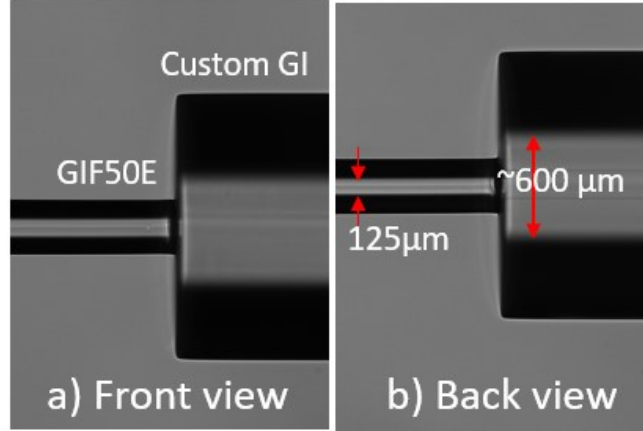


Figure 5. Images of the spliced fibre (SP) package. The spliced part of the sample is shown in (a) front and (b) back view with an almost seamless joint. The left fibre is the untapered end of conventional 50 μm Thorlabs graded-index fibre and the right fibre is a Bath manufacturer custom GI taper.

3.2 Condition of the coupling and transmission characterisation

To get the highest transmission of light through the fibre requires two main parameters to be adjusted to optimise the coupling conditions if it is assumed the alignment of the beam is on the optical axis. Firstly, the image of the laser spot should not exceed the fibre core size and secondly, the NA of the input beam is 90% of the NA manufacturer.¹³ In this section, we identify beam conditions as if the NA_{eff} is 90% of the NA_{manu} or NA_{taper} , the beam is in ‘filled’ condition. If $NA_{\text{eff}} < 90\%$ and $> 90\%$, the beam is in the ‘under-filled’ and ‘over-filled’ condition, respectively. We picked 90% based on the transmission experimentation results that most fibre transmission optimised around 90%. The misalignment test in this section used the same setup as in the previous work.³

The fibres used in the test are 50 μm GI Thorlabs untaper (GIF50E), 10 μm Thorlabs untaper (FG010LDA), Custom Bath manufactured GI taper and Thorlabs 50 μm GI taper (GIF50E). The transmission was plotted as a function of offset position in μm . Figure 6 shows the results from different coupling conditions.

It is clear to point out that overall transmission is optimised when the light coupling ‘filled’ the NA of fibre optics. The peak transmission is less when the fibre is underfilled or overfilled. The tapered fibres also have a higher transmission for a given offset position than the 10 μm step-index fibre as the width of the light profile is larger even though the tapers and SM fibre have the same core size. This is probably caused by light leaking out into the cladding with the tapered fibres. The custom GI taper has both higher transmission and larger profile width compared to Thorlabs GI taper. When the fibre is tapered down the taper always increases and so it is inevitably larger in the custom GI taper as the NA of the original fibre was larger (0.3 as opposed to 0.2 for the Thorlabs fibres GIF50E).

3.3 Results of transmission vs effective numerical aperture

Many factors can be adjusted to optimise transmission in tapered fibres e.g. taper length, fibre type and original fibre NA. Results from previous studies^{3,4} show both through experimentation and simulations that for 25, 50 and 100 mm taper length samples, the highest transmission occurred with the shortest length of 25 mm. This provided the motivation to perform a similar set of experiments with 25 mm taper length stand alone single tapered fibres as well as a 25mm-taper-length spliced taper package. This would enable a comparison to a 10 μm step-index conventional fibre as previously.

This section shows the results (Figure 7) of transmission dependence on NA_{eff} of the taper fibre as listed in Table 1. The test was carried out using a 635 nm coherent light source as shown in Figure 3. We call these results ‘direct coupling’ as the light source was coupled onto the test fibres directly from the objective lens.

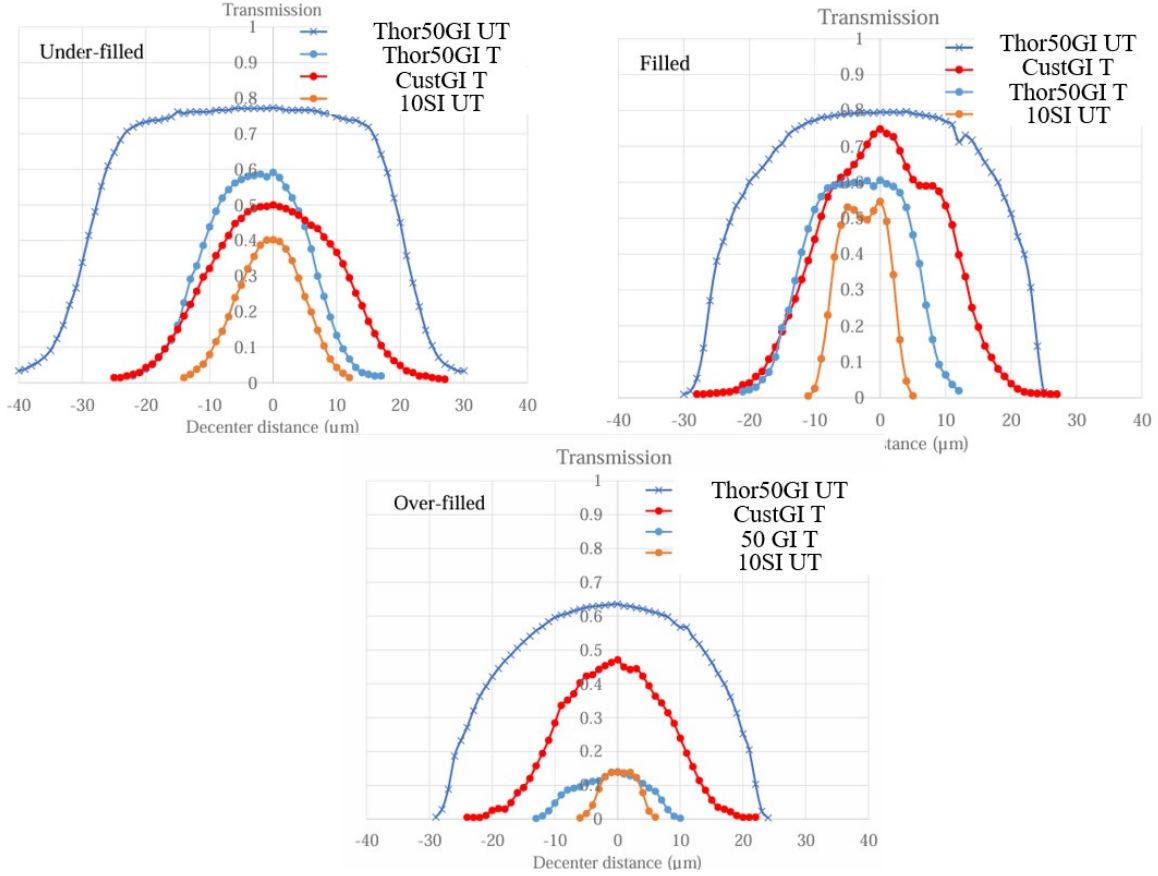


Figure 6. Results of misalignment impact on transmission of untapered (UT) and tapered (T) fibre. Test fibres are 10 μm SI untaper, 50 μm GI untaper, Thorlabs 50 μm GI taper and custom GI taper. There were three launched conditions presented for each test fibre: under-filled, filled and over-filled.

The experimental results (solid lines) are shown along with the COMSOL results (dash lines with no marks). CustGI T, 62.5GI T, Thor50GI T denote as Custom GI taper, 62.5 μm GI taper, and 50 μm GI taper, respectively. All fibre transmission tests drop from a maximum at around 90% of NA of manufacturer fibre/ taper fibre for this setup. The results suggest that the tapered custom GI taper performs has the best transmission for NA_{eff} . While the second best are Thorlabs 62.5 and 50 μm GI tapers. The trends of these two fibres are similar. However results from all the stand alone single tapers in Figure 7 are without any cladding light suppression. Because of the transmission of cladding light, all these taper samples therefore show higher transmission than the 10 μm step-index fibre.

The results of the spliced taper package show similar transmission trends to the 10 μm step-index fibre. One reason is this packaged taper cladding light was stripped away by the UV-cured epoxy covering the taper section, thereby reducing the transmission. Another reason for the lower transmission in the spliced taper package may be due to the fact that the Bath custom GI fibre was actually found to have a slightly elliptical core shape rather than circular. Hence when splicing this fibre to the circular core of the 50 μm Thorlabs GI fibre (GIF50E), the coupling will not be as good as if both cores had been circular.

Despite this, we further investigated transmission trends if the light was ‘butt-coupled’ to the spliced taper with the same condition of the fibre link system of EXOhSPEC i.e., 50 μm graded-index fibre is used as the input light coupling. Results in Figure 8 show the comparison transmission trends between spliced taper package and 10 μm step-index untapered fibre with two coupling conditions: direct coupling (solid lines) and butt-coupling (dash lines with solid marks). Direct coupling is when input light is passed through an objective lens before it

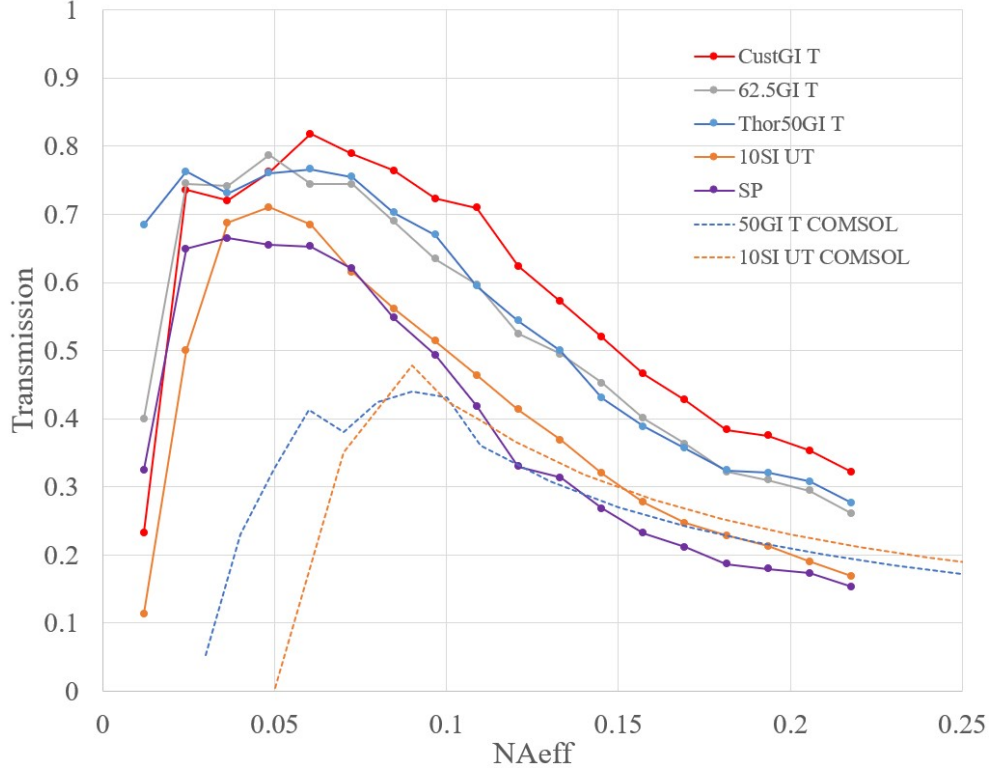


Figure 7. Results of transmission dependant on NA_{eff} . T states for Taper and UT states for Untapered fibre. COMSOL states for model results. The light source is a 635 nm laser. The results are from ‘direct’ coupling test.

is incident to the test fibre/ taper as shown in Figure 3. Butt coupling is when the fibre tips are just placed very close to each other to transfer the light from one to the other. The change in the transmission between the two coupling conditions is significantly less in the spliced taper package than for the 10 μm step-index fibre. For the spliced taper package the transmission drops less than 5% over the range of NA_{eff} . For 10 μm step-index untapered fibre there is a significant loss from 10 to 28% over the range of NA_{eff} .

4. DISCUSSION

The work focuses on how to optimise the light coupling from a 50 μm core fibre output from a telescope into a 10 μm core input of a spectrograph. With tapered and untapered fibre, considering the spot size does not exceed the core size, the light injection requires NA to be more than 90% of the fibre optics NA (manufacturer NA/ taper NA) from the current setup. This work showed that a custom GI taper has the highest transmission over the widest range of NA_{eff} , although the transmission of light leaking into the cladding needs to be carefully addressed before using with the spectrograph. Additionally, the elliptical core of the Custom GI fibre decreased the transmission through a splice between this fibre and another untapered fibre with a circular core with the same core size. Other samples with Thorlabs GI fibre with 50 μm and 62.5 μm are shown to be adequate choices for the spectrograph link system. Despite their slightly lower transmission in overall NA_{eff} compared to the custom GI fibre, their transmission is still higher than 10 μm step-index fibre.

In addition, the simulations from COMSOL suggest that the taper has better transmission compared to a 10 μm step-index fibre at small NA_{eff} (less than 0.9 in this case), which is consistent to the experimental results. The main focus of the study presented here is to be able to improve the overall throughput to the spectrograph. All tapers tested show reasonably good transmission with both direct coupling and ‘butt-coupling’ methods. We have shown both robust and reasonably good transmission, and that a taper is suitable for the light pipe for the EXOhSPEC. Though the spliced taper package here shows no improvement over the transmission trends with

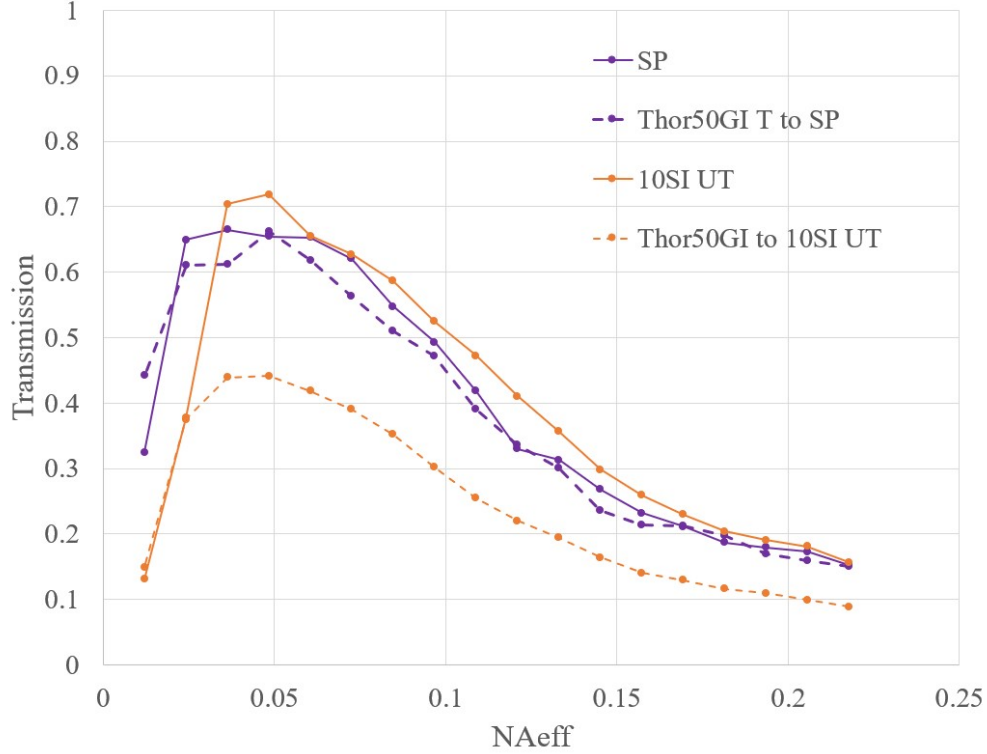


Figure 8. Results of butt-coupling transmission dependant on NA_{eff} . The spliced and 10 μm SI untaper were coupling with 50 μm graded-index fibre with coherent light 635 nm. Butt-coupling (dashed line) and direct coupling (solid line) results are shown in dash and solid lines, respectively.

the direct coupling test compared to a 10 μm step-index fibre. ‘Butt-coupling’ with 50 μm graded-index fibre results gives significant improvement for the spliced taper package compared to the 10 μm step-index fibre. The concept of a splice taper package shows that the transmission loss is significantly better than the current light link configuration where a 50 μm core fibre from the telescope is butt coupled to the 10 μm core fibre input to the EXOhSPEC spectrograph.

5. CONCLUSION

This work explains the concept of tapered fibre and also the custom design of a spliced taper package as a light link between a telescope and spectrograph. We show results on how to optimise the coupling efficiency for untapered and tapered fibre from the core size of 10 to 50 μm . The optimum condition is coupling into the fibre of an NA when is 90% of the NA of the manufactured bared fibre or taper NA. This is based on the latest transmission characterisation setup. As mentioned in previous studies^{3,4} and this work, a GI tapered fibre with 25 mm taper length is the best candidate for a light link because it has a higher overall transmission compared to 10 μm step-index fibre. The spliced taper package designed to suppress the cladding light and optimise coupling was tested. Poor coupling efficiency was seen and this was thought to be due to from different core shapes of the fibres in the package. Although the transmission trends are similar to a 10 μm SI fibre with direct coupling, the ‘butt-coupling’ results show a marked improvement of the spliced taper package over the 10 μm SI fibre. Due to the issues with obtaining reliable circular core shape with the custom GI fibre the Thorlabs taper with 62.5 μm core size is the most suitable approach for the fibre link to the spectrograph. This is because the Thorlabs 62.5 μm GI taper with its circular core shape has an acceptable loss and is more robust than a custom GI fibre i.e. less fragile and easier to handle during both fabrication and in the lab or on-sky observation. This leads to the possibility of a further improved design with a 62.5 μm core Thorlabs taper.

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REFERENCES

- [1] Hill, J. M., Angel, J. R. P., Scott, J. S., Lindley, D., and Hintzen, P., “Multiple object spectroscopy: the medusa spectrograph,” **242**, L69–L72 (Dec. 1980).
- [2] Jones, H. R. A., Martin, W. E., Anglada-Escudé, G., Errmann, R., Campbell, D. A., Baker, C., Boonsri, C., and Choochalerm, P., “A Small Actively Controlled High-resolution Spectrograph Based on Off-the-shelf Components,” **133**, 025001 (jan 2021).
- [3] Choochalerm, P., Martin, W. E., Jones, H. R., Errmann, R., Yerolatsitis, S., Wright, T. A., and Buisset, C., “Transmission properties of tapered optical fibres: Simulations and experimental measurements,” *Optical Fiber Technology* **66**, 102632 (2021).
- [4] Choochalerm, P., Martin, W. E., Jones, H. R., Usher, S., Wright, T. A., and Yerolatsitis, S., “Incoherent light in tapered graded-index fibre: A study of transmission and modal noise,” *Optical Fiber Technology* **75**, 103140 (2023).
- [5] Ghosh, S., Boonsri, C., Martin, W., Jones, H. R. A., Choochalerm, P., Usher, S., Yerolatsitis, S., Wocial, T., and Wright, T., “Mitigating modal noise in multimode circular fibres by optical agitation using a galvanometer,” *RAS Techniques and Instruments* **3**, 8–18 (01 2024).
- [6] Avila, G., “Tests of optical fibres for astronomical instrumentation at ESO,” in [*Fiber Optics in Astronomy*], Barden, S. C., ed., *Astronomical Society of the Pacific Conference Series* **3**, 63–73 (Jan. 1988).
- [7] Marcel, J., Haynes, R., and Bland-Hawthorn, J., “Application of fiber tapers in astronomy,” in [*Optomechanical Technologies for Astronomy*], Atad-Ettinger, E., Antebi, J., and Lemke, D., eds., **6273**, 1231–1240, International Society for Optics and Photonics, SPIE (2006).
- [8] Haynes, D. M., Haynes, R., Olaya, J. C., and Leon-Saval, S. G., “Optical fibre tapers: focal reduction and magnification,” in [*Modern Technologies in Space- and Ground-based Telescopes and Instrumentation II*], *The proceeding of SPIE* **8450**, 84503J (sep 2012).
- [9] Birks, T. A. and Li, Y. W., “The shape of fiber tapers,” *Journal of Lightwave Technology* **10**, 432–438 (April 1992).
- [10] Casperson, L. and Kirkwood, J., “Beam propagation in tapered quadratic index waveguides: Numerical solutions,” *Journal of Lightwave Technology* **3**, 256–263 (April 1985).
- [11] Tovar, A. A. and Casperson, L. W., “Beam propagation in parabolically tapered graded-index waveguides,” *Appl. Opt.* **33**, 7733–7739 (Nov 1994).
- [12] “Comsol multiphysics programming reference manual.”
- [13] guide, T., “Thorlabs Lab Facts: Modifying Beam Profiles with Multimode Fibers.” Thorlabs https://www.thorlabs.com/newgrouppage9.cfm?objectgroup_id=6838. Accessed: 16 August 2024).