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Lessons learned from the detection of wide companions by radial velocity and astrometry

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ABSTRACT

The detection and constraint of the orbits of long-period giant planets is essential for enabling their further study through direct imaging. Recently, it has been highlighted that there are discrepancies between different orbital fitting solutions. We address these concerns by re-analysing the data for HD 28185, GJ 229, HD 62364, HD 38529, 14 Her, ϵ Ind A, HD 211847, HD 111031, and GJ 680, offering explanations for these discrepancies. Based on the comparison between our direct modelling of the astrometric catalogue data and the ORVARA code, we find the discrepancies are primarily data-related rather than methodology-related. Our re-analysis of HD 28185 highlights many of the data-related issues and particularly the importance of parallax modelling for year-long companions. The case of eps Ind A b is instructive to emphasize the value of an extended radial velocity baseline for accurately determining orbits of long-period companions. Our orbital solutions highlight other causes for discrepancies between solutions including the combination of absolute and relative astrometry, clear definitions of conventions, and efficient posterior sampling for the detection of wide-orbit giant planets.

Key words: methods: data analysis – methods: statistical – techniques: radial velocities – stars: individual: HD 28185.

1 INTRODUCTION

The detection and characterization of cold Jupiters have become feasible with high-precision data from radial velocity (RV) facilities, *Gaia* astrometry, and the imaging by the mid-infrared instrument (MIRI; Rieke et al. 2015) installed on the *JWST*. As RV, astrometry, and imaging techniques all target planets around nearby stars, they can be used synergistically to gather more comprehensive information than each technique would individually provide.

Since *Gaia*'s second data release, several research groups have developed methods for jointly analysing *Hipparcos* and *Gaia* data (Snellen & Brown 2018; Kervella et al. 2019; Feng et al. 2019b; Li et al. 2021; Xiao et al. 2023). Among the many orbital fitting packages, such as OCTOFITTER (Thompson et al. 2023), GAIAPMEX (Kiefer et al. 2024), and NII-C(Jin, Jiang & Wu 2024), one widely used tool is ORVARA (Brandt et al. 2021a). This package constrains reflex motion using the Hipparcos–Gaia Catalog of Accelerations (HGCA; Brandt 2018, 2021). HGCA provides three proper motion values from the *Hipparcos* and *Gaia* reference epochs and the mean proper motion. Combined with HTOF (Brandt et al. 2021b), ORVARA can model the reflex motion's orbit using *Gaia* epoch data from

Gaia Observation Forecast Tool (GOST) and *Hipparcos* Intermediate Astrometric Data.

While Brandt, Dupuy & Bowler (2019) and Brandt (2021) transform the calibrated *Hipparcos* and *Gaia* catalogue data into proper motions, Feng et al. (2019b, hereafter F19) and Feng et al. (2022, hereafter F22) directly model the astrometric catalogue data. They incorporate five parameters for barycentric motion and seven parameters for reflex motion, astrometric jitter, and offsets to account for potential biases in the catalogue data. The F19 method was later enhanced to model both *Gaia* DR2 and DR3 data (Feng et al. 2023, hereafter F23), utilizing the GOST. RV data primarily constrain five orbital parameters, while astrometric data help determine the inclination (*I*) and longitude of the ascending node (Ω). Instead of calibrating the *Hipparcos* and *Gaia* data a priori, the F19 and F23 methods use astrometric jitters and offsets to account for potential biases in the *Hipparcos* catalogue and infer these parameters a posteriori.

Recently, Venner et al. (2024, hereafter V24) argued that the method developed by F19 and applied in wide companion detections by F22 is unreliable, citing discrepancies between F22 and other studies. This paper addresses V24's criticisms, contending that these discrepancies stem not from methodological issues but from (1) differences in data sets and conventions, (2) insufficient posterior sampling, (3) partial RV coverage, and (4) the influence of shortperiod companions. With partial RV coverage of the orbital phase of

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long-period planets, degeneracies may arise between mass, orbital period, and eccentricity. These degeneracies can be resolved by incorporating relative astrometry, extending the RV baseline, and ensuring adequate posterior sampling.

This paper is structured as follows: in Section 2, we compare the F19 method and its updated implementation with ORVARA. Next, we re-analyse the HD28185 data and compare our findings with those in V24 (Section 3). We then address discrepancies between F22 and other studies in Section 4. We also examine the case of eps Ind A b to highlight key lessons in Section 5. Finally, we present our discussion and conclusions in Section 6.

2 COMPARISON OF DIFFERENT METHODS

Upon examining the differences between ORVARA and the F19 method, we agree with V24 that the two approaches are nearly equivalent. The key distinction is that F19 uses astrometric jitter to account for systematic bias, while ORVARA employs calibrated HGCA data. We define $r_{\rm H}$ and $r_{\rm G}$ as the reference position vectors for *Hipparcos* and *Gaia*, respectively, with $\mu_{\rm H}$ and $\mu_{\rm G}$ as the corresponding proper motions, and $\mu_{\rm HG}$ as the mean proper motion, calculated as $(r_{\rm G} - r_{\rm H})/\Delta T$, where $\Delta T = t_{\rm G} - t_{\rm H}$ is the interval between *Hipparcos* and *Gaia* reference epochs. Both positions and proper motions are utilized in the F19 method.

For convenience, we define \mathbf{r}_0 and $\boldsymbol{\mu}_b$ as the barycentric position at the *Hipparcos* reference epoch and the barycentric proper motion. Additionally, we define $\boldsymbol{\mu}_X^r$ and \boldsymbol{r}_X^r as the proper motion and position at epoch X, which could be either *Gaia* (G) or *Hipparcos* (H) epochs. The position model is thus:

$$\boldsymbol{r}_{\mathrm{H}} = \boldsymbol{r}_{0} + \boldsymbol{r}_{\mathrm{H}}^{r} \,, \tag{1}$$

$$\boldsymbol{r}_{\rm G} = \boldsymbol{r}_0 + \boldsymbol{\mu}_b \Delta T + \boldsymbol{r}_{\rm G}^r \,. \tag{2}$$

The difference $\mathbf{r}_{\rm G} - \mathbf{r}_{\rm H}$, when divided by ΔT , yields:

$$\hat{\boldsymbol{\mu}}_{\mathrm{HG}} \equiv \frac{\boldsymbol{r}_{\mathrm{G}} - \boldsymbol{r}_{\mathrm{H}}}{\Delta T} = \boldsymbol{\mu}_{\mathrm{b}} + \frac{\boldsymbol{r}_{\mathrm{G}}^{\prime} - \boldsymbol{r}_{\mathrm{H}}^{\prime}}{\Delta T} \,. \tag{3}$$

By setting $\boldsymbol{\mu}_{\text{HG}} \equiv \frac{r_{\text{G}}^{r} - r_{\text{H}}^{r}}{\Delta T}$, we obtain: $\hat{\boldsymbol{\mu}}_{\text{HG}} = \boldsymbol{\mu}_{b} + \boldsymbol{\mu}_{\text{HG}}$, (4)

matching the model for observed mean proper motion $\mu_{\rm HG,o}$ as defined by Brandt et al. (2021a). Both F19 and ORVARA model *Gaia* and *Hipparcos* proper motions, $\mu_{\rm G}$ and $\mu_{\rm H}$, similarly, making them equivalent for orbit constraints, though F19 can also yield barycentric positions at reference epochs. Thus, the difference between F19 and ORVARA lies solely in the parametrization and not in the physical content.

Both methods share a limitation in that they cannot resolve the degeneracy between $\pi - I$ and I, leading to two possible inclination solutions, one for a retrograde and one for a prograde orbit. This ambiguity is illustrated in Fig. 1 by two face-on circular orbits. In ORVARA, the observed quantities $\mu_{H,o}$, $\mu_{G,o}$, and $\mu_{HG,o}$ help constrain the reflex motion components μ_H , μ_G , and μ_{HG} as well as the barycentric proper motion μ_b . For a circular orbit, where the orbital period is primarily constrained by RV data, the radius of the orbital path is fixed. Thus, for a circular and face-on orbit, $\mu_{H,o}$ and $\mu_{G,o}$ primarily determine the direction of orbital motion, while $\mu_{HG,o}$ constrains μ_{HG} . With two identical orbital circles that intersect two fixed points (i.e. D and F in Fig. 1), two orbital solutions can result, representing either rotation direction when using ORVARA or F19 methods.

To resolve the inclination or rotation degeneracy, one can utilize data from both *Gaia* DR2 and DR3, along with *Hipparcos*, to better constrain orbits. In the simplest case illustrated in Fig. 1, a circular orbit can be uniquely determined with three fixed points in 2D space. As shown in Fig. 1, an additional point is needed to distinguish between the two solutions represented by the blue and black circles. This approach is demonstrated by comparing the orbital solutions for HD 222237 b provided by the F23 and ORVARA methods (Xiao et al. 2024). An additional advantage of the F23 method over both F19 and ORVARA is its ability to model the parallax contribution to abscissae using GOST, while simultaneously fitting both barycentric motion and reflex motion models to data from *Hipparcos* and multiple *Gaia* data releases.

3 ORBITAL ANALYSIS OF HD 28185

HD 28185 is a G-type star (Gray et al. 2006) hosting at least two substellar companions, designated HD 28185 b and c. While prior studies, including V24 and F22, concur on the minimum masses and orbital parameters for the inner planet HD 28185 b (Santos et al. 2001; Minniti et al. 2009; Wittenmyer et al. 2009; Stassun, Collins & Gaudi 2017; Rosenthal et al. 2021), they differ in their interpretations of HD 28185 c. F22, using a combined analysis of RV and *Hipparcos–Gaia* astrometry, arrives at an orbital solution for HD 28185 c similar to that based on RV data alone from Rosenthal et al. (2021). V24, however, presents a contrasting solution, with significant differences highlighted in fig. 4 of V24. This discrepancy likely arises from two main factors.

First, F22 and V24 use different RV data sets. Unlike V24, F22 omits the CORALIE (Udry et al. 2000) data collected before JD2452500, leading to a baseline about 700 d shorter than V24. The CORALIE data show a negative RV trend before a turnaround around JD2452000 (see panel a of Fig. 2). Because this turning point provides a strong constraint on the orbital period in V24, both F22 and Rosenthal et al. (2021) determine a longer orbital period and higher mass for HD 28185 c without long enough RV baseline to resolve this turning point. In this case, *Hipparcos–Gaia* astrometry offers limited constraint on the period due to mass–period degeneracy in the astrometric signal.

Second, the presence of a year-long inner planet complicates the astrometric fit. As noted by V24, the inner companion induces an astrometric signal of at least 0.14 mas, comparable to the outer companion's influence. Additionally, the 9000-d orbital period derived for HD 28185 c by V24 closely aligns with the 24.75-yr *Hipparcos–Gaia* baseline, which could significantly diminish the observed astrometric signal if this solution is accurate.

V24 argue that the inner planet's orbital period, being similar to Earth's annual motion, would cause its astrometric effect to average out over the *Gaia* and *Hipparcos* observation spans. While this may hold for the differences in proper motion and position between *Hipparcos* and *Gaia*, it does not apply to parallax. The discrepancy between the *Hipparcos* parallax (23.62 ± 0.87 mas) and *Gaia* DR3 parallax (25.48 ± 0.02 mas) – a difference of about 2σ – suggests a bias possibly introduced by the inner companion. Notably, *Gaia* DR2 gives a parallax of 25.36 ± 0.04 mas, differing from the *Gaia* DR3 value by about 3σ .

We apply the F23 method, which accounts for the astrometric signal of the inner companion, to analyse data from *Gaia* DR2, DR3, and *Hipparcos* (collectively referred to as 'HG23'). We use RV data from CORALIE along with RV data from the Carnegie Planet Finder Spectrograph (PFS; Crane et al. 2010), the High-Resolution Spectrograph (HRS; Tull 1998) on the Hobby-Eberly Telescope



Figure 1. Schematic view of the two orbital solutions in proper motion space. The black and blue circles with directed arrows represent two equivalent orbital solutions. Grey and green vectors denote the proper motion vectors associated with stellar reflex motion, constrained by the observed proper motion vectors shown in orange. Point *A* marks the origin of the coordinate system, while *B* and *C* represent the barycentres for the two solutions. The vectors $AD \equiv \mu_{G,o}$ and $AF \equiv r_{H,o}$ denote the observed proper motions at the *Gaia* and *Hipparcos* reference epochs, respectively, and $AE \equiv \mu_{HG,o}$ represents the observed mean motion calculated from the positional difference between *Hipparcos* and *Gaia*. $BD \equiv \mu_G$ and $CD \equiv \mu_H$ indicate the stellar position relative to the barycentre at the *Gaia* and *Hipparcos* reference epochs, respectively from the positions of the reflex motion at the *Gaia* and *Hipparcos* epochs. Quantities with a prime symbol correspond to the alternate solution.

(Ramsey et al. 1998), the Magellan Inamori Kyocera Echelle (MIKE) spectrograph (Bernstein et al. 2003), and the High Accuracy Radial velocity Planet Searcher (HARPS; Pepe et al. 2000).

Using the F23 method, we analyse the data and present two solutions: one including the inner companion's astrometric signal (Model 1) and one excluding it (Model 2). Additionally, we include V24's solution (their Model 1) in Table 1. The corresponding fitting results for Model 1 are displayed in Figs 2 and 3, while the astrometric fitting results for Model 2 are shown in Fig. 4. As shown in Table 1, using the same RV data and excluding the inner companion's astrometric signal from the solution, Model 2 aligns with V24's results. The 1.5σ discrepancy in ω_c between Model 2 and V24 likely arises from their differing approaches to systematic bias in astrometric catalogues.

As shown in Fig. 3, the inner companion induces a significant parallax offset of approximately 0.4 mas, consistent with the observed discrepancies among *Hipparcos*, *Gaia* DR2, and DR3. This contribution from the inner companion to the total astrometric signal results in an improvement in the model's fit, with an increase in the

log-likelihood by 8.4. This corresponds to a decrease in the Bayesian Information Criterion by 12, assuming two additional parameters (I_b and Ω_b) compared to the model that excludes the inner companion's influence. The improvement in orbital fit is also evident in the comparison between Models 1 and 2 applied to the *Gaia* DR2 and DR3 data (see Figs 3 and 4).

Additionally, including the astrometric signal from the inner companion increases the uncertainties in the estimates of I_c and Ω_c (see Fig. B1). This is likely because some astrometric variation is attributed to the inner companion, reducing the signal-to-noise ratio for the outer companion. Despite these differences, Models 1 and 2 produce nearly identical solutions for the outer companion are consistent with those presented in V24. Since V24 and ORVARA follow the first convention, while we use the astrometric convention defined in Feng et al. (2019a), our value of Ω differs from theirs by 180 deg.

While the use of both DR2 and DR3 does not fully resolve the *I* and $\pi - I$ degeneracy for the outer companion with a 25-yr orbit,



Figure 2. RV + HG23 fits to HD 28185 RVs from Model 1. Panel (a) shows the best-fitting Keplerian orbit (thick black line) to the RV measurements and panel (b) shows their residuals. Panel (c) shows the phase-folded orbit of the inner planet HD 28185 b, with the signal of the outer planet HD 28185 c being subtracted. Likewise, panel (d) shows the phase-folded orbit of HD 28185 c after correcting the signal of HD 28185 b.

it provides a unique inclination solution for the inner companion, as shown in Table 1 and Fig. B1. This demonstrates the importance of using both *Gaia* DR2 (J2015.5) and DR3 (J2016.0) in uniquely determining the inclination of orbits with periods comparable to, or not much longer than, the half-year difference between the DR2 and DR3 reference epochs.

Based on the analyses above, we conclude that the discrepancy between the solutions for HD 28185 c reported by F22 and V24 is primarily due to differences in the RV data. A limited RV baseline can result in degeneracies between a companion's mass and orbital period. These degeneracies may be mitigated by extending the RV baseline or incorporating relative astrometry data, as demonstrated in Mesa et al. (2023), Philipot et al. (2023a), and El Morsy et al. (2024). This issue will be explored in greater detail in the following sections.

4 OTHER DISCREPANCIES IN F22'S ORBITAL SOLUTIONS

Aside from our detailed worked example of HD28185 in the previous section, we also discuss several other targets illuminated by V24 as showing discrepancies between F22 and other studies. We divide the causes of discrepancies into four categories, including different conventions (Section 4.1), insufficient posterior sampling (Section 4.2), inner companions (Section 4.3), and limited RV coverage (Section 4.4). We will also address the mass–period degeneracy in Section 4.5 and explore the anisotropic inclination distribution in Section 4.6. Since both ORVARA and F23 adopt the same parallel-tempering (PT) Markov Chain Monte Carlo (MCMC) sampler PTEMCEE (Vousden, Farr & Mandel 2016), we set 30 temperatures, 100 walkers, and 50 000 steps (80 000 for 14 Her) per chain to

Table 1. Parameters for HD 28185 system. Model 1 includes the astrometric contribution from the inner planet HD 28185 b, while Model 2 ignores it. Values in brackets represent alternative solutions.

Parameter	N	Iodel 1	Μ	Iodel 2			
	HD 28185 b HD 28185 c		HD 28185 b	HD 28185 c	HD 28185 b	HD 28185 c	
Fitted parameter							
Orbital period P (d)	$385.858^{+0.054}_{-0.055}$	9229^{+331}_{-222}	$385.859^{+0.053}_{-0.052}$	9236^{+319}_{-222}	$385.92\substack{+0.06 \\ -0.07}$	9090^{+460}_{-390}	
RV semi-amplitude K (m s ⁻¹)	$163.67_{-0.71}^{+0.74}$	$52.8^{+2.8}_{-2.4}$	$163.67_{-0.73}^{+0.71}$	$52.7^{+3.0}_{-2.3}$	$164.8_{-0.09}^{+0.09}$	$53.3^{+5.1}_{-4.7}$	
Eccentricity e	$0.0634\substack{+0.0030\\-0.0030}$	$0.140^{+0.024}_{-0.026}$	$0.0634^{+0.0029}_{-0.0030}$	$0.139_{-0.024}^{+0.023}$	$0.063^{+0.004}_{-0.004}$	$0.15\substack{+0.04 \\ -0.04}$	
Argument of periapsis ^{<i>a</i>} ω (deg)	$358.4_{-2.7}^{+2.8}$	$148.4_{-5.1}^{+4.5}$	$358.4_{-2.7}^{+2.7}$	$148.2^{+4.5}_{-5.3}$	$355.1^{+3.9}_{-3.9}$	162^{+8}_{-8}	
Mean anomaly at JD2451463 M_0 (deg)	$341.3_{-2.9}^{+2.8}$	$6.7^{+10}_{-4.9}$	$341.3_{-2.9}^{+2.8}$	$6.7^{+10}_{-4.8}$	-	_	
Inclination I (deg)	$156.5^{+6.1}_{-9.5}$	$73.0^{+10}_{-8.4} \ (109.5^{+9.1}_{-12})$	_	$64.9^{+7.2}_{-5.9} (111.8^{+6.8}_{-10})$	-	$66^{+11}_{-9} (114^{+9}_{-11})$	
Longitude of ascending node ^b Ω (deg)	28^{+29}_{-22}	$107^{+19}_{-22} (40^{+28}_{-27})$	_	$93^{+13}_{-14} (-5^{+17}_{-13})$	_	$271^{+15}_{-21} (178^{+18}_{-14})$	
Derived parameter							
Orbital period P (yr)	$1.05642^{+0.00015}_{-0.00015}$	$25.27^{+0.91}_{-0.61}$	$1.05643^{+0.00014}_{-0.00014}$	$25.29^{+0.87}_{-0.61}$	$1.0566\substack{+0.0002\\-0.0002}$	$24.9^{+1.3}_{-1.1}$	
Semimajor axis a (au)	$1.0282\substack{+0.0063\\-0.0064}$	$8.54_{-0.14}^{+0.21}$	$1.0285\substack{+0.0063\\-0.0064}$	$8.54_{-0.15}^{+0.20}$	$1.034\substack{+0.006\\-0.006}$	$8.50^{+0.29}_{-0.26}$	
Companion mass ^{c} m (M_{Jup})	$13.3^{+5.0}_{-3.9}$	$5.68^{+0.44}_{-0.36}$	_	$5.90^{+0.41}_{-0.38}$	-	$6.0\substack{+0.6\\-0.6}$	
Periapsis epoch $T_{\rm p} - 2450000 ({\rm JD})$	$1868.8^{+3.1}_{-3.0}$	10500^{+169}_{-162}	$1868.7^{+3.1}_{-3.0}$	10502^{+169}_{-162}	$1870.2_{-4.5}^{+4.5}$	10790^{+350}_{-280}	
Barycentric offset							
$\alpha * \text{ offset } \Delta \alpha * \text{ (mas)}$	$1.07_{-0.22}^{+0.16} \ (0.65_{-0.52}^{+0.36})$		$1.28^{+0.12}_{-0.11}$	$(-0.22^{+0.41}_{-0.30})$	-		
δ offset $\Delta\delta$ (mas)	$-0.50^{+0.48}_{-0.37}\ (0.95^{+0.23}_{-0.45})$		$-0.19^{+0.00}_{-0.00}$	$^{.35}_{.31}(1.23^{+0.13}_{-0.11})$	-		
$\mu_{\alpha*}$ offset $\Delta\mu_{\alpha*}$ (mas yr ⁻¹)	$-0.107\substack{+0.022\\-0.024}$		-0.	$103^{+0.024}_{-0.023}$		_	
μ_{δ} offset $\Delta \mu_{\delta}$ (mas yr ⁻¹)	$-0.098\substack{+0.017\\-0.017}$		$-0.094^{+0.018}_{-0.018}$		-		
ϖ offset $\Delta \varpi$ (mas)	$0.280\substack{+0.090\\-0.081}$		$0.017\substack{+0.021\\-0.021}$			-	
Instrumental parameter							
RV offset for CORALIE (m s ^{-1})	503	$06.4^{+3.4}_{-3.4}$	503	$06.3^{+3.6}_{-3.3}$	5030	$05.9^{+8.2}_{-8.2}$	
RV offset for HARPSpre $(m s^{-1})$	$71.8^{+1.7}_{-2.6}$		$71.8^{+1.6}_{-2.5}$		$75.6^{+4.0}_{-4.3}$		
RV offset for HRS $(m s^{-1})$	93	$3.7^{+2.2}_{-2.8}$	93	$3.7^{+2.2}_{-2.8}$	$95.1^{+4.3}_{-4.4}$		
RV offset for PFS (m s^{-1})		$18.6^{+1.9}_{-2.6}$	—4	$18.7^{+2.0}_{-2.6}$	$-31.5^{+4.0}_{-4.2}$		
RV offset for KECK (m s ^{-1})	89	$9.4^{+1.9}_{-2.6}$	89	$9.4^{+2.0}_{-2.6}$	-		
RV offset for MIKE $(m s^{-1})$	53	$3.1^{+1.7}_{-2.6}$	$53.1^{+1.7}_{-2.5}$		$55.3^{+4.6}_{-4.9}$		
RV jitter for CORALIE $(m s^{-1})$	9	$.7^{+2.0}_{-1.7}$	$9.6^{+2.0}_{-1.7}$		$9.0^{+2.1}_{-1.8}$		
RV jitter for HARPSpre $(m s^{-1})$	$5.7^{+1.9}_{-1.3}$		$5.7^{+2.1}_{-1.3}$		$6.0^{+2.3}_{-1.6}$		
RV jitter for HRS $(m s^{-1})$	1	$.6^{+1.6}_{-1.2}$	$1.6^{+1.6}_{-1.1}$		$1.8^{+1.7}_{-1.2}$		
RV jitter for PFS $(m s^{-1})$	$3.95^{+0.69}_{-0.56}$		$3.95^{+0.69}_{-0.57}$		$4.1^{+1.0}_{-0.7}$		
RV jitter for KECK (m s^{-1})	$1.21^{+0.37}_{-0.36}$		1.1	$21^{+0.38}_{-0.35}$	_		
RV jitter for MIKE $(m s^{-1})$	12	$2.3^{+3.3}_{-2.7}$	12	$2.3^{+3.3}_{-2.6}$	12	$.2^{+3.4}_{-2.6}$	
Jitter for <i>Hipparcos J</i> ^{hip} (mas)	$2.01^{+0.52}_{-0.54}$		2.0	$03^{+0.51}_{-0.51}$	-		
Error inflation factor S ^{gaia}	1.0	$79_{-0.054}^{+0.076}$	1.1	$50_{-0.076}^{+0.081}$		-	

Notes. ^{*a*} The argument of periastron of the stellar reflex motion, differing by π with planetary orbit, i.e. $\omega_p = \omega + \pi$.

 b The values of Ω should be increased by 180 deg for comparison with those reported by V24.

 c The stellar mass of 0.974 \pm 0.018 M_{\odot} is adopted from V24 and is assigned a Gaussian prior.

generate posterior distributions for all the fitting parameters, with the first 25 000 steps being discarded as burn-in. The priors can be found in Brandt et al. (2021a) and Feng et al. (2023), respectively. By default, the HGCA version for ORVARA is chosen to EDR3. The RV data sets used here for HD 38529, 14 Her, and GJ 229 are the same as Xuan et al. (2020), Bardalez Gagliuffi et al. (2021), and Brandt et al. (2021c), respectively.

4.1 Different conventions

HD 38529 hosts at least two companions, HD 38529 has been studied by Benedict et al. (2010), Xuan et al. (2020), and F22.

Using *Hubble Space Telescope (HST)* Fine Guidence Sensor (FGS) data, Benedict et al. (2010) derived $I_c = 48.8 \pm 4.0 \deg$ and $\Omega_c = 37.8 \pm 8.2 \deg$ for the outer companion. In contrast, Xuan et al. (2020) report $I_c = 135^{+8}_{-14} \deg$ and $\Omega_c = 217^{+15}_{-19} \deg$ based on *Hipparcos* and *Gaia* data. Notably, HGCA-based methods often yield two solutions for *I* and Ω , mirrored around $I_c = 90 \deg$ (see fig. 3 of V24). V24 suggest adjusting (I_c, Ω_c) given by Benedict et al. (2010) to align with Xuan et al. (2020) considering their use of different conventions. Without this adjustment, Ω_c determined by Benedict et al. (2010) is consistent with F22's $\Omega_c = 37.8^{+16.2}_{-14.9} \deg$, and I_c estimated by Xuan et al. (2020) differs from F22's $104.6^{+6.4}_{-8.7} \deg$ by 2σ .



Figure 3. Comparing the five-parameter astrometry of the Model 1 prediction to GDR2 and GDR3 astrometry. The barycentric motion of the HD 28185 system has been subtracted for both catalogue *Gaia* data (square) and the predictions (box plot). The inner thick line, edge of box, and whisker, respectively, denote the median, 1σ and 3σ uncertainty. The uncertainty is the product of the observed uncertainty and the error inflation factor. The subscripts of the label of the *x*-axis correspond to the *Gaia* release number.



Figure 4. Comparing the five-parameter astrometry of the Model 2 prediction to GDR2 and GDR3 astrometry. Symbols are the same as Fig. 3.

The ~ 2σ discrepancy between the inclinations given in F22 and Xuan et al. (2020) is likely due to the former using *Gaia* EDR3, while the latter used *Gaia* DR2. This is demonstrated by comparing the solutions from Xuan et al. (2020) and F22 with the ORVARA solutions derived from *Gaia* DR2 and EDR3 data in Fig. 5. Additionally, Xuan et al. (2020) determined a disc inclination of 71^{+10}_{-7} deg or equivalently 109^{+7}_{-10} deg, which agrees with F22 but differs from the planet c inclination reported by Xuan et al. (2020) by ~ 2σ . Given that the *HST*-derived inclination may be biased toward face-on solutions (Benedict et al. 2022), the inclination uncertainty given by Benedict et al. (2010) may be underestimated.

Considering that ORVARA uses the first convention defined in Feng et al. (2019a) and F22 use the third convention (or astrometric convention) defined in Feng et al. (2019a), the longitude of ascending nodes given by F22 and ORVARA differ by 180 deg. This difference in convention explains the so-called 11σ discrepancy in Ω reported by F22 and Xuan et al. (2020). Figure B2 shows the posterior

distributions from our updated solution for HD 38529, based on both *Gaia* DR2 and DR3 data.

4.2 Insufficient posterior sampling

14 Her, with two wide companions, has been studied by Bardalez Gagliuffi et al. (2021), who derived inclinations of $32.7^{+5.3}_{-3.2}$ and 101^{+31}_{-33} deg using HGCA data analysed with ORVARA. Benedict et al. (2023) found inclinations of 35.7 ± 3.2 and 82 ± 14 deg for 14 Her b and c, while F22 reported 144^{+6}_{-3} and 120^{+6}_{-29} deg. Missing the alternative inclination solution for 14 Her b from Bardalez Gagliuffi et al. (2021) (147.3 deg) creates an apparent discrepancy, yet this solution is consistent with F22's value. In Fig. 6, we compare the solutions from F22 and Bardalez Gagliuffi et al. (2021) with the ORVARA solutions rederived in this work. It is evident that both studies missed the alternative solution, and their inclination estimates are mirrored around 90°. This is further supported by



Figure 5. Marginalized posteriors of inclination *I*, longitude of ascending node Ω and mass *m* for HD 38529 c using ORVARA HGCA DR2 (grey; Brandt 2018) and EDR3 (blue; Brandt 2021), respectively. Their median and 1 σ confidence intervals are shown with the corresponding colours in each panel. The *y*-axis is scaled between 0 and 1. The results of Xuan et al. (2020) and Feng et al. (2022) are respectively denoted with the red square and the green diamond. It should be note that the value of Ω reported by Feng et al. (2022) is advanced by 180 deg to match the convention of ORVARA. It is clear that the main discrepancy between two studies stems from the use of different *Gaia* data.



Figure 6. Marginalized posteriors of inclination *I* and longitude of ascending node Ω for 14 Her b and c using ORVARA (EDR3 version). The posteriors are separately displayed based on the inclination of 14 Her b, i.e. $I_b < 90^\circ$ (blue) and $I_b > 90^\circ$ (grey). The results of Bardalez Gagliuffi et al. (2021, BG2021) and Feng et al. (2022) are also shown with the red square and the green diamond. The same adjustment for Ω of Feng et al. (2022) has been done for fair comparison. It seems that both studies only found one of the two solutions.

our reanalysis of 14 Her using both *Gaia* DR2 and DR3 data (see Fig. B3). Such insufficient posterior sampling for inclination occurs in many ORVARA-based studies and F22. For example, the solution $I \sim 45^{\circ}$ is only partially resolved by Xuan et al. (2020) using PT MCMC (see the right panel of their fig. 2) and is not presented as an alternative inclination solution. In our analyses, we employ PTEMCEE to sample the posterior and use the Gelman–Rubin statistic $\hat{R} < 1.1$ (Gelman & Rubin 1992) as a convergence criterion. However, even with $\hat{R} < 1.1$, chains may fail to sample other modes of a multimodal posterior distribution. Therefore, future studies should consider running multiple chains with diverse initializations and utilize a broader range of posterior sampling techniques to ensure thorough exploration of the posterior distribution.

4.3 Multiple companions

GJ 229 B is a well-known brown dwarf analysed by Brandt et al. (2020, 2021c) and F22. V24 noted that F22's estimated mass of $60.4^{+2.3}_{-2.4}$ M_{Jup} is lower than the 71.4 ± 0.6 M_{Jup} given by Brandt et al. (2021c), likely because F22 accounted for RV variations from two inner planets with periods of approximately 120 and 520 d (Feng et al. 2020), while other studies did not. To demonstrate this, we constrain the mass and orbit of GJ 229 B using the same data as in Brandt et al. (2021c), with the corner plot shown in Fig. B4. Our best-fitting mass of $71.73^{+0.55}_{-0.56}$ M_{Jup} aligns perfectly with the value reported in Brandt et al. (2021c). However, discrepancies in other parameters, such as eccentricity, likely arise from our treatment of

HARPS RV data before and after the fibre change as two independent data sets, whereas Brandt et al. (2021c) combine them into a single set. The mass estimation for GJ 229 B (a binary system comprising GJ 229 Ba and Bb, as reported by Xuan et al. 2024), derived without including potential inner companions, aligns with the dynamical mass inferred by Xuan et al. (2024) using the data from the Very Large Telescope (VLT) GRAVITY (interferometer in General Relativity Analysis via VLT InterferometrY) Wide mode and the CRyogenic InfraRed Echelle Spectrograph Upgrade Project (CRIRES +). This suggests that the existence of inner companions cannot be confirmed by the current astrometric and RV data.

HD 62364 hosts at least one wide companion, HD 62364 has been examined by F22 and others (Frensch et al. 2023; Xiao et al. 2023; Philipot et al. 2023a). While F22 identified two companions using HARPS data before 2019, subsequent studies with additional HARPS data found only one. By examining the one- and twocompanion solutions from F22, we identified that the two solutions used different RV data sets. The one-companion solution included 14 additional RVs compared to the two-companion solution. Due to an oversight in the data set change, F22 incorrectly calculated the logarithmic Bayes factor using the same number of data points, which led to the erroneous model selection. While such errors are rare, they can be avoided through careful management of databases and orbital solutions, particularly when analysing large samples of targets.

By re-analysing the updated RV data, we find that the onecompanion solution (Model 1) is strongly favoured over the twocompanion solution (Model 2), with a $lnBF_{21}$ of approximately -8. To compare with the results for HD 62364 b from Xiao et al. (2023), Frensch et al. (2023), and Philipot et al. (2023b), we re-analysed the updated HARPS RV data and *Hipparcos–Gaia* astrometric data. The best-fitting parameters are presented in Table A1, and the corresponding posterior distributions are shown in Fig. B5. As seen from Fig. B5, our results are consistent with previous solutions but with higher precision due to use of multiple *Gaia* DRs.

Therefore, the investigations of these discrepancies highlight the critical role of additional data, accurate model inference, and the determination of the correct number of planetary signals in deriving consistent orbital solutions.

4.4 Limited RV coverage and lack of relative astrometry

HD 211847, GJ 680, and HD 111031 host low-mass stellar companions identified through imaging Ward-Duong et al. (2015), Moutou et al. (2017), Gonzales et al. (2020), and Dalba et al. (2021). F22 initially classified them as brown dwarfs due to RV-only data, though recent studies including relative astrometry identified them as lowmass stars (Philipot et al. 2023a, b). Without relative astrometry from imaging data, Philipot et al. (2023a) acknowledge that they cannot obtain the correct solution using only RV and absolute astrometry, due to the limited RV time span. The oversight of the relative astrometry data for these three targets by F22 is likely due to the challenges of comprehensively searching the literature for such a large sample of targets, combined with their primary emphasis on analysing combined RV and astrometry data.

Using relative and absolute astrometry alongside RV data from Philipot et al. (2023a, b), we derive orbital solutions for HD 211847 B, GJ 680 B, and HD 111031 B using both the F23 and ORVARA methods (Table A2 and Figs B7–B9). For HD 211847, we added 51 HARPS RVs from the European Southern Observatory (ESO) archive to the RV data set used in Philipot et al. (2023a, b).

The solutions for HD 211847 B derived using F23 and ORVARA are consistent with each other and with the solution provided by Philipot et al. (2023a) within 2σ . For GJ 680 B, while the F23 and ORVARA solutions are consistent, the F23 solution exhibits significantly smaller uncertainty, likely due to the incorporation of *Gaia* DR2 data. Similarly, the F23 and ORVARA solutions for HD 111031 B are consistent within 1σ , as illustrated in Fig. B9. This further demonstrates that the discrepancies between F22 and other studies regarding these three targets stem from F22's oversight of relative astrometry data, particularly when RV coverage is limited, rather than from methodological differences.

4.5 Mass-period degeneracy

A key reason behind the discrepancies is that RV trends induced by long-period companions mainly constrain the host star's acceleration (g) due to the companion. For a companion of mass m on a circular orbit with semimajor axis a and inclination I, the RV acceleration is $g \propto m \sin I/a^2$. Thus, RV trends primarily constrain $m \sin I/a^2$, with astrometry from *Hipparcos–Gaia* contributing information on I and the longitude of the ascending node Ω . This limitation explains why values of m/a^2 reported by F22 align with other studies, despite different (m, a) values. However, longer RV coverage or relative astrometry is essential to constrain eccentric orbits and break the m - a (or mass–period m - P) degeneracy. Even if relative astrometry is not available from direct imaging (DI) data, the null results from such imaging can still constrain the potential solutions for the companions (Mawet et al. 2019). Our detailed comparison in Section 2 confirms that the F19 method used in F22 is equivalent to ORVARA. V24 suggested that the discrepancies in solutions are period-dependent. As discussed, in most cases, the root cause of discrepancies for long-period companions is the partial RV coverage rather than methodology. This 'evolution' of solutions for long-period companions, as detailed in Section 5, largely arises from short RV baselines.

4.6 Inclination distribution

V24 also argued that inclination discrepancies for long-period companions result from the non-isotropic inclination distribution observed by Benedict et al. (2023). However, as shown in Benedict et al. (2023), neither the *HST* exoplanet sample nor F22's sample follows the cos *I*-uniform distribution seen in the 6th Visual Binary Star Catalog (Hartkopf, Mason & Worley 2001). This is because RV data, sensitive to *m* sin *I*, favours high-inclination orbits, while astrometric signals inversely relate to inclination. Consequently, the inclination distribution for RV and astrometric detections differs from that of binaries with more prominent signals unaffected by detection limits.

5 EVOLUTION OF SOLUTIONS FOR EPS IND A B

Following initial indications of a potential wide-orbit companion in the RV data (Endl et al. 2002), continued efforts have been made to monitor and image this companion, culminating in the first successful image captured by Matthews et al. (2024) using *JWST*/MIRI. The imaging of this system was guided by combined analyses of RV and *Hipparcos–Gaia* data (Feng et al. 2019b, 2023; Philipot et al. 2023a). The solutions and data from these studies are summarized in Table 2.

In this work, we use the RV data collected by the Coudé Echelle Spectrometer Long Camera (LC) and the Very Long Camera (VLC, Zechmeister et al. 2013), the ESO UV-visual echelle spectrograph (Dekker et al. 2000), and HARPS before (HARPSpre) and after (HARPSpost) fibre change, and HARPS during covid pandemic (HARPSpost2). The data set of HARPSpost2 were released by Barbieri (2023) which extends the time baseline to ~ 29 yr. We bin the data each night to eliminate the high-frequency signal.

Solutions A and B are derived, respectively, using the F23 method without and with DI data. The DI data were collected by the VISIR (VLT Imager and Spectrometer for the mid-InfraRed)/NEAR (New Earths in the α Cen Region experiment) instrument (Pathak et al. 2021; Viswanath et al. 2021) at VLT and the MIRI mounted in *JWST*. The RV data used in this study largely aligns with that of Matthews et al. (2024). However, the perspective acceleration has been subtracted from the LC and VLC RVs in accordance with the recommendations of Janson et al. (2009). Comparisons among F23, and Solutions A and B are illustrated in Figs 7 and 8.

As shown in Table 2, the RV time span (ΔT_{rv}) is crucial for enhancing the accuracy of predictions derived from combined analyses of RV and astrometry. Solution A provides a predicted location for ϵ Ind A b consistent with observations, whereas Philipot et al. (2023a) and F23 (with a 24.8-yr RV baseline) yield shorter period solutions due to the limited RV time span (without HARPSpost2 data), which does not fully capture the RV variation's turn-over (see panel a of Fig. 7). When the RV baseline is short, the posterior sampler may also favour a shorter period solution, interpreting red noise in the RV data as orbital curvature.

The comparison of the companion's position, as predicted by F23 with a 24.8-yr RV baseline, and Solutions A and B, against the

Table 2. Parameters given in various studies for eps Ind A b and the data used by them.

$\overline{\Delta T_{rv}{}^a}$ yr	Astrometry	Imaging	Method	$m_b \ M_{ m Jup}$	P _b yr	e_b	I_b deg	Ω_b deg	Reference
24.8	HG2	_	F19	$3.25^{+0.39}_{-0.65}$	$45.20^{+5.74}_{-4.77}$	$0.26\substack{+0.07\\-0.03}$	$64.25^{+13.80}_{-6.09}$	$250.20^{+14.72}_{-14.84}$	Feng et al. (2019b)
24.8	HG3	_	ORVARA-like ^b	3.0 ± 0.1	$29.9_{-0.6}^{+0.7}$	0.48 ± 0.01	91^{+4}_{-5}	58 ± 5	Philipot et al. (2023a)
24.8	HG23	_	F23	$2.96\substack{+0.41\\-0.38}$	$42.92\substack{+6.38\\-4.09}$	0.26 ± 0.04	$84.41_{-9.94}^{+9.36}$	$243.38^{+14.36}_{-13.41}$	Feng et al. (2023)
29.2	HG3	MIRI + NEAR	ORVARA	$6.31\substack{+0.60\\-0.56}$	_	$0.40\substack{+0.15 \\ -0.18}$	103.7 ± 2.3	_	Matthews et al. (2024)
29.2	HG23	_	F23	$5.8^{+1.0}_{-1.0}$	162_{-40}^{+59}	$0.470\substack{+0.081\\-0.10}$	99^{+11}_{-10}	$234.5_{-7.3}^{+6.1}$	Solution A
29.2	HG23	MIRI + NEAR	F23	$7.29\substack{+0.60\\-0.61}$	180^{+32}_{-31}	$0.399\substack{+0.059\\-0.076}$	$105.4_{-2.4}^{+2.5}$	$228.3^{+1.7}_{-1.6}$	Solution B

Notes. ^a T_{rv} is the time span of all RV data sets.

^b The algorithm is largely based on ORVARA (Philipot et al. 2023a).



Figure 7. RV + HG23 fits to the RVs, *Hipparcos*, and *Gaia* astrometry of eps Ind A (Solution B). Panel (a) shows the RV curve, with the best-fitting Keplerian orbit indicated by the thick black line. Residuals (O–C) between the observed RVs and the model are depicted below. Panel (b) plots the best-fitting astrometric orbit of the star. The black dashed line inside the orbit connects the ascending node and the descending node. The plus symbol denotes the system's barycentre, and the grey line connects it with the periapsis. The post-fit *Hipparcos* abscissa residuals are projected into the RA and Dec. axes (grey dots) and have been binned into single points with colours. The brightness of these points gradually increases with observation time (the temporal baseline of each satellite is set to 1). The orientations of the error bars of each point denote the along-scan direction of *Hipparcos*. The arrow denotes the orientation of the orbital motion. Panel (c) shows the residual (O–C) of *Hipparcos* abscissa. Panels (c) and (d) are, respectively, the enlargement of the fitting to *Hipparcos* and *Gaia* astrometry. The former magnifies the square region of panel (b) which depicts the best fit to *Gaia* GOST data and the comparison between best fit and catalogue astrometry (positions and proper motions) at GDR2 and GDR3 reference epochs. The shaded regions represent the uncertainty of catalogue positions and proper motion and position offsets induced by the planet for *Gaia* DR2 and DR3.

true position observed by *JWST*/MIRI, is illustrated in Fig. 8. For simplicity, the solution provided by Philipot et al. (2023a) is not included in Fig. 8, though their predicted position is similar to that of F23. As demonstrated in Fig. 8 and summarized in Table 2, the discrepancy between the predicted and observed position of ϵ Ind A b is independent of the specific algorithm used (F19, F23, or ORVARA). This is particularly relevant for extremely long-period companions, where the RV data's acceleration derivative must be substantial to resolve degeneracies among *P*, *e*, and *m*. As shown in Table 2, both ORVARA and F23 give consistent orbital solutions if using RV data with a 30-yr baseline.

Compared to the solution provided by Matthews et al. (2024), and Solutions A and B incorporate all available RV data along with the HG23 data, resulting in an orbital solution with higher precision (see Table 2). Therefore, we recommend Solution B for future studies of this planet. The corresponding parameter table and corner plot are presented in Table A2 and Fig. B6 in Appendix B.

6 CONCLUSION

In this work, we demonstrate that the F19 method is similar to ORVARA, with the difference being that the former models astrometric jitter a posteriori, while the latter uses calibrated HGCA data. Compared to F19 and ORVARA, the F23 method offers the advantage of incorporating the parallax contribution to GOST-generated abscissae and utilizing multiple *Gaia* data releases. This approach better constrains the orbits of short-period companions and resolves the degeneracy in *I* and Ω .

We revisit the orbital solution of HD 28185 using the method developed in Feng et al. (2023) and applied in Xiao et al. (2024). By incorporating the astrometric contribution from the inner planet, HD 28185 b, with its year-long orbit, we derive the inclination and absolute mass of the inner companion. This approach provides a better fit to the astrometric data from *Hipparcos*, and *Gaia* DR2 and DR3 compared to V24's model that considers only the astrometric signal from the outer companion. This demonstrates the importance



Figure 8. Comparison of the position of ϵ Ind A b as predicted by F23 with a 24.8-yr RV baseline, Solution A, and Solution B, with the observed position from *JWST*/MIRI. The shade regions of panel (a) denote the predicted location of the planet (1 σ uncertainty) on 2023 July 3. Panels (b) and (c), respectively, show the best fit of our Solution B to the relative separation (ρ) and position angle (θ).

of simultaneously modelling both short- and long-period companions in multicompanion systems, as well as the necessity of accounting for parallax in *Gaia* and *Hipparcos* data analysis when constraining yearlong orbits. The discrepancy between F22 and V24 in the solution for HD 28185 c is likely due to the use of different RV data sets and insufficient posterior sampling.

We address V24's concerns regarding discrepancies between the solutions provided by F22 and other studies for HD 28185, HD 38529, 14 Her, HD 62364, GJ 229 B, HD 211847, GJ 680, and HD 111031. These discrepancies arise mainly for four reasons. First, the use of different conventions results in different values of Ω and I (e.g. HD 38529). In particular, the first and third conventions defined in Feng et al. (2019a) differs by 180 deg. Second, insufficient sampling of the double-peaked inclination posterior distribution leads to the discrepancy in the inclination given by various studies for 14 Her. This issue affects both F22 and many studies using ORVARA. Third, the multiple companions would complicate the analyses of RV and astrometric data. The solutions with and without considering potential companions may lead to discrepancies (e.g. GJ 229 B and HD 62364). Fourth, discrepancies in orbital periods are primarily due to differences in the time span of RV data sets (e.g. HD 28185 and HD 211847). The estimation of periods for wide-orbit companions is highly sensitive to the RV baseline, and this type of discrepancy is data-dependent, regardless of the method used. Underestimations of companion mass for extremely long-period companions (e.g. GJ 229 B, HD 211847 B, GJ 680 B, and HD 111031 B) occur when using only RV and astrometry without incorporating relative astrometry from imaging. This leads to a degeneracy among orbital period, companion mass, and eccentricity, and can result in misinterpreting RV jitter as reflex motion. We re-analyse these systems with imaged stellar companions by modelling their photocentric motion and achieve consistent solutions using the F23 and ORVARA methods.

Using ϵ Ind A b as an example, we further emphasize the critical role of the RV baseline in precisely constraining companion mass and orbit. We compare solutions obtained with older and newer RV data, with and without using relative astrometry from *JWST*/MIRI and the imaging of ϵ Ind A b conducted by the NEAR. We conclude that discrepancies between various solutions are primarily due to partial RV coverage of the orbital phase, rather than differences in the methods used. A sufficient RV baseline is crucial for accurately

estimating the mass and orbital parameters of companions with decades-long orbits.

Based on our investigation of the discrepancies and issues raised by V24, the following lessons are pertinent for future detections of long-period planets. First, caution is needed when reporting mass and orbital parameters for companions with decades-long orbits, especially when the RV coverage spans less than one-quarter of the full orbital period. However, if relative astrometry is available, the RV baseline can be shorter, provided the acceleration in the RV data is still significant. Second, adequate posterior sampling with multiple samplers using different initial parameters is essential to fully explore multimodal posteriors and achieve consistent solutions (e.g. Jin et al. 2024). Third, the inclination degeneracy may be resolved by using multiple Gaia data releases, which is crucial for precisely determining the mutual inclination between multiple giant companions in a system. Finally, the conventions for orbital parameters should be clearly defined, as this is essential for future comparative and statistical studies.

Despite existing discrepancies in the solutions for long-period companions, the combined use of RV and astrometry has successfully guided the DI of several giant planets, including AF Lep b (Franson et al. 2023), HIP 99770 (Currie et al. 2023), and eps Ind A b (Matthews et al. 2024). Therefore, we are optimistic about the future synergy between RV, astrometry, and DI, particularly with the expected release of *Gaia* epoch data in DR4.

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DATA AVAILABILITY

The RV and imaging data are available in the literature, whereas the *Gaia* and *Hipparcos* data are publicly accessible.

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APPENDIX A: PARAMETERS OF COMPANIONS

Parameter ^a	$GJ 229 B^b$	HD 62364 B	HD 3	38529 c	14 Her b	14 Her c	
	F23	F23	ORVARA (DR2)	ORVARA (EDR3)	(EDR3) ORVARA		
Fitted parameter							
Orbital period P (d)	98863^{+2348}_{-2245}	5163^{+21}_{-20}	$2127.8^{+3.1}_{-3.3}$	$2127.8^{+3.3}_{-3.2}$	$1763.63\substack{+0.80\\-0.84}$	51748^{+33266}_{-18353}	
RV semi-amplitude K (m s ⁻¹)	375^{+11}_{-12}	$170.3^{+2.2}_{-2.2}$	$169.8^{+1.1}_{-1.1}$	$169.8^{+1.1}_{-1.1}$	$90.32\substack{+0.41 \\ -0.42}$	$44.2^{+2.7}_{-2.5}$	
Eccentricity e	$0.8121\substack{+0.0044\\-0.0054}$	$0.6092\substack{+0.0045\\-0.0044}$	$0.3505\substack{+0.0055\\-0.0053}$	$0.3507\substack{+0.0057\\-0.0051}$	$0.3698^{+0.0032}_{-0.0034}$	$0.644^{+0.092}_{-0.11}$	
Argument of periapsis ω (deg)	$325.0^{+1.0}_{-1.0}$	$-0.26\substack{+0.76\\-0.77}$	$22.3^{+1.3}_{-1.4}$	$22.1^{+1.3}_{-1.3}$	$22.79_{-0.48}^{+0.48}$	$14.6^{+5.6}_{-5.5}$	
Mean anomaly ^{c} M_0 (deg)	$322.92\substack{+0.64\\-0.64}$	$44.8^{+1.4}_{-1.4}$	$134.1^{+1.1}_{-1.1}$	$134.2^{+1.0}_{-1.1}$	$59.91^{+0.38}_{-0.39}$	$20.6^{+12}_{-8.3}$	
Inclination I (deg)	$31.1^{+1.7}_{-1.5}$	$133.1^{+1.3}_{-1.3}$	$129.3_{-14}^{+9.4}$	$104.2^{+8.9}_{-11}$	$33.1_{-3.8}^{+6.5} (144.0_{-8.8}^{+7.3})$	$100^{+32}_{-33} (135^{+10}_{-26})$	
Longitude of ascending node Ω (deg)	$170.42\substack{+0.46\\-0.47}$	$98.4_{-3.0}^{+3.0}$	218^{+16}_{-15}	238^{+37}_{-17}	$236^{+14}_{-14}(35^{+13}_{-12})$	$311^{+32}_{-60} (26^{+301}_{-17})$	
Derived parameter							
Orbital period P (yr)	$270.7^{+6.4}_{-6.1}$	$14.135_{-0.056}^{+0.057}$	$5.8256^{+0.0085}_{-0.0090}$	$5.8256^{+0.0091}_{-0.0087}$	$4.8285^{+0.0022}_{-0.0023}$	142^{+91}_{-50}	
Semimajor axis <i>a</i> (au)	$36.10\substack{+0.47\\-0.47}$	$6.236\substack{+0.070\\-0.071}$	$3.607\substack{+0.039\\-0.039}$	$3.604_{-0.042}^{+0.043}$	$2.843\substack{+0.040\\-0.040}$	$27.0^{+11}_{-6.8}$	
Companion mass m (M_{Jup})	$71.73_{-0.56}^{+0.55}$	$17.90\substack{+0.54\\-0.53}$	$16.2^{+2.9}_{-2.3}$	$12.93\substack{+0.70\\-0.49}$	$8.9^{+1.1}_{-1.5}$	$7.3^{+2.3}_{-1.1}$	
Periapsis epoch $T_p - 2400000 \text{ (JD)}$	-38253^{+2177}_{-2278}	52405^{+23}_{-23}	$54405.2_{-6.3}^{+6.4}$	$54404.4_{-5.9}^{+6.3}$	$54904.0^{+1.9}_{-1.9}$	52224^{+159}_{-158}	
Barycentric offset							
$\alpha * \text{ offset } \Delta \alpha * \text{ (mas)}$	$-52.18\substack{+0.64\\-0.65}$	$0.117\substack{+0.031\\-0.031}$		—	—	—	
δ offset $\Delta\delta$ (mas)	$594.9^{+7.3}_{-7.3}$	$-0.282\substack{+0.029\\-0.028}$	—	—	_	—	
$\mu_{\alpha*}$ offset $\Delta\mu_{\alpha*}$ (mas yr ⁻¹)	$9.89_{-0.12}^{+0.12}$	$0.439^{+0.017}_{-0.016}$	_	—	_		
μ_{δ} offset $\Delta \mu_{\delta}$ (mas yr ⁻¹)	$-13.47\substack{+0.14\\-0.14}$	$0.673^{+0.016}_{-0.016}$	—	—	_		
ϖ offset $\Delta \varpi$ (mas)	$-0.036\substack{+0.018\\-0.018}$	$0.021\substack{+0.022\\-0.022}$	—		—	—	

Table A1. Parameters for GJ 229 B, HD 62364 B, HD 38529 c, and 14 Her system. Stellar mass: HD 62364 ($1.19 \pm 0.16 M_{\odot}$, Feng et al. 2022), HD 38529 ($1.36 \pm 0.02 M_{\odot}$, Xuan et al. 2020), and 14 Her ($0.98 \pm 0.04 M_{\odot}$, Bardalez Gagliuffi et al. 2021).

Notes.^{*a*} The fitted parameters used by ORVARA differ from ours, so we convert their values to the same form using MCMC posteriors.

^b We adopt a uniform prior for the stellar mass of GJ 229. The best-fitting value is $0.576 \pm 0.007 M_{\odot}$, comparable with the value of $0.579 \pm 0.007 M_{\odot}$ given by Brandt et al. (2021c).

^c The reference epoch for our model is set to the minimum Julian day of RVs, while for ORVARA, it is 2010.0 yr or JD = 2455197.50.

Table A2.	Parameters for eps Ind A b, HD 211847 B, HD111031 B, and GJ 680	B. Stellar mass: eps Ind A (0.76 \pm 0.04 $M_{\odot},$ Matthews et al. 2024), HD 21184
$(0.94 \pm 0.0$)4 $M_{\odot},$ Sahlmann et al. 2011, HD 111031 (1.17 \pm 0.06 $M_{\odot},$ Kervella,	Arenou & Thévenin 2022), and GJ 680 (0.47 \pm 0.01 $M_{\odot},$ Kervella et al. 2022

Parameter ^a	eps Ind A b		HD 21	HD 211847 B		HD 111031 B		GJ 680 B	
	Solution A (no DI)	Solution B (with DI)	F23	ORVARA	F23	ORVARA	F23	ORVARA	
Fitted parameter									
Orbital period P (d)	59001^{+21636}_{-14428}	65692^{+11580}_{-11189}	6201^{+16}_{-17}	6205^{+19}_{-18}	$49618\substack{+4339\\-3276}$	51160^{+4430}_{-3745}	95842^{+1139}_{-1339}	$143676^{+113121}_{-43902}$	
RV semi-amplitude K (m s ⁻¹)	$39.6^{+4.8}_{-4.8}$	$45.6^{+3.4}_{-3.2}$	275^{+19}_{-16}	$254.3^{+7.2}_{-7.6}$	543^{+24}_{-24}	550^{+35}_{-33}	986^{+13}_{-13}	896^{+125}_{-116}	
Eccentricity e	$0.470^{+0.081}_{-0.10}$	$0.399^{+0.059}_{-0.076}$	$0.5565^{+0.0095}_{-0.010}$	$0.563\substack{+0.011\\-0.011}$	$0.541\substack{+0.024\\-0.023}$	$0.531\substack{+0.033\\-0.037}$	$0.0479^{+0.0075}_{-0.0067}$	$0.42^{+0.20}_{-0.23}$	
Argument of periapsis ω (deg)	28^{+16}_{-15}	$17.8^{+12}_{-8.8}$	$165.5_{-4.2}^{+4.8}$	$171.1_{-1.9}^{+3.2}$	$82.0^{+5.1}_{-3.7}$	$83.4_{-5.6}^{+6.4}$	$185.7_{-4.7}^{+4.2}$	263^{+37}_{-43}	
Mean anomaly ^b M_0 (deg)	303^{+17}_{-22}	306^{+10}_{-12}	$263.1^{+1.2}_{-1.3}$	$31.3^{+3.3}_{-3.2}$	$58.44_{-1.0}^{+0.89}$	$65.5^{+5.6}_{-5.6}$	$161.2^{+5.1}_{-4.6}$	41^{+57}_{-25}	
Inclination I (deg)	99^{+11}_{-10}	$105.4_{-2.4}^{+2.5}$	$171.29_{-0.65}^{+0.57}$	$172.04_{-0.36}^{+0.32}$	$147.4_{-1.3}^{+1.3}$	$147.6^{+1.7}_{-1.7}$	$115.48^{+1.0}_{-0.94}$	$123.8^{+10}_{-8.9}$	
Longitude of ascending node ^{c} Ω (deg)	$234.5_{-7.3}^{+6.1}$	$228.3^{+1.7}_{-1.6}$	$-3.5^{+4.8}_{-4.2}$	$2.5^{+3.3}_{-1.8}$	$160.0^{+1.2}_{-1.2}$	$160.2^{+1.7}_{-1.6}$	$140.61\substack{+0.32 \\ -0.32}$	$139.1^{+1.6}_{-2.0}$	
Derived parameter									
Orbital period P (yr)	162^{+59}_{-40}	180^{+32}_{-31}	$16.978^{+0.045}_{-0.045}$	$16.990^{+0.051}_{-0.049}$	$135.8^{+12}_{-9.0}$	140^{+12}_{-10}		393^{+310}_{-120}	
Semimajor axis a (au)	$27.0^{+6.6}_{-4.5}$	$29.2^{+3.3}_{-3.4}$	$6.567^{+0.063}_{-0.062}$	$6.839^{+0.061}_{-0.061}$	$27.53^{+1.2}_{-0.88}$	$29.2^{+1.4}_{-1.2}$	$32.05_{-0.31}^{+0.29}$	46^{+22}_{-10}	
Companion mass $m(M_{Jup})$	$5.8^{+1.0}_{-1.0}$	$7.29^{+0.60}_{-0.61}$	$147.8^{+3.1}_{-3.1}$	$146.7^{+3.4}_{-3.3}$	$183.5^{+5.4}_{-5.2}$	$186.9^{+7.2}_{-6.7}$	$185.5^{+2.7}_{-3.2}$	$192.3_{-4.6}^{+4.4}$	
Periapsis epoch $T_{\rm p} - 2400000$ (JD)	-647^{+14940}_{-21953}	-6945^{+11272}_{-11673}	48016_{-29}^{+30}	54190^{+18}_{-16}	46133^{+241}_{-251}	45910^{+332}_{-384}	10235^{+1370}_{-1238}	38512^{+4231}_{-7891}	
Barycentric offset									
$\alpha * \text{ offset } \Delta \alpha * (\text{mas})$	$-23.5^{+6.6}_{-7.6}$	$-33.1^{+3.5}_{-3.5}$	$4.53_{-0.24}^{+0.25}$	_	$-114.7^{+4.1}_{-4.6}$	-	$677.9^{+9.1}_{-12}$	_	
δ offset $\Delta\delta$ (mas)	$-17.2^{+5.3}_{-7.5}$	$-29.0^{+2.7}_{-3.1}$	$26.79^{+0.60}_{-0.58}$	-	$77.2^{+3.6}_{-3.4}$	-	-787^{+12}_{-12}	-	
$\mu_{\alpha*}$ offset $\Delta\mu_{\alpha*}$ (mas yr ⁻¹)	$0.77^{+0.36}_{-0.39}$	$0.76_{-0.15}^{+0.16}$	$-3.539^{+0.043}_{-0.044}$	-	$-3.36^{+0.17}_{-0.21}$	-	$6.89^{+0.29}_{-0.30}$	-	
μ_{δ} offset $\Delta \mu_{\delta}$ (mas yr ⁻¹)	$0.21\substack{+0.41\\-0.55}$	$-0.457^{+0.085}_{-0.087}$	$0.421\substack{+0.029\\-0.029}$	-	$-2.12\substack{+0.17\\-0.16}$	-	$7.14_{-0.24}^{+0.27}$	-	
ϖ offset $\Delta \varpi$ (mas)	$-0.11\substack{+0.11\\-0.11}$	$-0.11\substack{+0.11\\-0.11}$	$0.014\substack{+0.034\\-0.034}$	-	$0.009^{+0.027}_{-0.025}$	-	$0.015\substack{+0.026\\-0.026}$	-	

Notes.^a The fitted parameters used by ORVARA differ from ours, so we convert their values to the same form using MCMC posteriors.

^b The reference epoch for our model is set to the minimum Julian day of RVs, while for ORVARA, it is 2010.0 yr or JD = 2455197.50.

 c We shift a π from Ω of ORVARA to comfort to our adopted coordinate system convention (see Convention III of appendix A of Feng et al. 2019a).





Figure B1. Posterior distributions of the selected orbital parameters for HD 28185 system. The median is denoted by a vertical dashed line.



Figure B2. Posterior distributions of the selected orbital parameters for HD 38529 system.



Figure B3. Posterior distributions of the selected orbital parameters for 14 Her system.



Figure B4. Posterior distributions of the selected orbital parameters for GJ 229 system.



Figure B5. Posterior distributions of the selected orbital parameters for HD 62364 system.



Figure B6. Posterior distributions of the selected orbital parameters for eps Ind A system.



Figure B7. Posterior distributions of the selected orbital parameters for HD 211847 system.



Figure B8. Posterior distributions of the selected orbital parameters for GJ 680 system.





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