

# **A Fatigue Life Assessment Methodology for Rolling-Element Bearing Under Irregular Loading**

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## **Abstract**

The paper presents a methodology for estimating the fatigue life of rolling-element bearing under irregular loading conditions. This method overcomes the limitations encountered by rolling-element bearing life models based on a constant bearing load assumption, when used in applications where bearing load varies over time with also changes in rotational speed. To include these irregular loading effects, a load-slice averaging methodology is applied to the loading history; in which the loading history is assumed to be composed of many thin slices of loading conditions. The operating conditions within each loading slice are averaged, and with the aid of linear damage rule and Lundberg-Palmgren load-life correlation for rolling-element bearings, each loading slice fatigue damage contribution is determined. The cumulative loading slice fatigue damage is used to estimate rolling-element bearing life. This approach can also be used as a tool for real-time life prognosis of rolling-element bearings. The method is demonstrated with simulated loading histories acting on a Cooper split cylindrical roller bearing and life prediction comparison is made between several approximate closed form bearing life expressions for different types of loading.

## **1. Introduction**

Rolling-element bearings perform a vital role in rotating equipment operation; where they aid relative motion between parts and also provide support for loads which are oftentimes dynamic in nature, resulting in shorter service life than expected.

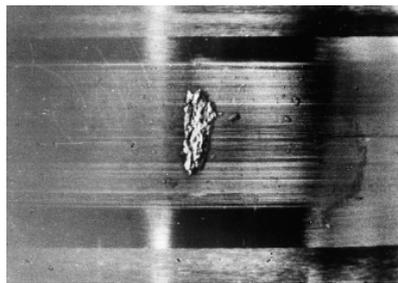
In practice, rolling-element bearings can experience shorter service life as a result of a complex mix of design faults, manufacturing errors, poor assembly, improper operation and maintenance <sup>(1)</sup>.

However, if a rolling-element bearing is correctly installed, lubricated, loaded and free from foreign contaminations during operation; every other failure mode can be avoided, with an exception of rolling contact fatigue (RCF) <sup>(2, 3)</sup>.

As such, RCF, becomes the life limiting failure mode, and has being the basis for the development of most rolling-element bearing life models reported in literature <sup>(4)</sup>.

Rolling-element bearing life can be defined as the number of inner race revolution or the number of hours of operation at a certain speed of rotation until the first sign of fatigue or spalling appears <sup>(2)</sup>.

Fatigue failure in rolling-element bearing is caused by the repeated stresses generated at the contact between the rolling element and race way, which facilitates the initiation and propagation of cracks forming spalls such as shown in Figure 1.



**Figure 1. Fatigue spall on a bearing raceway <sup>(1)</sup>**

Bearing fatigue failures can be classified as either subsurface initiated fatigue or surface initiated fatigue. Subsurface initiated fatigue occur when micro-cracks form underneath the bearing surface at material inhomogeneities or inclusions driven primarily by the alternating maximum Hertzian shear stress at depth; the microcracks propagate to the surface, causing a breakaway of the bearing material and finally forms a spall. Subsurface initiated fatigue occur more often when the bearing is operating under an elastohydrodynamic lubrication mode. On the other hand, surface initiated fatigue do occur for several reasons: as a result of poor lubrication, allowing metal to metal contact, damaging the bearing surface, leading to the formation of points of high stress concentration, thereby causing fatigue spalling or as a result of the presence of small hard foreign contaminants, scratching the bearing surface, forming points of high stress concentration leading to fatigue spalling.

Early interest into rolling-element bearing life prediction can be traced back to the works by Stribeck in 1896 who conducted fatigue tests on ball and roller bearing <sup>(5)</sup>. Stribeck's experimental data formed the basis upon which Goodman in 1912, formulated expressions to calculate the safe operating loads of rolling-element bearings <sup>(6)</sup>.

The rudiments for an analytical approach to predict rolling-element bearing life was laid down by Weibull in 1939 in his statistical theory for the strength of a material <sup>(7; 8)</sup>. The theory for the first time, accounted for the dispersion in fatigue life being observed in experiments, when a number of identical rolling-element bearings were fatigue tested under the same load.

During any such component test, Weibull postulated that, the probability of material survival  $S$ , the material property function  $f(x)$ , and the stressed volume  $V$ , is governed by the following relationship<sup>(7; 8)</sup>:

$$\ln \frac{1}{S} = \int_v f(x) dV \quad \dots\dots\dots(1)$$

Weibull proposed a material property function  $f(x)$ , for rolling-element bearings to take the form<sup>(9)</sup>:

$$f(x) = \tau^c N^e \quad \dots\dots\dots(2)$$

Where  $\tau$  is the critical stress,  $N$  is life in million stress cycles,  $e$  is the Weibull slope, and  $c$  is critical stress exponent determined experimentally.

Thus,

$$\ln \frac{1}{S} = A \tau^c N^e V \quad \dots\dots\dots(3)$$

Where  $S$  is the probability of material survival,  $\tau$  is the critical stress,  $N$  is life in million stress cycles,  $e$  is the Weibull slope,  $c$  is critical stress exponent determined experimentally and  $A$  is a bearing material constant.

Lundberg and Palmgren<sup>(10; 11)</sup> further improved on Weibull's model; they observed that for rolling-element bearings, several cracks occurred during fatigue testing that did not propagated to the surface and cause failure; as such, Weibull's assumption that the first crack would always lead to a failure did not hold true for rolling-element bearings; Rather, they found that for rolling-element bearings, the probability that a crack would propagate to the surface was dependent on the depth at which the most critical stress occurred.

With this insight, they proposed a material property function for rolling-element bearing of the form:

$$f(x) = \frac{\tau^c N^e}{z_0^h} \quad \dots\dots\dots(4)$$

Where  $\tau$  is the critical stress,  $N$  is life in million stress cycles,  $z_0$  is the depth to the maximum critical stress,  $e$  is the Weibull slope,  $c$  is critical stress exponent determined experimentally and  $h$  is the critical stress depth exponent determined experimentally.

Thus,

$$\ln \frac{1}{S} = A \frac{\tau^c N^e V}{z_0^h} \quad \dots\dots\dots(5)$$

Where  $S$  is the probability of material survival,  $\tau$  is the critical stress,  $N$  is life in million stress cycles,  $z_0$  is the depth to the maximum critical stress,  $e$  is the Weibull slope,  $c$  is

critical stress exponent determined experimentally, h is the critical stress depth exponent determined experimentally and A is a bearing material constant.

They further derived a load-life equation for rolling-element bearing, expressed as <sup>(10; 11)</sup>.

$$L_{10} = \left( \frac{C}{F_{eq}} \right)^p \dots\dots\dots(6)$$

Where  $L_{10}$  is the fatigue life in millions of inner race revolution that 90 percent of a group of identical rolling-element bearings would exceed before failure, C is the basic dynamic load rating, defined as the load that 90 percent of a group of identical bearing would be able to withstand for 1 million revolutions before failure,  $F_{eq}$  is the equivalent bearing load and p is the load-life exponent, experimentally shown to be 3 for ball bearings and 10/3 for roller bearings.

The equivalent bearing load  $F_{eq}$  can be expressed as:

$$F_{eq} = XF_r + YF_a \dots\dots\dots(7)$$

Where  $F_r$  is the radial component of the load,  $F_a$  the axial component of the load, X and Y are the radial and axial load factors respectively.

Lundberg-Palmgren model forms the basis of the rolling-element bearing lifing model proposed in the ISO 281 standard <sup>(12)</sup>.

However, to accommodate for different reliabilities and include the effect of lubricant regime, contamination, and bearing material fatigue stress limit used in modern rolling-element bearing life estimation, ISO 281 recommends an improved system approach, employing modification factors to the  $L_{10}$  estimates.

The ISO 281 method expresses the load-life equation in the form:

$$L_{nm} = a_1 a_{iso} \left( \frac{C}{F_{eq}} \right)^p \dots\dots\dots(8)$$

Where  $L_{nm}$  is the modified fatigue life in millions of inner race revolutions,  $a_1$  is life modification factor for reliability and  $a_{iso}$  accounts for the effect of lubrication, contamination and bearing material fatigue load limit.

Ioannides and Harris <sup>(13)</sup> proposed a new model to overcome the limitation they observed in Lundberg-Palmgren model based on experimental data of rolling-element bearing fatigue life. They found that, below certain Hertzian stresses, fatigue failure was not possible; this led them to propose a material property function of the form:

$$f(x) = \frac{(\tau - \tau_u)^c N^e}{z_0^h} \dots\dots\dots(9)$$

Where  $\tau$  is the critical shearing stress (von Mises stress),  $\tau_u$  is the fatigue limiting stress,  $N$  is life in million stress cycles,  $z_0$  is the depth to the maximum critical stress,  $e$  is the Weibull slope,  $c$  is critical stress exponent determined experimentally and  $h$  is the critical stress depth exponent determined experimentally.

Thus,

$$\ln \frac{1}{S} = AN^e \int_v \frac{(\tau - \tau_u)^c}{z_0^h} dV \dots\dots\dots(10)$$

Zaretsky <sup>(9; 14)</sup> proposed yet another material property function for rolling-element bearings of the form:

$$f(x) = \tau^{ce} N^e \dots\dots\dots(11)$$

Where  $\tau$  is the critical stress,  $N$  is life in million stress cycles,  $e$  is the Weibull slope, and  $c$  is critical stress exponent determined experimentally.

Thus,

$$\ln \frac{1}{S} = A \tau^{ce} N^e V \dots\dots\dots(12)$$

Zaretsky's model completely decouples the dependence of the critical stress exponent  $c$  on the Weibull slope  $e$  evident in the models by Weibull, Lundberg-Palmgren and Ioanides-Harris.

Zaretsky's model also eliminates the dependence of fatigue life on the depth to maximum critical stress. This is due to the argument that, rolling-element fatigue being a high cycle fatigue should have majority of its fatigue life dominated by the fatigue initiation stage; however, considering depth to maximum critical stress as an important factor in the fatigue life expression, implies otherwise.

Yu and Harris <sup>(15)</sup> proposed a stress-based bearing life model, to overcome the under prediction of bearing life by Lunderberg-Palmgren model and Ioanides-Harris model at low load applications. Their stress-based model ignores the effect of depth to critical stress and also decouples the dependence of the stressed volume integral on the life dispersion or Weibull slope. This model also considers the presence of a bearing infinite life and introduces this effect through a new fatigue intitiating stress exponent constant  $c$ .

The base model equation takes the form of:

$$\ln \frac{1}{S} = AN^e \left( \int_v \tau^c dV \right)^e \dots\dots\dots(13)$$

Where  $\tau$  is the fatigue initiating stress,  $N$  is life in million stress cycles,  $e$  is the Weibull slope,  $c$  is the new fatigue initiating stress exponent determined experimentally and  $A$  is the material constant determined experimentally.

## 2. Fatigue Life Analysis Under Irregular Loading

In practice, rolling-element bearing load and speed vary with time, and as such, bearing life calculations need to factor in their fatigue damaging contributions.

A widely used approach for considering fatigue damage contributions by irregular loadings on component life is the linear damage rule.

The linear damage rule proposed independently by Palmgren <sup>(2)</sup>, Langer <sup>(16)</sup> and Miner <sup>(17)</sup> as applied to rolling-element bearings states that if a bearing has a life of  $L_1$  million revolutions of inner race under a certain constant load and speed, after  $m_1$  million revolutions of inner race at the same constant load and speed, a portion  $m_1/L_1$  of its life has been consumed; if the same bearing is further exposed to another constant load and speed for  $m_2$  million revolutions of inner race and the life at this same constant load and speed is  $L_2$  million revolution of inner race, a further  $m_2/L_2$  portion of its life has been consumed. If this process continues, this rule proposes that failure would occur when the sum of the ratios  $m/L$ , the cumulative fatigue damage reaches a constant  $D$ ; which is generally assumed to be equal to 1.

The linear damage rule is expressed as <sup>(16; 17)</sup>:

$$\frac{m_1}{L_1} + \frac{m_2}{L_2} + \frac{m_3}{L_3} + \dots + \frac{m_k}{L_k} = 1 \quad \dots\dots\dots(14)$$

Equation (14) can be re-written in a new form, by introducing  $L$ , represent the total remaining bearing life:

$$\frac{m_1}{LxL_1} + \frac{m_2}{LxL_2} + \frac{m_3}{LxL_3} + \dots + \frac{m_k}{LxL_k} = \frac{1}{L} \quad \dots\dots\dots(15)$$

By introducing  $X_i$ , a non-dimensional life fraction into equation (15) which is defined as the ratio  $m_i/L$ , then, equation (15) simplifies to:

$$\frac{X_1}{L_1} + \frac{X_2}{L_2} + \frac{X_3}{L_3} + \dots + \frac{X_k}{L_k} = \frac{1}{L} \quad \dots\dots\dots(16)$$

### 2.1 Determination of The Non-Dimensional Life Fraction ( $X_i$ )

The non-dimensional life fraction  $X_i$  is defined as:

$$X_i = \frac{m_i}{L} = \frac{N_i x \Delta t_i}{N_n x t} \dots\dots\dots(17)$$

Where  $N_i$  is the inner race rotational speed at the  $i$ th load,  $\Delta t_i$  is the duration of time spent at the  $i$ th load and speed,  $t$  is the total duration of bearing operation and  $N_n$  is the nominal inner race rotational speed, determined by:

$$N_n = \frac{\sum_{i=1}^k (N_i x \Delta t_i)}{t} \dots\dots\dots(18)$$

Combining equation (17) and (18),

$$X_i = \frac{N_i x \Delta t_i}{\sum_{i=1}^k (N_i x \Delta t_i)} \dots\dots\dots(19)$$

From equation (19), the following condition is implied for the non-dimensional life fraction:

$$X_1 + X_2 + X_3 + \dots + X_k = 1 \dots\dots\dots(20)$$

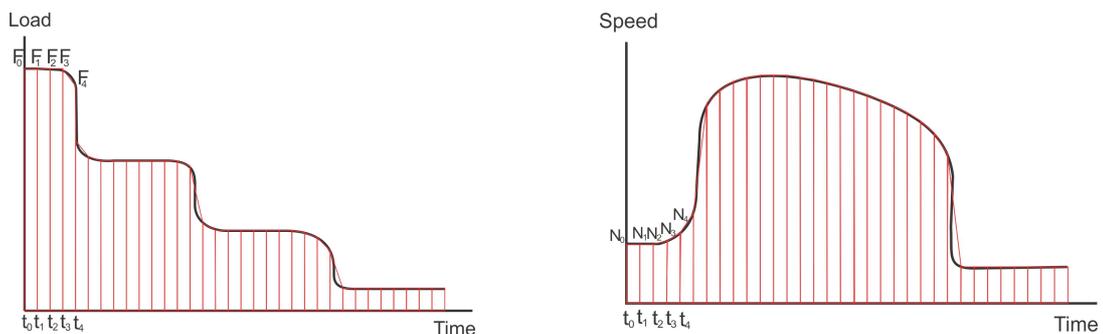
If the rotational speed is constant, equation (19) simplifies to:

$$X_i = \frac{\Delta t_i}{t} \dots\dots\dots(21)$$

### 3. Load-Slice Averaging Methodology

The load-slice averaging methodology is an algorithm, applying equation (16) and the Lundberg-Palmgren load-life relationship expressed in equation (6) as the basis to calculate the life of rolling-element bearing under any loading condition.

The methodology assumes the loading history to be composed of thin slices of loading conditions as shown in Figure 2. Within each loading slice, the loading conditions are averaged. The non-dimensional life fraction of each slice is a function of its time duration. Figure 3 shows a schematic implementation of the load-slice averaging methodology.



**Figure 2. Load History Slicing**

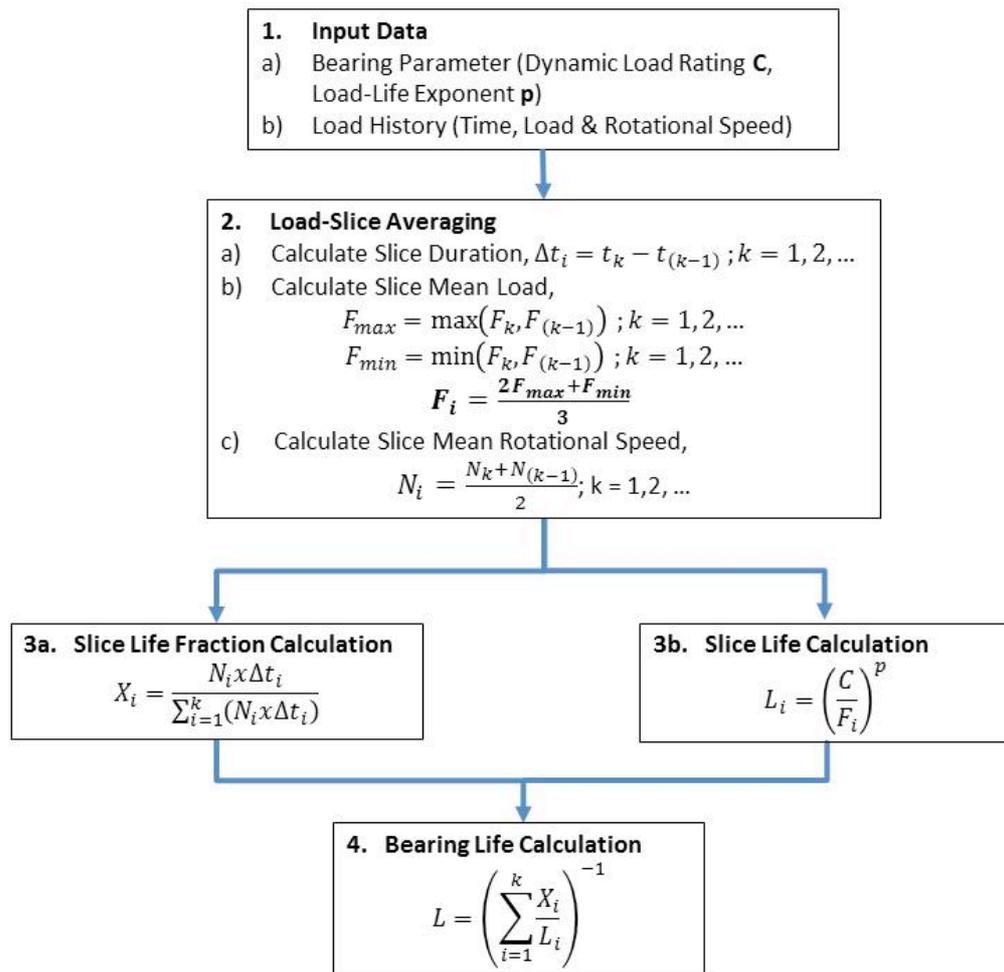


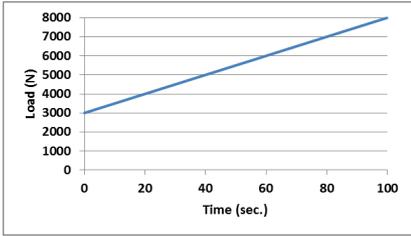
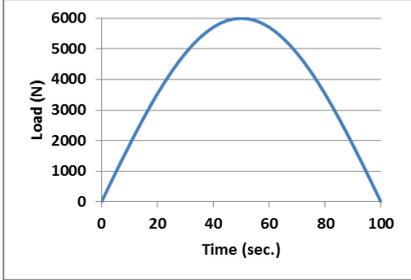
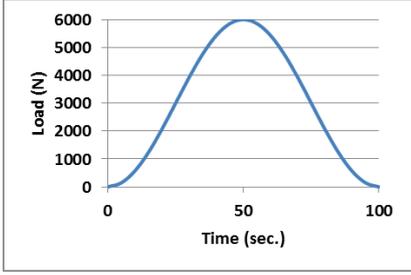
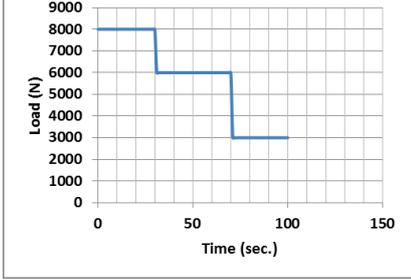
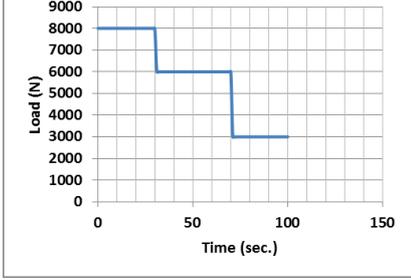
Figure 3. Load-Slice Averaging Methodology

#### 4. Results/ Discussion

The load-slice averaging methodology shown in Figure 3 is implemented in MATLAB and the validity of this technique is performed by comparison with several closed form solution for bearing life estimation under different loading condition shown in Table 1.

Table 1. Closed Form Bearing Life Expressions For Different Loading Cases

Load Case	Description	Load Profile	Closed Form Bearing Life Formula
1	Constant Load at Constant Speed (N = 1500 rpm)	<p>Figure 4. Load Case 1</p>	$F_{eq} = F_{max}$ $L_{10} = \left(\frac{C}{F_{eq}}\right)^p$

2	Linear Fluctuating Load at Constant Speed (N = 1500 rpm)	 <p style="text-align: center;"><b>Figure 5. Load Case 2</b></p>	$F_{eq} = \frac{F_{min} + 2F_{max}}{3}$ $L_{10} = \left( \frac{C}{F_{eq}} \right)^p$
3	Sinusoidal Fluctuating Load at Constant Speed (N = 1500 rpm)	<p>a)</p>  <p style="text-align: center;"><b>Figure 6. Load Case 3a</b></p> <p>b)</p>  <p style="text-align: center;"><b>Figure 7. Load Case 3b</b></p>	<p>a)</p> $F_{eq} = 0.75F_{max}$ $L_{10} = \left( \frac{C}{F_{eq}} \right)^p$ <p>b)</p> $F_{eq} = 0.65F_{max}$ $L_{10} = \left( \frac{C}{F_{eq}} \right)^p$
4	Fluctuating Stepped Load at Constant Speed (N = 1500 rpm)	 <p style="text-align: center;"><b>Figure 8. Load Case 4</b></p>	$F_{eq} = \left( \frac{\sum (F_i^p t_i)}{\sum t_i} \right)^{1/p}$ $L_{10} = \left( \frac{C}{F_{eq}} \right)^p$
5	Fluctuating Stepped Load at Variable Speed	 <p style="text-align: center;"><b>Figure 9. Load Case 5: Load</b></p>	$F_{eq} = \left( \frac{\sum (F_i^p N_i t_i)}{\sum (N_i t_i)} \right)^{1/p}$ $L_{10} = \left( \frac{C}{F_{eq}} \right)^p$

		<p><b>Figure 10. Load Case 5: Speed</b></p>	
6	<p>Rotating Load at Constant Speed (N = 1500 rpm)</p> <p><math>F_1</math> = Static Load <math>F_2</math> = Rotating Load</p> <p><b>Figure 13. Rotating Load</b> <sup>(18)</sup></p>	<p><b>Figure 11. Load Case 6</b></p> <p><b>Figure 12. Chart for <math>f_m</math></b> <sup>(18)</sup></p>	<p>a) SKF Method <sup>(18)</sup></p> $F_{eq} = f_m (F_1 + F_2)$ <p><math>f_m</math> determined from Figure 12.</p> $L_{10} = \left( \frac{C}{F_{eq}} \right)^p$ <p>b) NSK Method <sup>(19)</sup></p> $F_2 \geq F_1$ $F_{eq} = F_2 + 0.3F_1 + 0.2 \frac{F_1^2}{F_2}$ $F_2 \leq F_1$ $F_{eq} = F_1 + 0.3F_2 + 0.2 \frac{F_2^2}{F_1}$ $L_{10} = \left( \frac{C}{F_{eq}} \right)^p$

**Table 2. Bearing Data** <sup>(20)</sup>

Bearing Type	Cooper Split Cylindrical Roller Bearing (01 B 35M GR)
Dynamic Load Rating (C)	67 kN
Load-Life Exponent (p)	10/3

**Table 3. Comparison of Closed Form Bearing Life Solution and Load-Slice Averaging Technique Bearing Life Solution**

Load Case	Description	Closed Form Bearing Life (a) (million rev.)	Load-Slice Averaging Technique Bearing Life (b) (million rev.)	% Variation (b-a)/a*100
1	Constant Load at Constant Speed	3112.28	3112.28	0.00%
2	Linear Fluctuating Load at Constant Speed	2599.01	3208.10	23.44%

3	Sinusoidal Fluctuating Load at Constant Speed	a) 8119.72	a) 7432.16	a) -8.47%
		b) 13082.80	b) 10045.06	b) -23.22%
4	Fluctuating Stepped Load at Constant Speed	2566.95	2537.70	-1.14%
5	Fluctuating Stepped Load at Variable Speed	2993.58	2960.81	-1.09%
6	Rotating Load at Constant Speed	a) SKF Method 2549.48	2842.24	a) 11.48%
		b) NSK Method 2599.01		b) 9.36%

Table 3 provides a comparison of the results between the closed form bearing life solutions and the load-slice averaging methodology. The results show a good agreement for most of the load cases considered with an exception of load case 2 and load case 3b. These high variations are as a result of the rounding errors introduced by the cubic mean load averaging employed in the methodology which is more pronounced in these load cases.

## 5. Conclusion

This work shows that the load-slice averaging methodology can be used as a general approach for the fatigue life estimation of rolling-element bearing under any loading condition.

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