Batalin-Vilkovisky Formulation of $\mathcal{N}=1$ Supergravity in Ten Dimensions

Julian Kupka,* Charles Strickland-Constable,† and Fridrich Valach‡
Department of Physics, Astronomy and Mathematics, University of Hertfordshire, College Lane, Hatfield, AL10 9AB, United Kingdom

(Received 25 February 2025; accepted 24 April 2025; published 28 May 2025)

We present a full Batalin-Vilkovisky action in the component field formalism for $\mathcal{N}=1$ supergravity in ten dimensions coupled to Yang-Mills multiplets.

DOI: 10.1103/PhysRevLett.134.211602

Introduction—Ten-dimensional $\mathcal{N}=1$ supergravity coupled to super Yang-Mills was introduced in the early 1980s in [1–3]. Beyond serving as a "mother" theory for a host of lower-dimensional supergravity theories, it is particularly important for the low-energy description of the type I and heterotic superstring theory. Recently [4,5], the framework of generalized geometry was used as a convenient packaging tool to provide a simplified description of this theory (building on earlier works [6–9]). In particular, this simplifies significantly the structure of the four-fermion terms in the action, which are well known to be one of the major complications in formulating supergravity theories.

In this Letter we make the next step forward and present a full Batalin-Vilkovisky (BV) action of the theory. We partially leverage the fact that the BV formulation was already constructed for a very special sector in the generalized-geometric moduli space, namely, when the generalized metric (defined below) is frozen to be the identity operator. This leads to a topological theory ("dilatonic supergravity"), whose BV description was built in [10].

Nevertheless, generalizing this description to the nontopological case of physical supergravity is tricky, partly due to the higher-fermion terms in both the action and supersymmetry transformations, which make the calculation substantially more involved. Another complication is caused by the fact that the field space is naturally the total space of a vector bundle, and the fermionic fields (the dilatino, gravitino, and gaugino) do not correspond to coordinates on this field space, but rather describe elements of the fibers (this is a consequence of the fact that they are sections of the spinor bundle whose very definition requires a choice of metric). This problem is typically dealt with by passing to the vielbein description of the metric. Here we take a more direct geometric approach and simply work with the vector bundle structure directly. This leads to some significant simplifications—for instance, the algebra of local supersymmetries (13) does not feature any Lorentz terms on the rhs as these are canceled by the term coming from the nonzero curvature on this vector bundle.

Consequently, the BV action (19) is much simpler than one would *a priori* expect. Still, checking explicitly that it satisfies the classical master equation is not easy. Consequently, we do not give a full proof of this fact but only provide various evidence in favor of its validity. A complete proof is left for a future work.

Although immensely important for the purpose of quantization, the BV analysis of supergravity has so far been mostly restricted to the $D=4$ case [11] (more recently see also [12,13]). To the best of our knowledge the present Letter is the first instance when the BV action for a higher-dimensional supergravity has been constructed in the background independent component field formalism (as opposed to the pure spinor superfield approach, see [14] and references therein).

We conclude the Introduction by highlighting the fact that the BV formulation of the $\mathcal{N}=1$ $D=10$ case is particularly interesting in that it is directly linked to the work [15] of Costello-Li, since it provides the starting point for their twist of supergravity; for more details see the last section.

Bosonic field content—We first recall the generalized-geometric description of supergravity, based on [4,7]. We refer the reader to these works for more details and conventions.

The theory itself is defined in terms of a transitive Courant algebroid E [16,17] over a ten-dimensional base space M. Locally, this is given by a vector bundle

$$E \cong_{\mathrm{loc}} TM \oplus T^*M \oplus (\mathfrak{g}\times M), \qquad (1)$$

*Contact author: j.kupka@herts.ac.uk
†Contact author: c.strickland-constable@herts.ac.uk
‡Contact author: f.valach@herts.ac.uk

with $\mathfrak{g}$ a Lie algebra with an invariant pairing denoted by Tr. Sections of E thus correspond to formal sums of a vector field, a 1-form, and a $\mathfrak{g}$-valued function. This structure is equipped with a bracket, pairing, and a map $a: E \to TM$ given by

$$\begin{aligned}
[X+\alpha+s, Y+\beta+t] &= L_X Y + (L_X\beta - i_Y d\alpha + \mathrm{Tr}\, t\, ds) \\
&\quad + (L_X t - L_Y s + [s,t]_{\mathfrak{g}}), \\
\langle X+\alpha+s, Y+\beta+t\rangle &= \alpha(Y) + \beta(X) + \mathrm{Tr}\, st \\
a(X+\alpha+s) &= X.
\end{aligned}$$

Let H denote the line bundle of half-densities on M, and $\mathcal{H}^*$ be the space of its invertible (i.e., everywhere nonvanishing) sections. The bosonic field content of the theory then consists of the following fields: (i) a generalized metric $\mathcal{G}$, i.e., a symmetric endomorphism $E \to E$ satisfying $\mathcal{G}^2 = 1$; and (ii) an invertible half-density $\sigma \in \mathcal{H}^*$.

The generalized metric induces an orthogonal splitting $E = C_+ \oplus C_-$ into its ± 1 eigenbundles. We will denote the frames of C_+ and C_- by $e_a, e_b, \ldots$ and $e_\alpha, e_\beta, \ldots$, respectively. We shall make a further assumption that $\langle\cdot,\cdot\rangle|_{C_+}$ has signature (9, 1) and admits spinors, and $a|_{C_+}: C_+ \to TM$ is an isomorphism.

Denote the space of such generalized metrics by $\mathcal{M}$. The last two conditions provide the bridge to the ordinary description of the field content, as under the identification (1) any C_+ takes the form

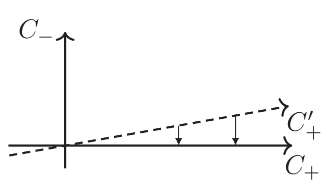
$$\left\{x + \left(i_x g + i_x B - \frac{1}{2}\mathrm{Tr} A i_x A\right) + i_x A \middle| x \in TM\right\} \subset E \qquad (2)$$

for some Lorentzian metric g, Kalb-Ramond 2-form B, and G-connection 1-form A. The dilaton function ϕ is encoded in σ via

$$\sigma^2 = \sqrt{|g|} e^{-2\phi}, \qquad (3)$$

where $\sqrt{|g|}$ stands for the standard metric density.

Consider now the "tautological" bundle $\mathcal{C}_+ \to \mathcal{M}$, whose fiber at $\mathcal{G}$ is the space $\Gamma(C_+)$. Any small change $\mathcal{G} \rightsquigarrow \mathcal{G}' \coloneqq \mathcal{G} + \delta\mathcal{G}$ induces a small deformation of the subbundle $C_+ \rightsquigarrow C'_+$. Since the orthogonal projection $E \to C_+$ gives an isomorphism $C'_+ \to C_+$ (see the following picture), we have an identification of the nearby fibers of $\mathcal{C}_+$, i.e., a connection.



A straightforward calculation shows that the curvature of this connection is

$$F(\delta_1\mathcal{G}, \delta_2\mathcal{G}) = \frac{1}{4}[\delta_1\mathcal{G}, \delta_2\mathcal{G}]: \Gamma(C_+) \to \Gamma(C_+), \qquad (4)$$

where $\delta_{1,2}\mathcal{G}$ are two infinitesimal variations of $\mathcal{G}$, i.e., vectors at $T_{\mathcal{G}}\mathcal{M}$. Analogously we obtain a connection and curvature on $\mathcal{C}_- \to \mathcal{M}$.

Fermions and supersymmetry—Denoting the Majorana-Weyl spinor bundles for C_+ by $S_\pm$, the fermionic field content of the theory is

$$\rho \in \Gamma(\Pi S_+ \otimes H), \quad \psi \in \Gamma(\Pi S_- \otimes C_- \otimes H), \qquad (5)$$

where Π denotes the parity shift. As shown in [4], these fields encode the usual dilatino and gravitino + gaugino, respectively. Note that we define the fermions as half-densities.

Since ρ and ψ are sections of bundles which themselves depend on the generalized metric, the classical field space has the structure of (the total space of) a vector bundle

$$\mathcal{S}_0 \to \mathcal{M} \times \mathcal{H}^*, \qquad (6)$$

whose fiber at $(\mathcal{G}, \sigma)$ is

$$\Gamma(\Pi S_+ \otimes H) \times \Gamma(\Pi S_- \otimes C_- \otimes H). \qquad (7)$$

Since the bundles $S_\pm$ are naturally associated to C_+, it follows that $\mathcal{S}_0$ carries a connection inherited from the ones on $\mathcal{C}_\pm \to \mathcal{M}$. Its curvature is

$$\begin{aligned}
F(\delta_1\mathcal{G}, \delta_2\mathcal{G})\rho &= \frac{1}{8}\delta_1\mathcal{G}_a{}^\beta \delta_2\mathcal{G}_{b\beta}\gamma^{ab}\rho, \\
F(\delta_1\mathcal{G}, \delta_2\mathcal{G})\psi^\alpha &= \frac{1}{8}\delta_1\mathcal{G}_a{}^\beta \delta_2\mathcal{G}_{b\beta}\gamma^{ab}\psi^\alpha + \frac{1}{2}\delta_{[1}\mathcal{G}_a{}^\alpha \delta_{2]}\mathcal{G}^a{}_\beta\psi^\beta.
\end{aligned} \qquad (8)$$

Note that this has the form of a Lorentz transformation.

In order to write down kinetic terms for the fermions we again recall the construction from [4,7]. A generalized connection D [18] is said to belong to the class $LC(\mathcal{G},\sigma)$ if it is torsion-free and preserves both $\mathcal{G}$ and σ. Such connection exist but are not unique [19]; however, there exist objects constructed out of $\mathcal{G}$, σ, and $D \in LC(\mathcal{G},\sigma)$, which are independent of the choice of the representative $D \in LC(\mathcal{G},\sigma)$ and thus only depend on $\mathcal{G}$ and σ. The most important are (i) the generalized scalar curvature $\mathcal{R}$, (ii) the generalized Ricci tensor $\mathcal{R}_{a\beta}$, (iii) the Dirac operator $\not{D} = \gamma^a D_a$ in $\not{D}\rho$ and $\not{D}\psi^\alpha$, (iv) the operator D_α in $D_\alpha\rho$ and $D_\alpha\psi^\alpha$, and (v) the operator D when acting on any $f \in C^\infty(M)$.

This allows us to construct the following action functional S_0 [4] on the space $\mathcal{S}_0$:

$$S_0 = \int_M \mathcal{R}\sigma^2 + \bar{\psi}_\alpha \not{D}\psi^\alpha + \bar{\rho}\not{D}\rho + 2\bar{\rho}D_\alpha\psi^\alpha - \frac{1}{768}\sigma^{-2}(\bar{\psi}_\alpha\gamma_{abc}\psi^\alpha)(\bar{\rho}\gamma^{abc}\rho) - \frac{1}{384}\sigma^{-2}(\bar{\psi}_\alpha\gamma_{abc}\psi^\alpha)(\bar{\psi}_\beta\gamma^{abc}\psi^\beta), \tag{9}$$

which is invariant under the local supersymmetries, i.e.,

$$\delta_\epsilon\mathcal{G}_{ab} = \delta_\epsilon\mathcal{G}_{\alpha\beta} = 0, \qquad \delta_\epsilon\mathcal{G}_{a\beta} = \delta_\epsilon\mathcal{G}_{\beta a} = \frac{1}{2}\sigma^{-2}\bar{\epsilon}\gamma_a\psi_\beta,$$
$$\delta_\epsilon\sigma = \frac{1}{8}\sigma^{-1}(\bar{\rho}\epsilon),$$
$$\delta_\epsilon\rho = \not{D}\epsilon + \frac{1}{192}\sigma^{-2}(\bar{\psi}_\alpha\gamma_{abc}\psi^\alpha)\gamma^{abc}\epsilon,$$
$$\delta_\epsilon\psi_\alpha = D_\alpha\epsilon + \frac{1}{8}\sigma^{-2}(\bar{\psi}_\alpha\rho)\epsilon + \frac{1}{8}\sigma^{-2}(\bar{\psi}_\alpha\gamma_a\epsilon)\gamma^a\rho, \tag{10}$$

The supersymmetry parameter ϵ is here a function on $\mathcal{S}_0$, which for any given field configuration $(\mathcal{G},\sigma,\rho,\psi)$ takes value in $\Gamma(\Pi S_- \otimes H)$ (note that this bundle itself depends on $\mathcal{G}$). Formula (10) thus define a vector field δ_ϵ on $\mathcal{S}_0$—more precisely the first and second pair of formulas express the horizontal and vertical parts of this vector field, respectively. This is the general meaning of the formulas for supersymmetry variations [20].

Similarly, we note that any section $\zeta \in \Gamma(E)$ induces an infinitesimal automorphism of E. This is usually expressed via the generalized Lie derivative operator $\mathcal{L}_\zeta$ and again produces a vector field δ_ζ on $\mathcal{S}_0$ which preserves the functional S_0. Note that this action is reducible, as $\mathcal{L}_{Df} = 0$ for any $f \in C^\infty(M)$.

Let us now turn to the algebra of the symmetries. As usual, its most interesting part corresponds to the commutator of two supersymmetries. On the bosonic fields we have

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}] = \delta_\epsilon + \delta_\zeta, \tag{11}$$

where we defined

$$\zeta^a \coloneqq \frac{1}{4}\sigma^{-2}\bar{\epsilon}_2\gamma^a\epsilon_1, \qquad \epsilon \coloneqq -\frac{1}{2}\not{\zeta}\rho + \delta_{\epsilon_1}\epsilon_2 - \delta_{\epsilon_2}\epsilon_1, \tag{12}$$

while on the fermions

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}]\rho = \delta_\epsilon\rho + \delta_\zeta\rho - \frac{1}{2}\not{\zeta}(\not{D}\rho + \ldots),$$
$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}]\psi^\alpha = \delta_\epsilon\psi^\alpha + \delta_\zeta\psi^\alpha + \left(\frac{1}{4}\epsilon_{[2}\bar{\epsilon}_{1]} - \frac{1}{2}\not{\zeta}\right)(\not{D}\psi^\alpha + \ldots), \tag{13}$$

where the last parentheses contain the equations of motion of ρ and ψ, respectively. Note that this provides a significant simplification when compared to the usual formulas (cf. [1]).

As usual, the calculations leading to (13) are relatively lengthy and involve a generous handful of Fierz identities. Notably, since the formula (10) really correspond to horizontal and vertical parts of the vector field δ_ϵ on $\mathcal{S}_0$, the corresponding commutator on fermions picks up an extra term (in addition to the "naive" commutator of variations), coming from the curvature (8). This removes all the terms which look like Lorentz transformations and which would be present in the vielbein formulation. [Note that Lorentz transformations are not expected to appear in (13) as they are *not* symmetries of the *metric tensor formulation* of the theory.] We postpone the details of the calculation to a future work [21].

BV field space—To construct the BV space we start with the classical field space, add ghosts and ghosts for ghosts corresponding to local symmetries, and then adjoin the corresponding antifields. This yields the BV space

$$\mathcal{F}_{\mathrm{BV}} \coloneqq T^*[-1]\mathcal{S}, \tag{14}$$

where $\mathcal{S} \to \mathcal{M} \times \mathcal{H}^*$ is the vector bundle whose fiber at $(\mathcal{G},\sigma)$ is

$$\Gamma(\Pi S_+ \otimes H) \times \Gamma(\Pi S_- \otimes C_- \otimes H) \times \Gamma(\Pi S_- \otimes H)[1] \times \Gamma(E)[1] \times C^\infty(M)[2]. \tag{15}$$

Here $[n]$ signifies the degree shift and corresponds to the ghost number. The overall parity is the sum of the superdegree (bosonic or fermionic) and the parity of the ghost degree. Elements of the fiber (15) correspond to the fermionic fields ρ and ψ, the supersymmetry ghost e, the diffeomorphism ghost ξ, and the ghost for ghost f, respectively. To summarize, our field content up to now consists of

$$\mathcal{G} \in \Gamma(E^* \otimes E) \text{s.t.} \times\ \mathcal{G}^2 = 1 \quad \text{and} \quad \mathcal{G}^* = \mathcal{G},$$
$$\sigma \in \Gamma(H)\ \text{every where nonvanishing},$$
$$\rho \in \Gamma(\Pi S_+ \otimes H,)$$
$$\psi \in \Gamma(\Pi S_- \otimes C_- \otimes H),$$
$$e \in \Gamma(\Pi S_- \otimes H)[1],$$
$$\xi \in \Gamma(E)[1],$$
$$f \in C^\infty(M)[2]. \tag{16}$$

In particular, the fields $\mathcal{G}$, σ, e, and f are even and the rest is odd.

The space $\mathcal{F}_{BV}$ also includes the dual antifields, whose most convenient description is as follows. We start by noting that, following the discussion above, $\mathcal{S}$ carries a natural connection and hence a splitting of its tangent spaces into horizontal and vertical parts. This gives an identification

$$T^*[-1]\mathcal{S} \cong \pi^*(T^*[-1](\mathcal{M} \times \mathcal{H}^*)) \oplus \pi^*\mathcal{S}^*[-1], \tag{17}$$

of bundles over $\mathcal{S}$, where $\pi: \mathcal{S} \to \mathcal{M} \times \mathcal{H}^*$ is the projection. We will describe the fibers of the first and second summand by the dual coordinates $\mathcal{G}^*$, σ^*, and ψ^*, ρ^*, ξ^*, e^*, f^*, respectively. More concretely, for any configuration $(\mathcal{G}, \sigma, \psi, \rho, \xi, e, f)$ we have

$$\begin{aligned}
\mathcal{G}^* &\in T^*_{\mathcal{G}}[-1]\mathcal{M} \cong \Gamma(C_+ \otimes C_- \otimes H^2)[-1],\\
\sigma^* &\in \Gamma(H)[-1],\\
\psi^* &\in \Gamma(\Pi S_+ \otimes C_- \otimes H)[-1],\\
\rho^* &\in \Gamma(\Pi S_- \otimes H)[-1],\\
\xi^* &\in \Gamma(E \otimes H^2)[-2],\\
e^* &\in \Gamma(\Pi S_+ \otimes H)[-2],\\
f^* &\in \Gamma(H^2)[-3].
\end{aligned} \tag{18}$$

Here we used the fact that infinitesimal deformations of $\mathcal{G}$ correspond to deformations of C_+; and any nearby deformed C'_+ is the graph of a vector bundle map $C_+ \to C_-$ (see the picture above). We also used the identifications $C_\pm \cong C^*_\pm$, $E^* \cong E$, and $S^*_\pm \cong S_\mp$. Note that ψ^*, ρ^*, and ξ^* are even and the rest is odd.

BV action—We now claim that the BV extension of the supergravity action (9) is

$$\begin{aligned}
S = \int_M & \mathcal{R}\sigma^2 + \bar\psi_\alpha \not{D}\psi^\alpha + \bar\rho\not{D}\rho + 2\bar\rho D_\alpha\psi^\alpha - \frac{1}{768}\sigma^{-2}(\bar\psi_\alpha\gamma_{abc}\psi^\alpha)(\bar\rho\gamma^{abc}\rho) - \frac{1}{384}\sigma^{-2}(\bar\psi_\alpha\gamma_{abc}\psi^\alpha)(\bar\psi_\beta\gamma^{abc}\psi^\beta)\\
&+ \sigma^*\left[\mathcal{L}_\xi\sigma - \frac{1}{8}\sigma^{-1}(\bar\rho e)\right] + \mathcal{G}^*_{\alpha\beta}\left[(\mathcal{L}_\xi\mathcal{G})^{\alpha\beta} + \frac{1}{2}\sigma^{-2}(\bar e\gamma^a\psi^\beta)\right] + \bar\rho^*\left[\mathcal{L}_\xi\rho + \not{D}e + \frac{1}{192}\sigma^{-2}(\bar\psi_\beta\gamma_{abc}\psi^\beta)\gamma^{abc}e\right]\\
&+ \bar\psi^*_\beta\left[(\mathcal{L}_\xi\psi)^\beta + D^\beta e + \frac{1}{8}\sigma^{-2}(\bar\psi^\beta\rho)e - \frac{1}{8}\sigma^{-2}(\bar\psi^\beta\gamma_a e)\gamma^a\rho\right] + \bar e^*\left[\mathcal{L}_\xi e + \frac{1}{16}\sigma^{-2}(\bar e\gamma_a e)\gamma^a\rho\right]\\
&+ \left\langle\xi^*, Df + \frac{1}{2}\mathcal{L}_\xi\xi\right\rangle - \frac{1}{8}\xi^*_a\sigma^{-2}(\bar e\gamma^a e) + \frac{1}{2}f^*\left(\mathcal{L}_\xi f + \frac{1}{8}\sigma^{-2}(\bar e\gamma_a e)\xi^a - \frac{1}{6}\langle\xi, \mathcal{L}_\xi\xi\rangle\right) - \frac{1}{64}\sigma^{-2}(\bar e\gamma_a e)(\bar\psi^*_\beta\gamma^a\psi^{*\beta})\\
&- \frac{1}{32}\sigma^{-2}(\bar e\psi^*_\beta)(\bar e\psi^{*\beta}) - \frac{1}{64}\sigma^{-2}(\bar e\gamma_a e)(\bar\rho^*\gamma^a\rho^*).
\end{aligned} \tag{19}$$

Following the usual BV machinery the form of this action is essentially read off from what was discussed before: the linear terms in antifields include both (10) and the generalized diffeomorphisms, as well as the "structure coefficients" of the symmetry algebra [first part of the rhs of (13)], while the terms quadratic in antifields quantify the failure of the symmetries to close off-shell [last part of the rhs of (13)]. Finally, an important nontrivial check is provided by the fact that when taking $\mathcal{G} = 1$ (and after a constant rescaling of ρ) the expression (19) matches the BV action for the dilatonic supergravity [10]. This can, in particular, be used to determine the terms in (19) containing the ghost-for-ghost f, which account for the reducibility of our description of generalized diffeomorphisms (which is the same regardless of whether $\mathcal{G} = 1$ or not) [22]. Our result also structurally matches the $D = 4$ supergravity BV analysis in [11].

That being said, we note that although highly suggestive, the above arguments do not provide a full proof that the classical master equation is indeed satisfied. However, even after performing additional nontrivial checks (which are too lengthy to report on here) we have not found any indication that the formula (19) is incomplete or incorrect in any way and hence we are highly confident in its validity. The full proof of this fact is left for a future work.

Conclusions and outlook—We have found the BV action for the $\mathcal{N} = 1$ supergravity in 10 dimensions, in general, coupled to a super Yang-Mills sector. It looks somewhat likely that with further effort one might succeed in performing a similar BV analysis for the type II supergravity. One should also be able to derive the corresponding results for the lower-dimensional supergravities via consistent truncations. Using our results one could proceed to look for a perturbative solution to the quantum master equation in order to investigate the quantum nature of the supergravity theories. It would also be interesting to relate the present BV formulation to superstring field theory. We leave these questions for future work.

In [23] Costello and Li suggested a procedure of twisting supergravity. Their twist starts by taking the BV formulation of supergravity and then expanding the BV action S around its critical point [24] which has a nonzero value of the supersymmetry ghost e. In particular it was conjectured in [15] that the (holomorphic) twist of type I supergravity on a Calabi-Yau fivefold is described by the $\mathbb{Z}_2$-fixed locus of the BCOV theory [25] coupled to the SO(32) holomorphic Chern-Simons theory. The present Letter could be used to put this conjecture on more solid ground by completing the BV description of (the two-derivative part of) its starting point, i.e., type I supergravity.

Following this philosophy, let us look more closely at the critical points of the BV action (19) (for any gauge group). The corresponding equations are easy to find and are shown in the Appendix. In particular setting to zero all the fields except for $\mathcal{G}$, σ, and e these equations reduce to

$$\mathcal{R}_{a\beta} = \not{D} e = D_\alpha e = \bar{e}\gamma^a e = 0. \tag{20}$$

This is the condition for the background $(\mathcal{G}, \sigma)$ to be supersymmetric, with the extra requirement that $\bar{e}\gamma^a e = \mathcal{R}_{a\beta} = 0$. (Note that the vanishing of the generalized scalar curvature $\mathcal{R}$ follows from the Lichnerowitz formula [4,7].) The Eq. (A4) can be therefore regarded as a generalisation thereof. This, in particular, suggests an interesting modification of the condition for the existence of a parallel spinor (leading in the classical case to, e.g., Calabi-Yau manifolds) to one of the form

$$D_\alpha e = \frac{1}{16}\sigma^{-2} e(\bar{\psi}^*_\alpha e). \tag{21}$$

We recall here that both spinors e and ψ^* are even (commuting).

Acknowledgments—C. S.-C. and F. V. are supported by an EPSRC New Investigator Award, Grant No. EP/X014959/1.

Data availability—No data were created or analyzed in this study.

[1] E. Bergshoeff, M. de Roo, B. de Wit, and P. van Nieuwenhuizen, Nucl. Phys. **B195**, 97 (1982).

[2] G. F. Chapline and N. S. Manton, Phys. Lett. **120B**, 105 (1983).

[3] M. Dine, R. Rohm, N. Seiberg, and E. Witten, Phys. Lett. **156B**, 55 (1985).

[4] J. Kupka, C. Strickland-Constable, and F. Valach, arXiv: 2410.16046.

[5] J. Kupka, C. Strickland-Constable, and F. Valach, Phys. Rev. Lett. **134**, 111601 (2025).

[6] W. Siegel, Phys. Rev. D **48**, 2826 (1993).

[7] A. Coimbra, C. Strickland-Constable, and D. Waldram, J. High Energy Phys. 11 (2011) 091.

[8] M. Garcia-Fernandez, Commun. Math. Phys. **332**, 89 (2014).

[9] A. Coimbra, R. Minasian, H. Triendl, and D. Waldram, J. High Energy Phys. 11 (2014) 160.

[10] J. Kupka, C. Strickland-Constable, and F. Valach, Phys. Rev. D **111**, 046020 (2025).

[11] L. Baulieu, M. P. Bellon, S. Ouvry, and J.-C. Wallet, Phys. Lett. B **252**, 387 (1990).

[12] A. S. Cattaneo, N. Moshayedi, and A. S. Funcasta, arXiv: 2412.14300.

[13] A. S. Cattaneo and F. Fila-Robattino, arXiv:2503.07373.

[14] M. Cederwall, Pure spinors in classical and quantum supergravity, in *Handbook of Quantum Gravity* (Springer, Singapore, 2023), 10.1007/978-981-99-7681-2_47.

[15] K. Costello and S. Li, Adv. Theor. Math. Phys. **24**, 1723 (2020).

[16] Z.-J. Liu, A. Weinstein, and P. Xu, J. Diff. Geom. **45**, 547 (1997).

[17] P. Ševera, arXiv:1707.00265.

[18] A generalized connection is a map $D\colon \Gamma(E)\times\Gamma(E)\to\Gamma(E)$ satisfying $D_{fu}v = fD_u v$, $D_u(fv) = fD_u v + (a(u)f)v$, and $D\langle\cdot,\cdot\rangle = 0$. Any such connection also naturally acts on half-densities via $D_u\sigma = L_{a(u)}\sigma - \frac{1}{2}\sigma\mathrm{tr}(Du)$, where L is the usual Lie derivative.

[19] M. Garcia-Fernandez, Adv. Math. **350**, 1059 (2019).

[20] In other words, notice that $\delta_\epsilon\rho$ corresponds to the difference of two sections of two different bundles (corresponding to $\mathcal{G}$ and $\mathcal{G}+\delta_\epsilon\mathcal{G}$, respectively). In order to make sense of this one needs to provide the identification of these two bundles, which is where the connection on $\mathcal{S}_0$ is used.

[21] J. Kupka, C. Strickland-Constable, and F. Valach (to be published).

[22] As seen in [4] this corresponds to the reducibility of the gauge transformations of the B field.

[23] K. Costello and S. Li, arXiv:1606.00365.

[24] By a point we mean a point in the even part $\mathcal{F}_{BV}^{\text{even}}$ of the supermanifold $\mathcal{F}_{BV}$, i.e., a configuration with all odd fields vanishing.

[25] M. Bershadsky, S. Cecotti, H. Ooguri, and C. Vafa, Commun. Math. Phys. **165**, 311 (1994).

End Matter

Appendix: Critical points of the BV action—We are interested in critical points of S on $\mathcal{F}_{BV}^{\text{even}}$. Since all the terms in S contain an even number of odd fields, we can first consistently set them all to zero before performing the variation. The equation of motion for ξ^* then becomes

$$D_a f - \frac{1}{8}\sigma^{-2}\bar{e}\gamma_a e = 0, \qquad D_\alpha f = 0. \tag{A1}$$

Since in our setup $a|_{C_-}$ is surjective it follows that

$$df = 0, \qquad \bar{e}\gamma_a e = 0. \tag{A2}$$

The equation of motion for ρ^* then reduces to

$$\not{D} e = 0. \tag{A3}$$

A straightforward (and very short) calculation using formulas from [4] then shows that the remaining equations are

$$\begin{aligned}
0 &= \mathcal{R}_{a\alpha}\sigma^2 + \frac{1}{4}\bar{\rho}^*\gamma_a D_\alpha e - \frac{1}{4}\bar{e}\gamma_a D_\alpha \rho^* - \frac{1}{2}\bar{\psi}^*_\alpha D_a e - \frac{1}{4}\bar{\psi}^*_\alpha \gamma_{ab} D^b e - \frac{1}{4}\bar{e}\gamma_{ab} D^b \psi^*_\alpha, \\
0 &= \mathcal{R}\sigma^2 + \frac{1}{2}\bar{\psi}^*_\alpha D^\alpha e - \frac{1}{2}\bar{e} D^\alpha \psi^*_\alpha + \frac{1}{32}\sigma^{-2}(\bar{e}\psi^*_\alpha)(\bar{e}\psi^{*\alpha}), \\
0 &= D_\alpha e - \frac{1}{16}\sigma^{-2} e(\bar{\psi}^*_\alpha e), \\
0 &= D^\alpha \psi^*_\alpha - \not{D}\rho^* - \frac{1}{4}\sigma^{-2}\xi^*_a \gamma^a e - \frac{1}{32}\sigma^{-2}\gamma_a e(\bar{\psi}^*_\alpha \gamma^a \psi^{*\alpha}) - \frac{1}{16}\sigma^{-2}\psi^*_\alpha(\bar{e}\psi^{*\alpha}) - \frac{1}{32}\sigma^{-2}\gamma_a e(\bar{\rho}^*\gamma^a \rho^*), \\
0 &= D^a \xi^*_a + D^\alpha \xi^*_\alpha.
\end{aligned} \tag{A4}$$