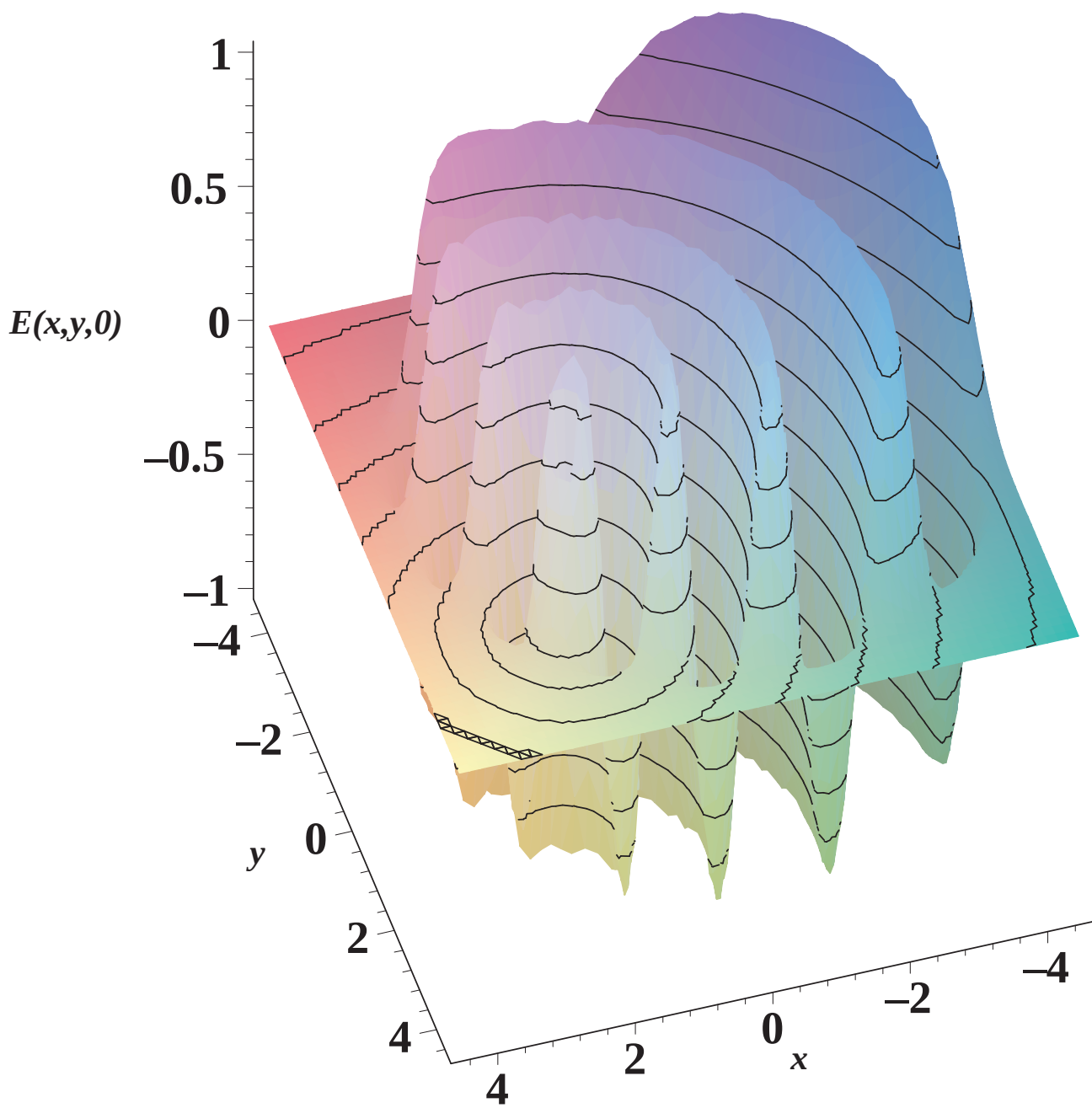




Coherent Transport and Concentration of Particles in Optical Traps using Varying Transverse Beam Profiles

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Tailored time-dependent variations of the transverse profile together with longitudinal phase shifts of laser beams are studied. It is shown theoretically that a standing wave setup and real-time beam forming techniques (e.g. by computer-addressed holograms) should make it possible to implement smooth transport across and along the beams employed in optical trapping schemes. Novel modes for the efficient collection, transport, and concentration of trapped particles should thus become realizable in optical trapping setups.

PACS numbers:

32.80.Lg Mechanical effects of light on atoms, molecules, and ions;

32.80.Pj Optical cooling of atoms, trapping;

32.80.Qk Coherent control of atomic interactions with photons

42.50.Vk Mechanical effects of light on atoms, molecules

42.60.Jf Beam characteristics: profile, intensity, and power; spatial pattern formation

I. INTRODUCTION

Trapping of objects with light is possible in all transparent media such as liquids, air and other gases, and vacuum. Laser beam trapping has become an established technique where the size of the trapped objects ranges over many orders of magnitude from atoms to particles of several hundred μm size [1]. There are two standard configurations: firstly counter-propagating plane waves form standing light field patterns yielding multiple traps arranged as crystals (i.e. periodic intensity patterns or 'light crystals' [2]), and secondly, strongly focussed laser beams form laser tweezers, whose foci serve as single trapping centers [1]. Because of their great power concentration, laser tweezers can levitate and hold small beads of many micrometers size. Smaller objects such as bacteria, nano-particles, molecules and atoms can also be held and moved by 'light crystals'. For several atomic species magneto-optical [3] and all-optical cooling schemes [4] have, moreover, allowed to create ultra-cold samples of dilute gas, some of them at fraction of nanokelvin temperatures in the Bose-Einstein-condensate state [5].

Although trapping (and cooling) particles with light is now a well established and mature field, moving such trapped particles with the help of the trapping fields is less refined. The main purpose of this paper is the introduction of a new approach to the collection, (coherent) transport, and spatial concentration of particles.

In particular the spatial concentration of particles with current schemes is not optimized: in the case of plane wave generated light crystals the crystal cells cannot be merged, laser tweezers suffer from small focal volumes, and optical washboard potentials [6] do not transport co-

herently. So far, spatial concentration towards one point is only achieved by changes to an auxiliary potential such as that of an assisting magnetic field [7]. Here, it is shown that modulation of the beam characteristic of a laser tweezer itself can open up new modes of coherent transport, capture, concentration, excitation, and release of particles.

For trapped atomic clouds this can help to increase their phase space density since they can be simultaneously cooled and spatially concentrated [4]. Likewise, larger particles suspended in viscous media can be concentrated in phase space. In the case of cold atomic clouds it might help to continuously replenish lossy traps [8] thus leading to continuous, Bose-Einstein-condensation-mediated, atom-laser operation [9, 10]. In the case of ions, fermions and other mutually repulsive particles their collection and spatial concentration might make it easier to reach unit-filling factors for the particle population trapped in an optical lattice [11]; useful, e.g., for grid-based quantum computing [12].

This paper will first review current setups and outline my approach in section II and the terminology for the description of paraxial beams in section III. Section IV and its subsections will deal with one- and two-dimensional modifications of the transverse mode profile of paraxial beams over time. This will demonstrate that manipulating the mode structure of a laser beam enables us to tailor its structure in such a way that controlled transport of trapped particles across the beam becomes possible and that this can be designed such that the particles are moved into a smaller volume by merging the cells of the effective light lattice that traps the particles. For further concentration an optical conveyor belt is introduced allowing us to concentrate particles towards a 'point' in space and unload them there. After that, coherence preserving transport is considered. Next, these ideas are generalized for the case of low-field seeking trap particles in section V followed by the conclusion.

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II. CURRENT SETUPS AND POSSIBLE MODIFICATIONS

For focussed beams several scenarios have been implemented: longitudinally moving the trapping center by refocussing together with redirection of the beam axis allows for three-dimensional movement of the focus [8]. But the focal volume is quite small and has to be moved in order to efficiently pick up more particles. Therefore multi-beam approaches in which many independent Gaussian foci are created have been demonstrated by holographical beam splitting [13, 14], including their independent movement [13, 15], merger [16], and application for size-selective particle deflection [17]. Yet, these methods [13–16] do not continuously collect over the entire beam volume but rather ‘pointwise’ at the various foci.

In focussed beams also the transverse beam profiles have been changed to generate annular high-order TEM-modes yielding ‘optical tubes’ [18, 19], ‘optical bottles’ [20] and beam centers surrounded by washboard potentials [6]. Moreover, beams have been equipped with orbital angular momentum thus allowing trapped particles to be turned [21] and, also, to use this freedom for quantum information coding [22]. Finally, tilted reflecting light sheets together with gravity have been used to implement atomic billiards [23].

For counter-propagating plane waves (wide laser beams) light crystals with different symmetries are realizable. Depending on the number and relative orientation of the employed beams, they form, e.g., cubic, tetrahedral, and super-lattices [24] in three dimensions. Effectively two-dimensional sheets formed by evanescent waves [25] and arrays of one-dimensional tubes [26] have also been implemented. Such crystals can be moved by detuning the frequency [27] or otherwise shifting the relative phase between beams, their lattice constants can be varied to some extent by changing the relative angles between interfering beams [2], but their unit cells cannot be merged. I therefore want to explore other avenues for the transport and concentration of trapped particles.

Currently established setups are quite static in the sense that the underlying optical beam shapes are kept unaltered [28]. The following changes to the beam structure could be considered though: one could change the longitudinal properties of the beam, but for beams propagating freely in homogenous media this is typically done by changes to their spectral composition, for instance by frequency-sweeping [27] or pulsing the beam [29]. Here, only gradual and slow changes which, in the case of quantum particles, will also allow us to preserve the trapped objects’ coherence [30, 31] are considered. Also, the polarization state of the trapping light fields could be changed over time, this will be further investigated in future work. Instead, the present paper concentrates on slow temporal variations to the beam’s transverse field and intensity profiles for uniformly focussed polarized light beams [32].

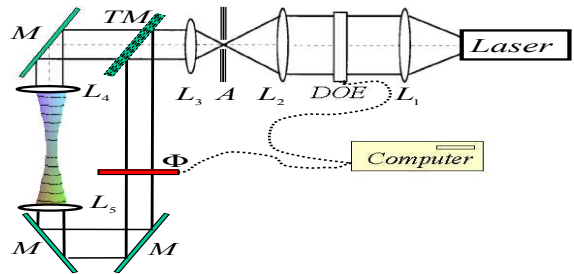


FIG. 1: Sketch of possible technical implementation (not to scale): a setup using laser, five lenses and aperture A , a semi-transparent balanced mirror TM to split the beam and three mirrors M to recombine the two parts into one counter-propagating standing wave beam. A computer-controlled diffractive optical element DOE generates the modulation of the trapping beam (which is shown as a solid hyperboloid). A phase-shifter Φ allows for computer-controlled longitudinal shift of the beam.

The field of diffractive optics is a mature field routinely using computer-generated holograms to alter light fields. Liquid crystal arrays and other spatial light modulators, have been developed for video-beamer technology but are now also used to implement diffractive optical elements with computer-generated holograms in real time [1, 13–16, 33, 34]. Typically, the diffractive element is positioned in a region where the laser beam is wide and its wave fronts parallel, see Figure 1. It imprints its amplitude information on the wide parallel beam the width of which then is suitably shrunk (using lenses L_2 and L_3 in Figure 1). Finally the beam is collimated using another set of lenses (L_4 and L_5 in Figure 1). Since the focus of the collimated beam is –up to rescaling and redirection– the Fourier-transform of the diffractive element’s amplitude pattern, we can easily calculate the required input with a computer that controls the diffractive optical element in real time.

Resolution of the diffractive optical elements are not a problem since mega-pixel LCD-screens are commercially available. Also the deviations of the diffractive element’s input from the ideal pattern due to its pixelated structure is not a problem. The regular pixelation gives rise to diffraction off axis which can be filtered out using an aperture A serving as an effective low-pass filter [34].

If the phase shift $\Phi(t)$ is implemented by shifting the frequency of one of the counter-propagating beams with respect to the other, very large phase differences can be accumulated very quickly. The same is currently not yet true for modifications of $I(x, y, z; t)$, beam formers were developed for video technology and only allow to modify the transverse intensity profile at video frame rates, i.e. on the order of some hundred Hertz [1, 13–16, 33, 34]. Fortunately, this is not a fundamental limit and it should be easily overcome in the near future [28].

III. HERMITE-GAUSSIAN BEAMS: TEM-MODES

In practical applications laser beams which are not too tightly focussed are very important. Although the ideas presented here are in principle applicable in more general cases, e.g. for very tightly focussed beams or very general fields created by intensity masks or holograms, we will only consider quasi-monochromatic beams in the paraxial scalar approximation [35, 36]. In this approximation the solutions are the familiar *transverse electromagnetic* or TEM_{mn} modes describing x -polarized beams propagating in the z -direction with a vector potential $\mathbf{A} = (A_x, A_y, A_z)$ whose only non-zero component is A_x with [36]

$$A_x(\mathbf{r}, t; k) = \psi_{mn}(\mathbf{r}) e^{i(kz - \omega t)}, \quad (3.1)$$

where the scalar function ψ_{mn} contains products of Gaussians and Hermite-polynomials, i.e. the familiar harmonic oscillator wave functions $\varphi_m(\xi) = H_m(\xi) \exp(-\xi^2/2)/\sqrt{2^m m! \sqrt{\pi}}$, ($m = 0, 1, 2, \dots$), and various phase factors [35, 36]

$$\begin{aligned} \psi_{mn}(\mathbf{r}) = & \frac{w_0}{w(z)} \varphi_m\left(\frac{\sqrt{2}x}{w(z)}\right) \varphi_n\left(\frac{\sqrt{2}y}{w(z)}\right) \\ & \times e^{\frac{ik}{2R(z)}(x^2+y^2)} e^{-i(m+n+1)\phi(z)}. \end{aligned} \quad (3.2)$$

The dispersion-relation of light in a homogenous medium $\omega = ck$ was used; x, y are the transverse and z the longitudinal beam coordinate, t is time and $w_0 = \sqrt{2b/k} = \sqrt{\lambda b/\pi}$ is the relation that links the minimal beam diameter w_0 with the Rayleigh range b . The beam diameter at distance z from the beam waist ($z = 0$) obeys $w(z) = \sqrt{w_0^2(1 + z^2/b^2)}$ and for large z shows the expected amplitude decay of a free wave $\propto 1/|z|$, the beam's opening angle in the far-field is $\arctan(\lambda/(\pi w_0))$. The corresponding wave front curvature is described by $R(z) = (z^2 + b^2)/z$, and the longitudinal phase shift (Gouy-phase) follows $\phi(z) = \arctan(z/b)$; according to the Gouy-phase factor $e^{-i(m+n+1)\phi(z)}$ it leads to relative dephasing between different modes.

The vector potential A_x of Equation (3.1) describing a beam travelling in the positive z -direction ($\mathbf{k} = k\hat{\mathbf{z}}$) yields an electric field which is polarized in the x -direction with a small contribution in the z -direction due to the tilt of wave fronts off the beam axis ($\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}$ are the unit-vectors). We omit this wavefront tilt and hence only deal with the scalar approximation

$$\mathbf{E} \approx E_x \hat{\mathbf{x}} = \Re \{ \omega A_x \hat{\mathbf{x}} \}. \quad (3.3)$$

Just like the paraxial approximation, the scalar approximation gets better the less focussed the beam (the larger the beam waist w_0) is.

Since the wave equation is linear and the harmonic oscillator wave functions form a complete orthonormal set for the transverse coordinates x and y , we are free to

FIG. 2: Plot of a possible transverse field configuration at $z = 0$, i.e. a slice across the laser beam. The field forms ring structures converging at position $(x, y) = (2, 2)$.

combine the above solutions to generate many interesting field and intensity configurations [37]

$$A_x(\mathbf{r}, t; \mathbf{k}) = \sum_{m,n=0}^{\infty} c_{mn}(t) \psi_{mn}(\mathbf{r}) e^{i(kz - \omega t)}. \quad (3.4)$$

The coefficients $c_{mn}(t)$ can be complex (i.e. change amplitude and phase of the beam), can be varied with time and do not obey normalization restrictions. Since we have trapping in mind let us also assume that we discuss standing wave fields formed from a superposition of (otherwise identical) counter-propagating beams, see Figure 1 above. In this case we have

$$A_x = \sum_{m,n=0}^{\infty} c_{mn}(t) \psi_{mn}(\mathbf{r}) e^{i(kz - \omega t + \Phi(t))} + c.c., \quad (3.5)$$

where $\Phi(t)$ represents the controllable, relative phase shift between the two beams forming the standing wave pattern and *c.c.* stands for complex conjugate. The resulting intensity distribution $I(x, y, z; t) \propto \mathbf{E}(x, y, z; t)^2$ only contains terms with a controllable (slow) time-dependence, namely $c_{mn}(t)$ and $\Phi(t)$ (see remarks at end of section II).

IV. EXAMPLES

A. Transverse 2-D Profiles

As an example Figure 2 specifies a possible field configuration of concentric waves emerging at the periphery of the trapping beam which then travel across the beam converging at one point [at $(x, y) = (2, 2)$] on the opposite edge thus concentrating all captured particles into a perl string, on the beam's fringe, such as that depicted

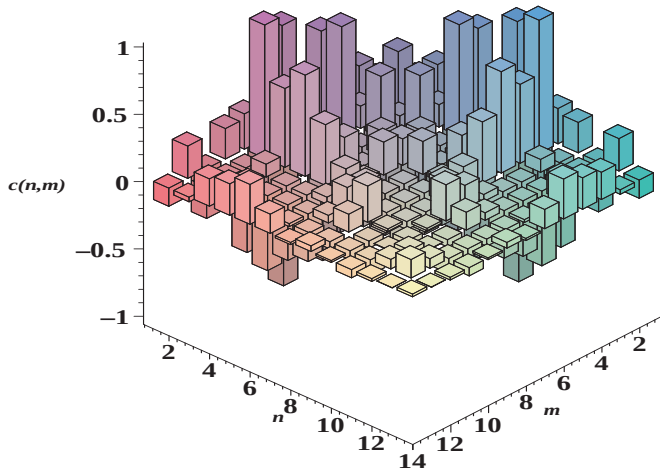


FIG. 3: Expansion coefficients c_{nm} up to 12-th order $n, m = 0, \dots, 12$ for the field shown in Figure 2. The coefficients are real numbers because the electric field is chosen to be real, the exchange-symmetry of the coefficient ($n \leftrightarrow m$) is due to the field's symmetry: $E(x, y) = E(y, x)$.

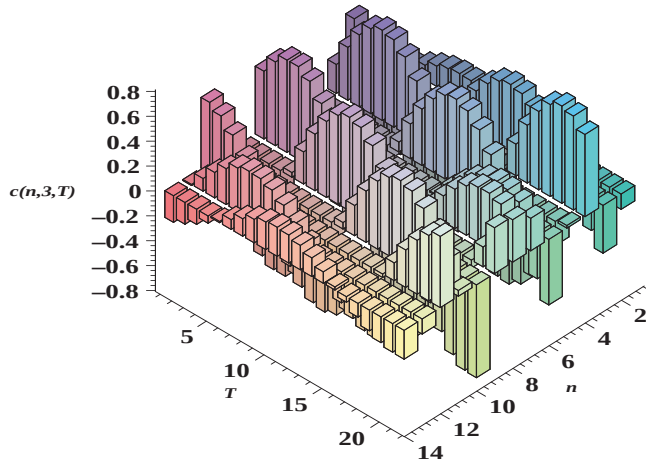


FIG. 4: Time-variation of a subset of the coefficients displayed in Figure 3. With this kind of (imposed) sinusoidal time-dependence the intensity pattern displayed in Figure 5 smoothly and periodically converges towards the concentration point on the beam edge.

in Figure 7 below. Figure 3 displays the expansion coefficients $c_{mn}(t)$ at one particular moment in time t up to twelfth order in m and n and Figure 4 depicts the time-development of a subset of the coefficients $c_{mn}(t)$ and displays the periodic motion underlying the concentration process portrayed above.

Figure 5 shows the intensity corresponding to the field configuration displayed in Figure 2 reconstructed using the expansion coefficients of Figure 3. In this case expansion up to twelfth order gives satisfactory results. This allows us to create tailored two-dimensional potential landscapes that can be changed over time. In particular particle concentration, tunnelling or classical escape scenarios could be implemented in this way [38, 39].

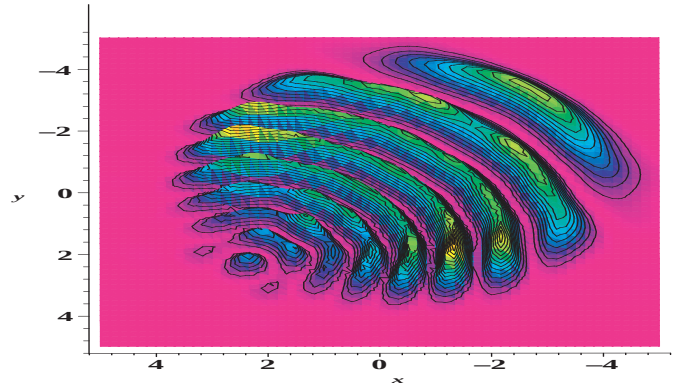


FIG. 5: Plot of the intensity distribution $I(x, y, 0; 0)$ associated with the transverse field displayed in Figure 2 where the transverse field modes have been determined up to 12th order: compare Figures 3 and 4.

Once the field is specified in this way at one beam plane this constitutes initial conditions which determine the shape along the rest of the beam. The analysis of the resulting overall beam behaviour and its possible applications are our next topics.

B. 3-D Concentration ‘in a Point’

Let us first consider some motivation for the following considerations: let us assume that we try to optically manipulate particles, we want to coherently transport, concentrate and, finally, release particles. For the particle release into a small volume we want to assume that there is some kind of background-trap into which we want to unload particles. We imagine that we have captured, concentrated, and transported them using a ‘foreground’-trap which relies on the methods described above.

The background-trap’s field must be sufficiently strong to hold particles but weak compared with the foreground field. Such a background-trap could be a single laser tweezer focus, a magnetic trap, or it could form a light crystal. We will see that even in the case of light crystals, with their rather uniform trapping power it is possible for the foreground beam to dominate the particles’ behavior throughout the transport and yet release the particles into a small area. This is achieved by an optical *conveyor belt* with a well defined end.

In section IV A we have just studied the transverse variation of the trapping beam structure which allows us to capture particles throughout the beam volume and within every transverse slice concentrate particles at the beam edge. Next, we assume that this concentration processes ceases and instead, we keep the particles we have concentrated fixed at the beam edge. Now, by changing the relative phase between the two beams that form the standing wave pattern (3.5), we can shift the entire structure. Let us concentrate on the side which moves towards the focus. Clearly, the foreground-trap’s strength

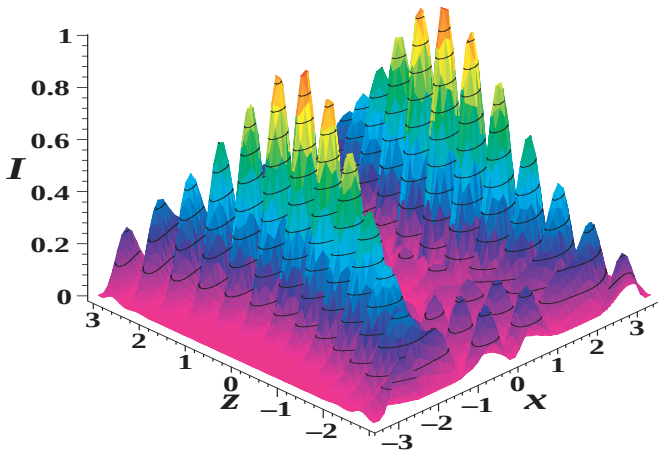


FIG. 6: Plot of the standing wave intensity distribution $I(x, 0, z)$ (arbitrary units) of a beam with parameter $b = 3$ and TEM-mode structure $(\varphi_3(\sqrt{2}x/w(z)) + \varphi_5(\sqrt{2}x/w(z))) \cdot \varphi_0(\sqrt{2}y/w(z))$ near the beam focus $z = 0$.

increases near the focus. This is illustrated by the increase in focal intensity displayed in Figure 6, and nothing much is won: if the foreground-trap manages to dominate the background-trap elsewhere, it will typically increase its dominance near the focus.

But, in the case of an intensity distribution which, unlike that of Figure 6, is asymmetric with respect to the beam axis, we expect according to ray-optics that the intensity is mapped through the beam focus; this intensity mapping can be exploited.

C. Gouy's phase flips the intensity at the focus: the optical conveyor belt

For illustration consider the effectively one-dimensional superposition $(\varphi_4(\sqrt{2}x/w(z)) + i\varphi_5(\sqrt{2}x/w(z))) \cdot \varphi_0(\sqrt{2}y/w(z))$. Its focal, standing-wave, intensity profile in the (x, z) -plane $I(x, 0, z)$ is shown in Fig. 7 and we see that the intensity pattern flips over when the beam passes the focal area. It does not uniformly weaken on one side for the light intensity to smoothly move over to the other side, instead, there is an interesting interference scenario at the focus by which the intensity ridge is effectively terminated on the beam edge and separated by interference nodes from the section across the beam where the same edge is 'resurrected' on the other side [32].

If we now assume that the relative phase $\Phi(t)$ in of the standing waves (3.5) is varied, we immediately see, that this can constitute a perl string of traps which is moved towards the focus where it suddenly weakens. At the point where the background trap is as strong as this diminishing perl string, it starts to take over from the foreground-trap, this combination thus forms an optical conveyor belt with a well defined exit point where the foreground beam's cargo is handed over to the background-trap.

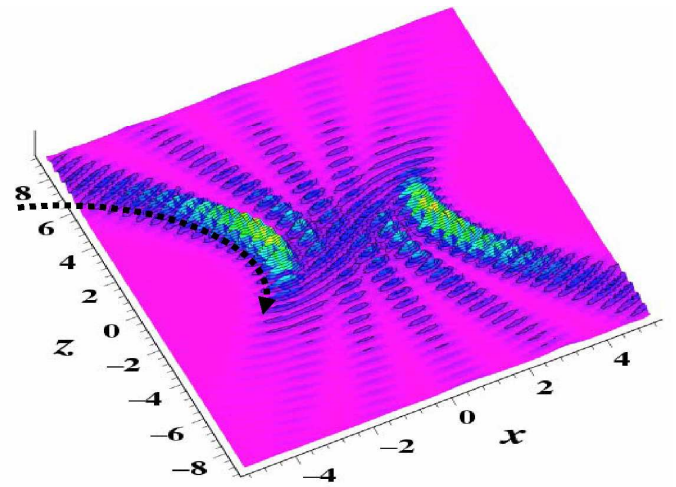


FIG. 7: Same as Figure 6 but with TEM-mode structure $(\varphi_4(\sqrt{2}x/w(z)) + i\varphi_5(\sqrt{2}x/w(z))) \cdot \varphi_0(\sqrt{2}y/w(z))$. The arrowhead indicates the unloading point if this field configuration is used as an optical conveyor belt.

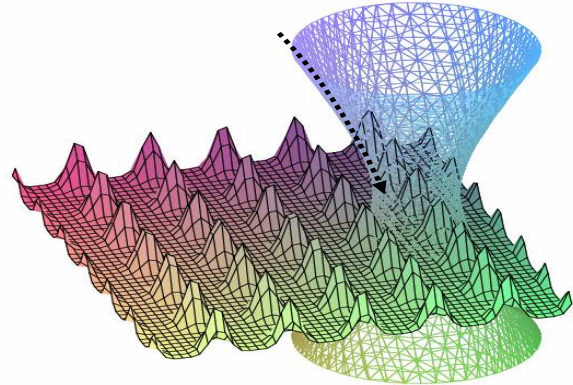


FIG. 8: Sketch of a possible setup: the foreground-trap shaped as a beam funnel, feeds the background-trap, an (effectively two-dimensional) array of laser traps formed by evanescent waves.

Fig. 8 sketches this scenario for the case of an evanescent-wave light crystal that acts as a stationary background-trapping field fed from above by the funnel-shaped foreground field.

D. Coherence-preserving Transport

Many examples of coherence-preserving transport of quantum particles [8, 31], their tunnelling [27, 38] and classical escape dynamics [17] have already been observed for optically trapped particles. With the greater variety of trapping potentials becoming available through the methods sketched here, it will be possible to implement new tailored potential and thus study such systems further.

Note, that tunnelling and classical escape processes de-

pend extremely sensitively (exponentially) on the potential barrier size (Gamov-effect) [39]. In this context it is worth mentioning that we can change the intensity ratio between foreground and background-trap field thereby modifying the barrier between them to make use of this exponential sensitivity. This allows us to fine-tune the transfer process from one to the other.

V. LOW-FIELD SEEKERS

The discussion in section IV B only applies to high-field seeking particles, but for some tasks we will want to trap low-field seekers [18, 20]. Our above discussion can be extended to serve the case of low-field seeking particles as well, using an altered field configuration providing us with an *optical 'bubble' or 'foam' beam*.

As a first step, the intensity profile discussed in section IV A would have to be surrounded by a light rim sealing off the beam edge and a modification of the beam such that it contains suitable dark areas which can house low-field seeking particles, see Fig 9. The beam would remain leaky though, since particles could escape through the nodes of the beams longitudinal standing wave pattern. In order to plug this escape route one can create a second standing wave beam acting as a stop-gap that is uniformly bright in the transverse plane and aligned with the rest of the trapping beam, but longitudinally shifted by a quarter wavelength. In order to avoid possible destructive interference between these two parts of the trapping beam they should be orthogonally polarized leading to a simple adding up of their respective intensities, see Fig 10. This way we can create a beam with dark inclusions surrounded by bright areas – a *'light-foam' or 'bubble' beam*.

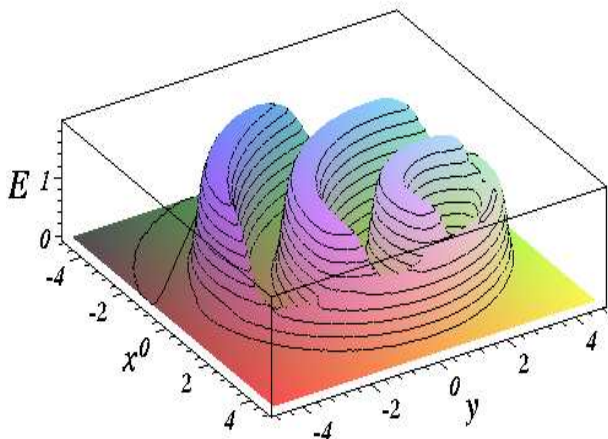


FIG. 9: Sketch of a possible field configuration for low field seeking particles: the field surrounds areas of low intensity with high intensity regions thus trapping particles in light bubbles (and concentrating them towards the area around $(x, y) = (2, 2)$).

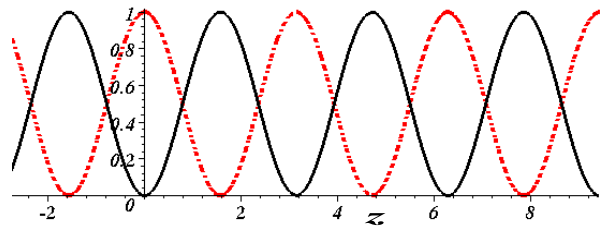


FIG. 10: Sketch, along the beam axis z , of the configuration of the transversely modulated field (red dotted line) in conjunction with a 90° -phase shifted orthogonally polarized field of equal strength (black line) that puts an intensity plug at the nodes of the former. According to $\sin^2 + \cos^2 = 1$ two waves of equal intensity can securely encase trapped low-field seeking particles.

VI. CONCLUSIONS

It was shown how to implement arbitrary transverse fields with arbitrary time-dependence, useful for trapping, coherent manipulation, concentration and release of particles. In particular a scheme for an optical conveyor belt with an end (in the focal region) is introduced. Use of another interlacing trap with orthogonal polarization was introduced in order to explain how the ideas discussed here can be generalized to low-field seeking trapped particles.

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