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Synchronization in Memristive Small-World Neural Networks Under Electromagnetic Radiation

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Abstract: The human brain is composed of a large number of neurons that work together to process the generation, transmission, reception, and processing of information. The topological structure of the human brain has small-world characteristics, and the synchronization and neuron firing are influenced by the electromagnetic field. In this paper, we use four-stable discrete memristors to simulate the external electromagnetic field, and construct a memristive small-world neural network (MSNN) model based on Rulkov neurons, and conduct numerical simulations. We have found that the MSNN exhibits multiple coexisting behaviors of synchronous, asynchronous, and chimeric states under different initial conditions of the discrete memristors. At the same time, changing the strength of electromagnetic induction can affect the synchronization performance of the MSNN. Finally, we find that increasing the electromagnetic induction strength can enhance the neuron firing action potential.

Key words: small-world neural network; synchronization; electromagnetic induction

1 Introduction

The brain is the command center of life activities, and various physiological activities such as cognition, memory, and emotion rely on the complex neural network in the brain[1]. Neural networks in various brain regions work together to process and transmit physiological information. Scientific evidence suggests that most brain functional disorders are closely related to abnormal neuron firing and abnormal neural network synchronization, such as Parkinson's disease[2–4], epilepsy[5–6], and tinnitus[7–8]. Synchronization is a typical feature of complex neural networks and the primary method for neurons to transmit information[9]. It can effectively represent the states and patterns of each neuron in complex neural networks and promote information transmission between neurons. Lu[10] researched the synchronization of discrete linear coupled networks and proposed sufficient conditions for their synchronization stability. Xin et al.[11] constructed a circular network comprising fractional-order memristive coupled neurons and found that the network exhibits synchronous transition phenomena with the change of fractional order. He et al. established a double-layer neural network model with different intra-layer topologies and found that the intra-layer structural parameters and intra-layer electromagnetic induction of the network can affect its synchronization performance. They also derived the condition for complete synchronization of the double-layer neural network[12]. Muni et al.[13] researched a ring star network based on Memristive Hindmarsh-Rose Neurons and revealed its synchronous, asynchronous, and chimera states. Studying neural networks helps us gain a deeper understanding of the essence of the human brain and intelligence, contributing to the development and innovation of technology. In fields such as physics and biology, it has become a focus of attention for experts and scholars[14–17].

In the 1990s, Duncan J. Watts and Steven Strogatz proposed the famous theory of 'small-world' networks.

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Neurological researchers discovered that the connection patterns between neurons in the brain have small-world characteristics and referred to this kind of neural network as the small-world neural network[18]. In recent years, research on small-world networks has focused on the impact of the small-world topology on the dynamic characteristics of systems. For example, Ferrari et al.[19] studied the cortical regions of the brain with small-world network characteristics. A network was constructed by Rulkov neurons to simulate the cortical regions, and the synchronization behavior of the network was explored and verified. In addition, the synchronization behavior of the delayed feedback control network was also considered. Shatnawi[20] constructed a small-world neural network model based on Hodgkin-Huxley neurons and explored in detail the coherent resonance and random synchronization phenomena. Lu[21] used the discrete memristor to simulate synapses to construct a small-world network based on Rulkov neurons, and studied the impact of network topology and memristive coupling strength on neural synchronization. Guo[22] proposed a small-world spike neural network, studied its anti-interference ability, and compared the anti-interference ability of spike neural networks with different topology structures. Peng studied the impact of autapses on the synchronization behavior of small-world neural networks based on the Hodgkin-Huxley neuron model[23], and found that the transmission delay of electrical autapses can suppress the synchronization of the neural network, while the transmission delay of chemical autapses can promote the synchronization of the neural network. Hu constructed a scale-free neural network with small-world characteristics[24], studied the impact of Spike-timing-dependent plasticity (STDP) on neural network synchronization, and compared the optimal STDP maximum weight range of scale-free networks with small-world characteristics, small-world networks, and scale-free networks. Tang et al.[25] constructed a delayed coupled small-world neural network in a noisy environment and analyzed its chimeric state. The firing behavior between neurons requires an extremely strict physiological electrical environment, which is shaped by various physical factors, among which the dynamic changes of electric and magnetic fields are particularly critical. According to Maxwell's electromagnetic field theory, there is a close interaction and influence between electric and magnetic fields, which play an indispensable role in the firing process of neurons. Therefore, it is necessary to study neural networks in electromagnetic field environments. However, previous research on small-world networks did not consider electromagnetic effects.

In May 2008, HP laboratory staff published a paper in Nature, successfully developing a physical memristor using TiO2 and nanoscale technology, proving the existence and feasibility of memristor components at the physical level[26]. The successful production of memristors marks a significant breakthrough, and this milestone event has attracted widespread attention from researchers and scientists in various fields. For example, Wang et al.[27] utilized InGaZnO material to fabricate neuronal synaptic elements with memory function. Adhikari et al.[28] proposed the three fingerprint characteristic of memristors, which has become one of the criteria for judging memristors. In addition, memristors can simulate external electromagnetic fields and establish neural network systems in corresponding electromagnetic field environments[29–31]. Scholar Quan Xu[32], based on the Wilson neuron model of a memristor, used various dynamic analysis methods and added the external electromagnetic field environment to discover that the memristive Wilson neuron model exhibits rich firing behavior. Numerical verification was achieved through hardware circuits. The scholar also proposed a dual neural network composed of heterogeneous neurons and conducted in-depth research on its dynamic behavior in electromagnetic field environments. Ma[33] used a four-state memristor to simulate the electromagnetic field and studied the dynamic behavior of scale-free networks in the electromagnetic field environment, discovering the coexistence behavior of multiple states in scale-free networks. Muni et al.[34] studied the impact of electromagnetic effects on the Hindmarsh-Rose(HR) neural network and discovered that the neural network shows synchronization of chimeric states. Yu et al.[35] proposed a non-polynomial memristor that satisfies the Lipschitz condition to solve the coupling between computational complexity and resource utilization in the circuit implementation of the memristor-based multiscroll Hopfield Neural Network, and proposed a new adaptive synchronization scheme to simulate neural network synchronization.

In this article, the four-state discrete memristors are used to simulate the external electromagnetic field. A memristive small-world neural network(MSNN) model is established based on Rulkov neurons, and numerical simulations are conducted. The main contributions of this article include:

(1) A small-world neural network influenced by the electromagnetic field is constructed based on the Rulkov neuron model and the four-state discrete memristors.

(2) The impact of memristor initial values on the dynamic behavior of the MSNN is explored.

(3) Explore the impact of the strength of electromagnetic induction on the synchronization and neuron firing action potential of the MSNN.

The remaining structure of this article is as follows: We introduce the model of the four-state discrete memristor and propose a MSNN model based on Rulkov neurons in the electromagnetic field in Section 2. We study the effects of different initial values of the discrete memristor and the strength of electromagnetic induction on the synchronization and firing behavior of the MSNN in Section 3. Finally, a summary is made in Section 4.

2 Model Construction

2.1 Discrete Memristor Model

A four-state memristor is proposed in reference [36]. In this article, the memristor is discretized and used to simulate the external electromagnetic field. The expression is as follows:

$$\begin{cases} i(n) = W(\phi(n))v(n) = \phi(n)v(n) \\ \phi(n+1) = 0.1(sgn(\phi(n)) + sgn(\phi(n)+2)+ \\ sgn(\phi(n)-2)) + 0.9\phi(n) + 0.01v(n) \end{cases} \tag{1}$$

Here, $i(n)$ represents current, $v(n)$ represents voltage, $W(\phi(n))$ represents the memductance, and $\phi(n)$ is the internal state variable of the discrete memristor.

When applying a sinusoidal voltage $V = Asin(2\pi Fn)$ to the discrete memristor, as shown in Fig. 1, gradually changing the voltage amplitude A from 2 to 4, the I-V curve of the discrete memristor becomes larger and larger. As shown in Fig. 1, as the frequency gradually increases, the hysteresis loop begins to contract gradually. When the frequency F approaches infinity, the hysteresis loop contracts into a single-value function. The results indicate that the discrete memristor conforms to the three fingerprint characteristics of memristors.

This kind of memristor can converge to different equilibrium points with different initial values. As shown in Fig. 2, when the initial values of the discrete memristor are set to −3, −1, 1, and 3, the discrete memristor stabilizes in four different states, confirming its characteristics as a four-stable discrete memristor.

2.2 MSNN Model

In this study, the WS small-world neural network model is used. Specifically, it is a nearest neighbor-coupled network consisting of N neurons, where all neurons form a loop, and each neuron is connected to K ($K<N$) nearest neurons on its left and right sides. Subsequently, all interconnected neurons randomly disconnect and reconnect based on probability p. Fig. 3 shows the topology of the WS small-world neural network simulated using Matlab, where N = 100 and K = 4.

In this article, a MSNN model under the electromagnetic field is constructed based on the Rulkov neuron model and the four-state discrete memristors and its model expression is as follows:

$$\begin{cases} x_i(n+1) = \dfrac{\alpha}{1+x_i(n)^2} + y_i(n) + k\displaystyle\sum_{\mathrm{j}=1}^{N} \varepsilon_{i,j}(x_j(n)- \\ x_i(n)) + w\varphi_i(n)x_i(n) \\ y_i(n+1) = y_i(n) - b(x_i(n)-\sigma) \\ \varphi_i(n+1) = 0.1(\mathrm{sgn}(\varphi_i(n)) + \mathrm{sgn}(\varphi_i(n)+2) + \mathrm{sgn} \\ (\varphi_i(n)-2)) + 0.9\varphi_i(n) + 0.01x_i(n) \end{cases} \tag{2}$$

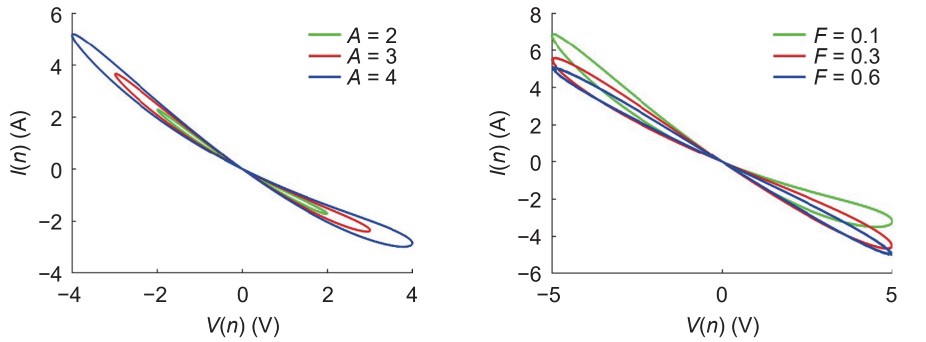

Fig. 1 The characteristic curve of the discrete memristor (a) Fixed *F*, *A* is 2,3,4; (b) Fixed *A*, *F* is 0.1, 0.3, 0.6

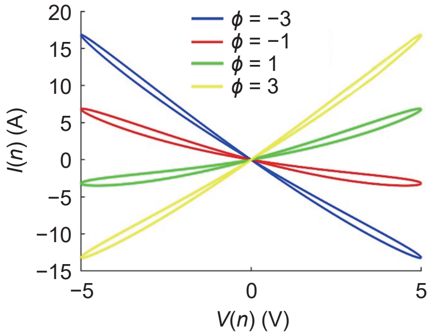

Fig. 2 Multi steady state characteristics of the discrete memristor

Here, k is the coupling strength, w is the electromagnetic induction strength, N represents the number of nodes, α and b are the control parameters and σ is the externally exerted influence. $\varepsilon_{i,j}$ is the connection matrix, $\varepsilon_{i,j}=1$ represents the connection between two neurons, and $\varepsilon_{i,j}=0$ represents no connection.

3 Dynamic Analysis of the MSNN

In this section, the dynamic behavior of small-world neural networks in an external electromagnetic field was studied. The dynamic behavior of the MSNN is explored from the aspects of the electromagnetic induction strength and initial values of the memristors. The coexistence phenomenon induced by different initial values of the memristors in neural networks, the enhancement and weakening of synchronization performance caused by electromagnetic induction strength, and the impact of electromagnetic induction strength on neuron firing action potential are analyzed using spatiotemporal patterns, time series diagrams, and node instantaneous state diagrams.

3.1 Coexistence phenomenon induced by initial values of the memristors

Firstly, we set the system parameters fixed to $\alpha = 3,\ b = 0.001,\ \sigma = -1,\ k = 0.01,\ w = 0.1$. The initial values of both fast and slow variables of neurons are set to 0. When adjusting the initial state of the memristors to 2, as shown in Fig. 4 (a1-b1), the neural network exhibits a chimeric state of asynchronous and synchronous coexistence. When the initial state of the memristors is 3, as shown in Fig. 4 (a2-b2), the neural network exhibits a special chimeric state of cluster synchronization and asynchrony coexistence. When the initial state of the memristors is −3, it can be seen from Fig. 4 (a3-b3) that it is in an asynchronous state.

3.2 Impact of electromagnetic induction on the MSNN

In addition, this study demonstrates that magnetic induction has a significant effect on firing behavior. In order to quantitatively study the spatiotemporal synchronization of the MSNN, we introduce a synchronization parameter s,

$$\begin{cases} s = \langle s(n) \rangle, \\ s(n) = \sqrt{\dfrac{\frac{1}{N}\sum_{i=1}^{n} (x_i(n))^2 - \left(\frac{1}{N}\sum_{i=1}^{n} (x_i(n))\right)^2}{N-1}}. \end{cases} \tag{3}$$

Here, N represents the number of nodes. It is obvious that the smaller the synchronization parameter s, the better the spatiotemporal synchronization behavior of the MSNN.

The parameters of the neuronal system are fixed at $\alpha = 3$, $b = 0.001$, and $\sigma = -1$. The initial values of both fast and slow variables of neurons are set to 0. We fix the initial value of the memristors to 2. As shown in Fig. 5, spatiotemporal diagrams and instantaneous state diagrams of the small-world neural network were plotted under the conditions of electromagnetic induction strength $w = 0.01$, 0.05, and 0.1, respectively. From the spatiotemporal diagrams, it can be seen that as the coupling strength increases, its regularity becomes more and more obvious, and the corresponding instantaneous state diagram dispersion decreases. The increase in the electromagnetic induction strength promotes the synchronization of the MSNN.

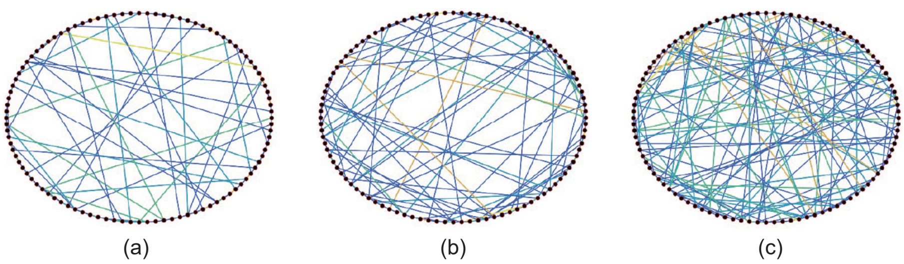

Fig. 3 Small-world neural network topology diagrams, when the reconnection probabilities are 0.2, 0.4, 0.6.

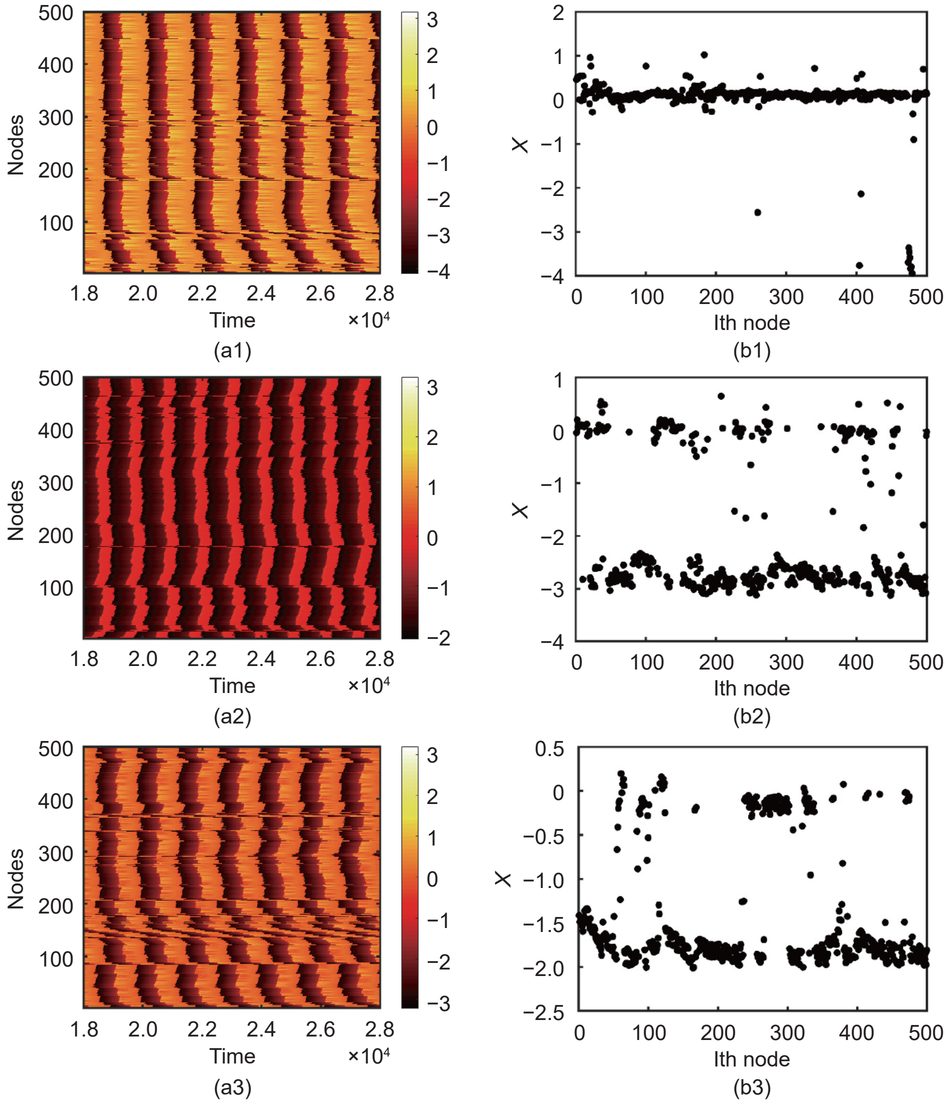

Fig. 4 Spatiotemporal diagrams (left) and instantaneous state diagrams (right), when the initial values of the memristors are 2, 3, and -3 from top to bottom.

We set the system parameters as α = 3, b = 0.001, and σ = −1. When the electromagnetic induction intensity is greater than 0.1, it has a destructive effect on neural network synchronization. As shown in Fig. 6, spatiotemporal diagrams and instantaneous state diagrams of electromagnetic induction strengths w = 0.15, 0.18, and 0.2 are shown. At this time, increasing the electromagneticinduction strength has a destructive effect on the synchronization of the MSNN.

As shown in Fig.7, From the synchronization coefficient and magnetic induction curve graph, When the electromagnetic induction strength is less than 0.1, the synchronization coefficient decreases as the electromagnetic induction strength increases, which promotes network synchronization. When the electromagnetic induction strength is greater than 0.1, the increase in magnetic induction intensity will suppress the synchronization of the MSNN.

In addition, as shown in the node state diagrams in Figs. 5 and 6, it can be seen that when the electromagnetic coupling strength w = 0.01, neuron firing action potentials range between [−0.1, 0.5]. As the electromagnetic strength increases, neuron firing action potentials also increase, as shown in Fig. 6 (c3). When the electromagnetic strength w = 0.2, neuron firing action potentials range between [0.1, 1.5].

4 Conclusion

In this article, a MSNN model under the electromagnetic field is constructed based on the Rulkov neuron model and the four-state memristors. Subsequently, considering the initial state and

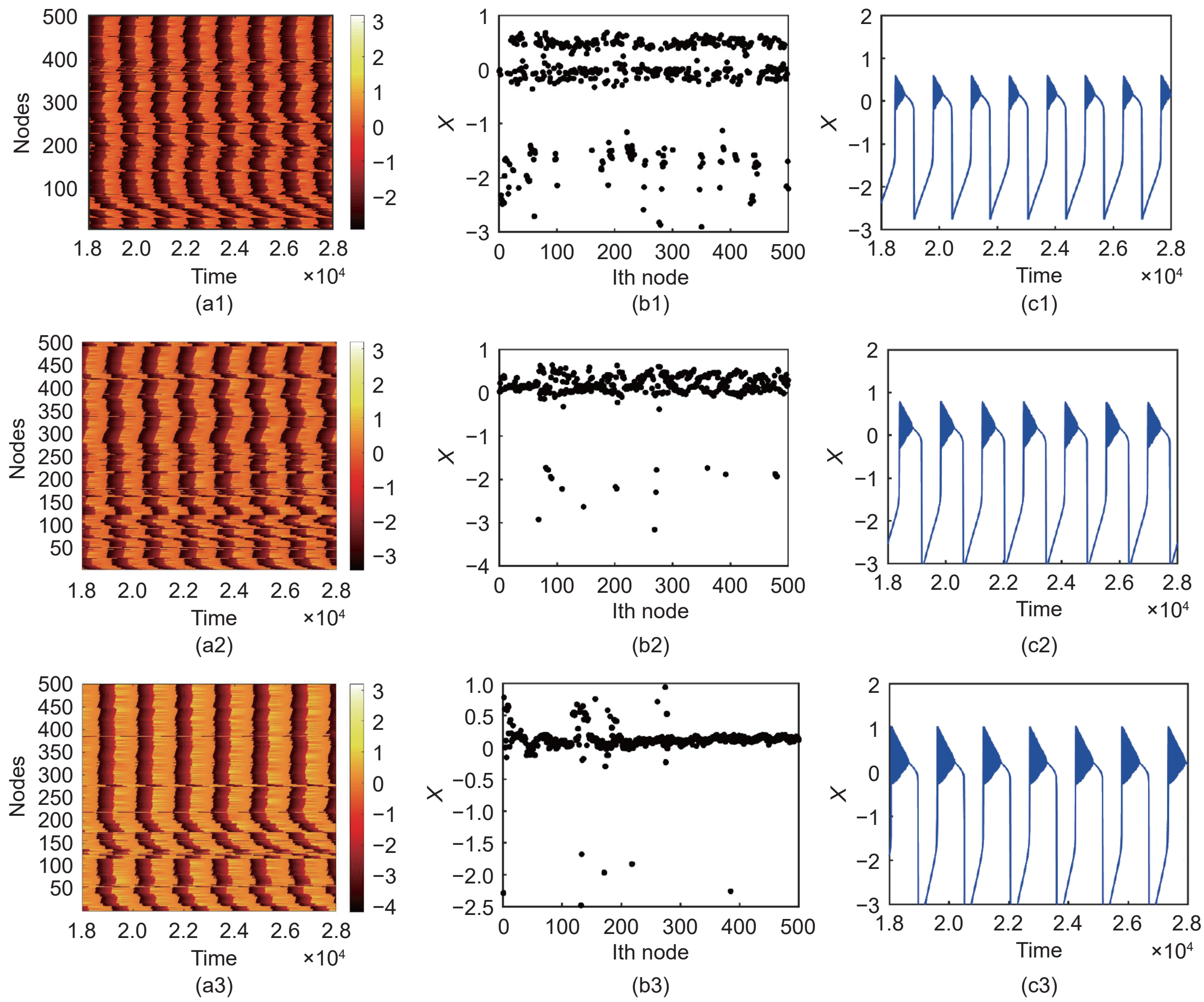

Fig. 5 Spatiotemporal diagrams (left), instantaneous state diagrams (middle), and node state diagrams (right), when the electromagnetic induction strengths are 0.01, 0.06, and 0.1, respectively.

electromagnetic coupling strength of memristive synapses, we use spatiotemporal diagrams, instantaneous state diagrams, and node state diagrams to analyze the synchronization and discharge behavior of the MSNN. Through numerical simulation, we find that different initial states of the memristor can induce synchronous, asynchronous, and chimeric states in the MSNN. Meanwhile, changing the strength of electromagnetic coupling can affect the synchronization of the MSNN and the neuron firing action potential. Considering the inconsistent coupling strength between neurons in real neural networks, future work will focus on constructing more realistic models of complex neural networks to explore their dynamic behaviors comprehensively.

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Conflict of Interest

The authors declare no conflict of interest.

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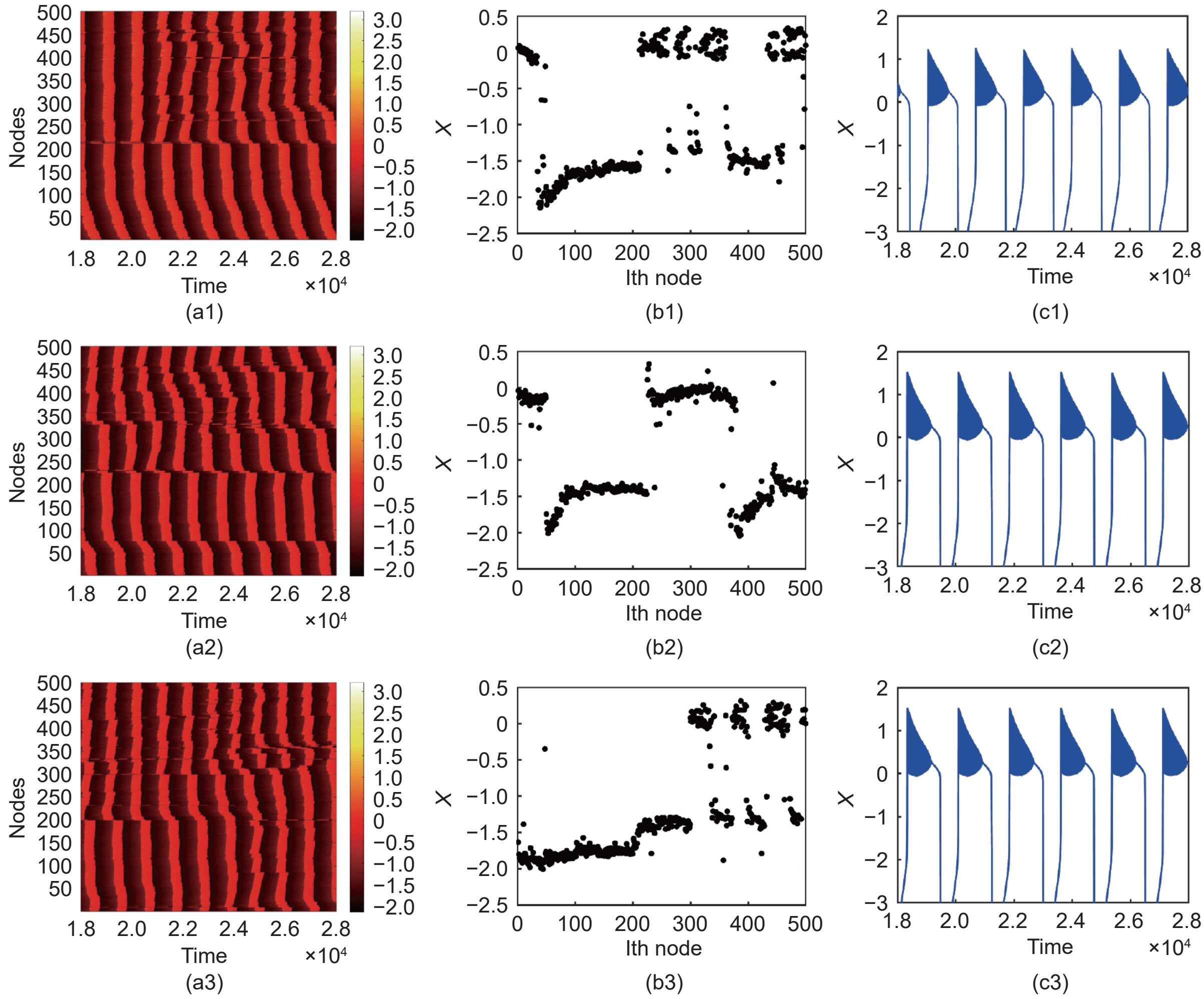

Fig. 6 Spatiotemporal diagrams (left), instantaneous state diagrams (middle), and node state diagrams (right), when the electromagnetic coupling strengths w = 0.15, 0.18, and 0.2, respectively.

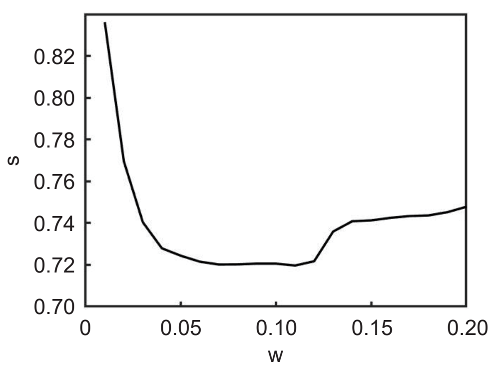

Fig. 7 Effect of the electromagnetic induction strength on the synchronization performance of the MSNN.

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