EXPLORING GEOMETRICAL TRANSFORMATION PROCEDURES THROUGH PHYSICAL MODELS AND HOLOGRAPHY

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The combination of digital and traditional methodologies in visualisation is an integral aspect of architectural and design practice. This integration is particularly evident in the generation of forms and shapes, from initial conception to realisation, facilitating the interplay of diverse fields. These hybrid methods involve volumetric transformations grounded in geometric principles, defining tools for generating and analysing shapes by incorporating 3d modelling and advanced manufacturing techniques.

This paper critically reflects on the relationship between geometry and form development by exploring the geometrical transformations of regular polyhedra. Using operative design as a tool for spatial formation, a catalogue of shapes is generated to examine how their combination can inform potential design processes. The physical and geometric qualities of the objects produced are analysed using 3D-printed models, complemented by traditional holographic representations. In particular, optical holography is employed as a visualisation medium, offering dynamic and immersive perspectives and revealing the complexity of geometric forms, engaging the viewer in a multidimensional experience.

This research highlights the fundamental role of geometry in architecture and design, encouraging the development of a structured framework of geometric principles and operations. This framework is designed to support iterative and dynamic design processes, enhancing the understanding of spatial relationships and serving as a heuristic tool for addressing the challenges of spatial complexity

Keywords: : Geometric Principles, Form Generation, Hybrid Visualisation, Regular Polyhedra, Holography.

Introduction

From hands-on design work to academic research, the way in which space is conceptualised by manipulating and transforming shapes and surfaces has produced many original ideas. These ideas have been extensively applied across various fields, fostering dynamic collaborations among designers, architects, scientists, and artists and resulting in multiple explorations, with geometry as a crucial tool in modelling and fabricating artistic and design pieces. These works transform geometric principles into tangible, real-world objects, continually pushing applied design boundaries and contributing to developing an effective relationship between the disciplines of representation and those of design. Building on this perspective, this paper examines how a predefined set of geometrical transformations of solids and their combinations can be used to develop a systematic design procedure. By employing the five regular polyhedra as case studies, this investigation aims to establish a set of three-dimensional forms through reproducible guidelines, beginning with fundamental shapes and progressing towards the design of sculptural objects.

Three theoretical approaches to spatial definition emerged in recent decades and inspired our paper. The first line is centred on the interaction between mathematics and the visual sciences, as explored by artists, scholars, and designers who have created three-dimensional design pieces, defining how shapes can be described through numerical relationships. An example of this methodology is the work of Bathsheba Grossman[1], who creates mathematical sculptures using computer-aided design and 3D modelling (Fig. 1).

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[1] Grossman, *Pure Math.*

1 | *K. Curtis, Bathsheba Grossman Sculptures, image from: https://www.flickr.com/photos/cassidy/506419373.*

2 | *A. Kudless, Spore Lamp, Matsys, image from: https://www.matsys.design/spore-lamps#0*

Grossman's works, produced in various materials, including steel and bronze, push beyond conventional fabrication methods. George Hart[2] reinterprets classical three-dimensional forms and their potential transformations by employing a similar process. His artworks utilise materials such as paper, wood, plastic, and even creations of everyday household objects. Harriet and David Brisson[3] have instead explored the relationship between art and science through the concept of hypergraphics – a term describing n-dimensional geometries examined beyond traditional image-making techniques. Their research in geometry includes mathematical models, stereographic projections, and physical sculptures, expanding the visual representation of higher-dimensional forms.

The relationship between geometry and human motion is, instead, the key element that Daniela Berthold explores through Platonic solids, particularly the icosahedron. Inspired by Rudolf Laban's theories, Berthold's research organises movement within geometric frameworks, influencing posture, alignment, and spatial awareness. Three-dimensional modelling and fabrication techniques facilitate the creation of "movement infrastructures" that integrate motion sequences with geometric constructions.

A diverse methodology relies on computational and algorithmic design[4], and it is exemplified by the work of scholars such as Marjan Colletti and Andrew Kudless (Fig. 2). Colletti[5] explores the relationship between geometry and digital design, emphasising hybrid methodologies, which merge post-digitality and studies on materiality. Kudless[6], on the other hand, investigates geometry as a product of the relationship between form and growth. His research employs digital fabrication, generative modelling, and coding to explore structural form-finding and craftsmanship.

Michael Hansmeyer[7] furtherly expands this discourse using algorithms to generate and fabricate architectural forms. Particularly relevant are the works investigating how subdivision and recursive transformations can produce unique, nature-inspired forms and continuously evolving architectural configurations.

Finally, a third approach is employed by Anthony Di Mari and Nora Yoo, who propose a systematic spatial creation method in their books *Operative Design*[8] and *Conditional Design*[9], describing transformation techniques applied to basic geometric solids. Their work presents an extensive catalogue of "spatial verbs", such as extrude, nest, carve, interlock, and fold, illustrating diverse methods for manipulating volumes, encouraging form generation as an evolving and interactive process rather than a predetermined or static outcome. Similarly, Joseph Choma presents a related perspective in *Études for Architects*[10], in which

[2] Hart, *Geometric Sculpture.*
[3] Brisson, *Visualization in Art and Science,* p. 257.
[4] Caetano, Santos, Leitão, *Computational design in architecture: Defining parametric, generative, and algorithmic design*, pp. 287-300.
[5] Cfr. Colletti, *Digital poetics: An open theory of design-research in architecture.*
[6] Cfr. Kudless, *Drawing Codes.*
[7] Hansmeyer, *Computational Architecture: Platonic Solids.*
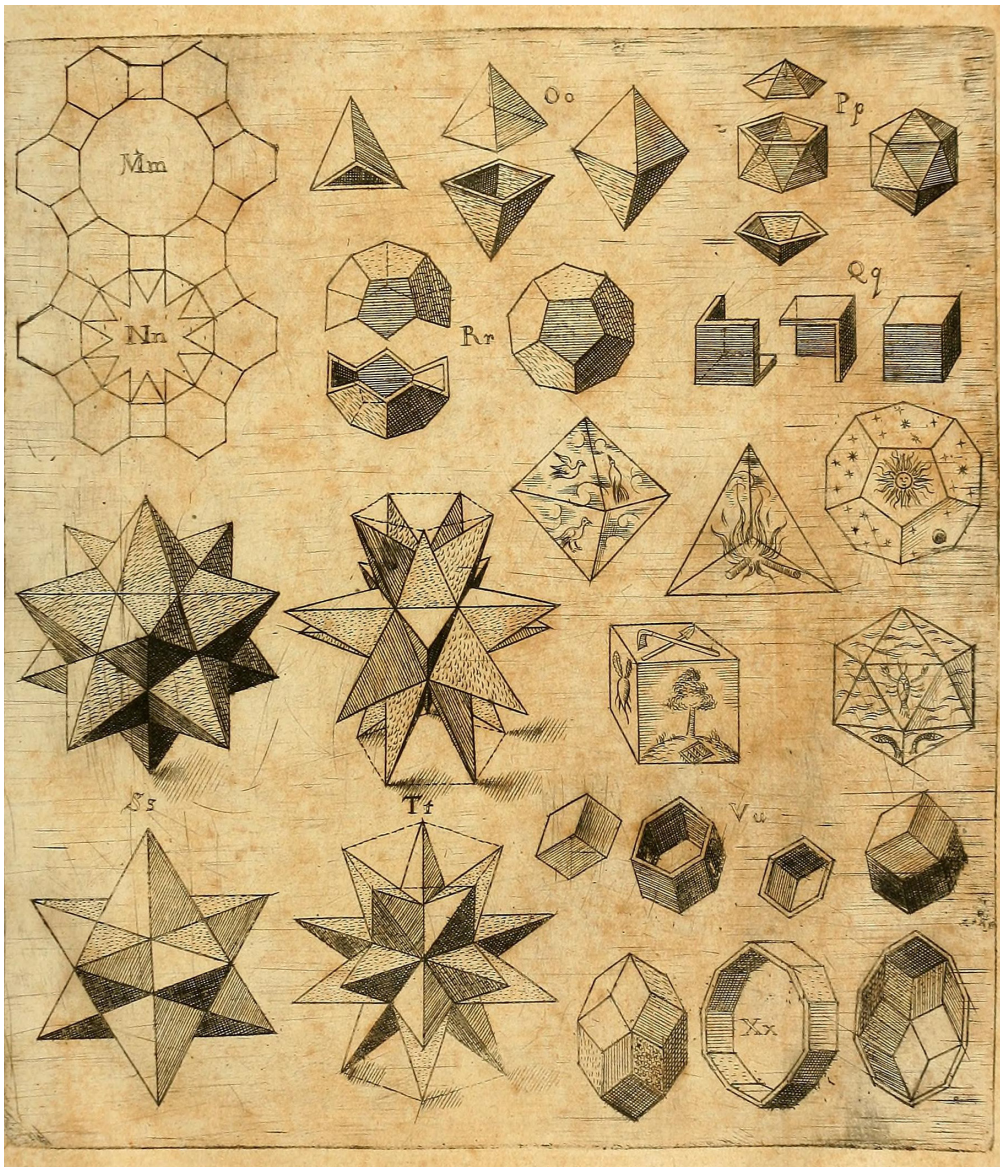[8] Cfr. Di Mari, Yoo, *Operative design.*
[9] Cfr. Di Mari, Yoo, *Conditional design.*
[10] Cfr. Choma, *Études for Architects.*

fundamental geometric shapes are the primary basis for pedagogical exercises. These exercises promote computational thinking and introduce design as an iterative, reflective, and exploratory process, enabling architects to engage critically with form, space, and structure through systematic experimentation.

Regardless of their differing methods, these experiences contribute to the broader geometric exploration proposed in this study, which integrates operative design and surface transformation, outlining a process that begins with regular convex polyhedra and generates a series of sculptural objects. These objects are fabricated using 3D printing and later reproduced through holography to establish a methodological framework that facilitates studying interactions between geometry and design while evaluating various representational techniques.

Regular Polyhedra
From Plato to Parametric Design

For millennia, people have been fascinated by geometric shapes and solid forms, particularly those with balanced proportions and symmetry. Throughout history, scientists, artists, and architects have drawn inspiration from these structures, with Regular Polyhedra holding a special place among them. The Pythagoreans were among the first to investigate the properties of these shapes, which later became known as Platonic solids[11].

Plato provided the first systematic description of these figures in his philosophical dialogue *Timaeus*, where he explored their mathematical and cosmic significance. Regular Convex polyhedra are defined as convex three-dimensional shapes composed entirely of congruent regular polygonal faces, with the same number of faces meeting at each vertex (Table 1). There are five Platonic solids, each distinguished by the type of polygonal face that composes them. The tetrahedron, octahedron, and icosahedron are all constructed from equilateral triangular faces, the cube is formed from square faces, and the dodecahedron consists of pentagonal faces (Table 1).

Plato attributed each of the four classical elements – earth, air, water, and fire –to a corresponding Platonic solid, establishing a symbolic relationship between geometry and the physical world. The cube was linked to earth for its stability, the octahedron to air because of its light-

ness, the icosahedron to water for its fluidity, and the tetrahedron to fire for its sharp, energetic structure. The fifth and final solid, the dodecahedron, was said to represent the cosmos itself[12].

Before the ancient Greeks, several earlier civilisations exhibited an understanding of these geometric forms through artefacts and symbolic representations. Archaeological findings suggest that Neolithic societies possessed knowledge of Regular Polyhedra. One of the most remarkable discoveries is a collection of intricately carved stone polyhedral objects unearthed in Scotland, dating back to approximately 2000 BC[13]. These artefacts, typically produced from stone or clay, exhibit symmetrical characteristics resembling Platonic solids, reflecting an early recognition of geometric principles and a refined appreciation for mathematical regularity.

3 | *J. Kepler, Harminices Mundi, Page with illustrations, image from: https://en.wikipedia.org/wiki/Harmonice_Mundi#/media/File:Ioanniskepplerih00kepl_0081.jpg.*

| Regular Polyedra	Faces (F)	Vertices (V)	Edges (E)
Tetrahedron	4	4	6
Cube	6	8	12
Octahedron	8	6	12
Dodecahedron	12	20	30
Icosahedron	20	12	30

Table 1 | *Number of faces (F), vertices (V), and edges (E) associated with Regular Polyhedra.*

11 Cfr. Calter, *The Platonic Solids.*
12 Emmer, *Art and Mathematics: The Platonic Solids*, p. 277.
13 Marshall, *Carved stone balls*, pp. 4-72.

4 | *M. C. Escher, Gravitation, lithograph and watercolour, image from: https://en.wikipedia.org/wiki/Gravitation_%28M._C._Escher%29#/media/File:Gravitation.jpg*

5 | *M. Brückner, Max Brückner's Collection of Polyhedral Models (1900), image from: https://publicdomainreview.org/collection/max-bruckner-s-collection-of- polyhedral-models-1900/*
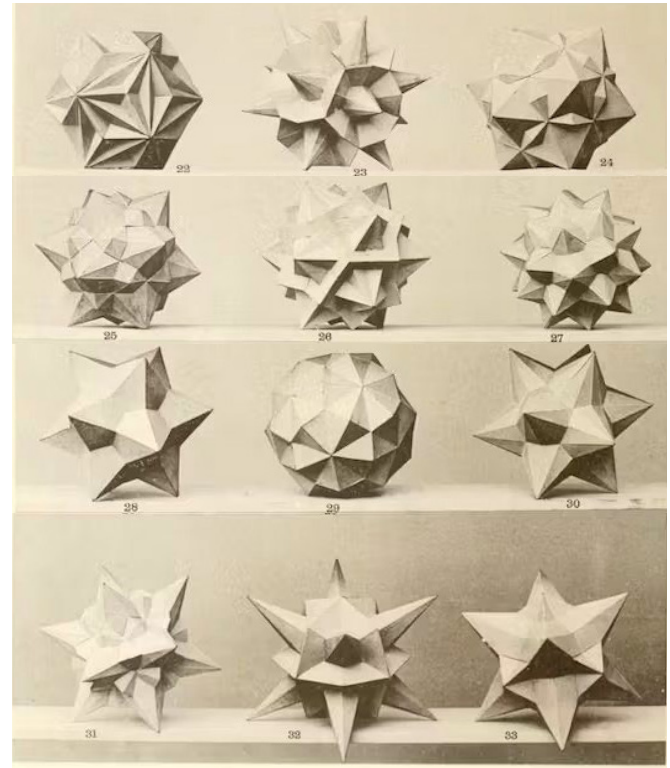
The interplay between geometric forms and scientific inquiry drove many graphical and visual explorations published between the 16th and 17th centuries. Renaissance scholars deeply studied perspective and its applications to regular solids. One of the most influential works on the subject was *De Divina Proportione* (1509)[14], authored by the Italian mathematician Luca Pacioli. This treatise discusses the mathematical principles underlying artistic beauty and features 60 plates illustrating Platonic solids and other unique polyhedra Leonardo da Vinci drew. His meticulous sketches are among the earliest known depictions of skeletal solids, making it possible to distinguish their front and back structures clearly.

Another significant contribution to the geometric representation of Polyhedra was *Perspectiva Corporum Regularium* (Perspective of Regular Solids), published in 1568 by the German goldsmith and printmaker Wenzel Jamnitzer. This remarkable study presents 120 variations of forms inspired by Platonic solids, exploring how they could be transformed through rotation, truncation, and combination. Jamnitzer's engravings illustrate an impressive array of three- dimensional objects that demonstrate the potential of these solids as fundamental building blocks of artistic and scientific endeavours. His work, exemplified in the Geometric Study, was influential in shaping future studies on geometric transformation. Johannes Kepler's book *Harmonices Mundi* (The Harmony of the World), published in 1619, explored mathematical relationships in nature[15] (Fig. 3), particularly those involving geometry, music, and astronomy. A significant part of this work is Kepler's attempt to relate the Platonic solids to the structure of the cosmos, building on ideas initially proposed by Plato. The volume features geometric illustrations, including truncated solids tiling, stellation of Polyhedra and drawings of the regular polyhedra and their transformations.

These early studies in geometric transformation laid the foundation for architectural and design principles that continue influencig contemporary practitioners. Architects and designers have long been attracted by the potential of combining geometric primitives to create modular and scalable systems.

A compelling example is the work of Zvi Hecker[16], who developed what is referred to as "Polyhedric Architecture"– a design philosophy incorporating polyhedral geometry as a core structural element. One of his most famous projects, the Ramot Housing Complex, employs interlocking cubes and dodecahedra to form a dynamic apartment unit arrangement that seamlessly integrates with the natural rocky landscape. A similar study in geometry is evident in the Synagogue in the Negev Desert, a building consist-

14 Giusti, Maccagni, *Luca Pacioli e la matematica del Rinascimento.*
15 Emmer, *Art and Mathematics: The Platonic Solids*, p. 280.
16 Hecker, T*he Cube and the Dodecahedron in My Polyhedric Architecture*, p. 272.
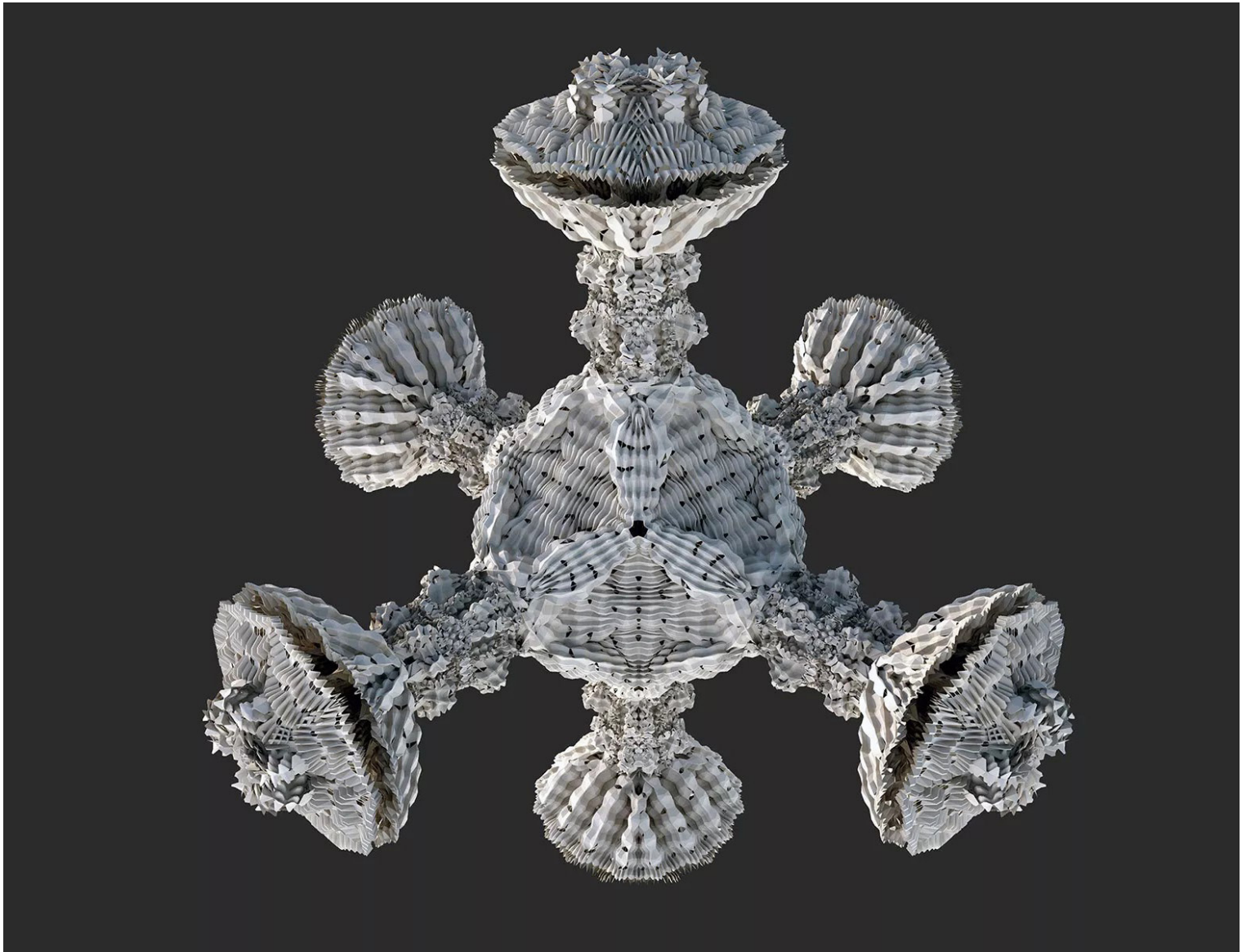
6 | *Hansmeyer, M., Computational Architecture: Platonic Solids, image from: https://www.michael-hansmeyer.com/platonic-solids.*

ing of a combination of truncated octahedra, truncated tetrahedra and Cuboctahedra.

In the realm of visual arts, numerous renowned artists have drawn inspiration from regular polyhedra. The Dutch graphic artist Maurits Cornelis Escher deeply explored mathematical structures, incorporating them into his intricate and thought-provoking illustrations. His lithograph *Gravitation* depicts a nonconvex regular polyhedron (Fig.4), specifically the small stellated dodecahedron[17].

Many of his works suggest a familiarity with Leonardo da Vinci's illustrations from *De Divina Proportione*, further reinforcing the historical continuity of interest in these geometric wonders. Similar studies in geometry were conducted by Max Brückner, who documented his research on stellated and uniform polyhedra in his 1900 book *Vielecke und Vielflache: Theorie und Geschichte* (Poly-

gons and Polyhedra: Theory and History)[18] (Fig. 5).

In The Sacrament of the Last Supper (1955), Salvador Dalí portrays a room structured as a hollow regular dodecahedron[19]. Similarly, Gerard Caris centred his artistic career on the regular dodecahedron and the pentagon, formulating a new art movement termed Pentagonism. With modern computational tools, designers can now use complex algorithms and scripts to generate shapes based on mathematical rules and parameters rather than purely aesthetic intentions. A key example of this digital approach is The Platonic Solids Project, developed by Michael Hansmeyer in 2008[20] (Fig. 6). It explores how iterative geometric processes generate intricate forms from basic primitives. Rather than combining solids, Hansmeyer's method recursively subdivides a polyhedron's faces to create increasingly complex structures, showcasing algorithmic design's poten-

17 Cfr. Bool, et al. *M.C. Escher: His Life and Complete Graphic Work*.
18 Brückner, *Vielecke und Vielflache, Theorie und Geschichte.*
19 *The Sacrament of the Last Supper.*
20 Hansmeyer, *Computational Architecture: Platonic Solids.*

7,8 | *Different types of extrusions and stellations (Image by B. Polimeni).*

9,10 | *Subdivision operators, Handles, rind modelling, fractal operators (Image by B. Polimeni).*

tial in architecture, sculpture, and product design. Other researchers have expanded on these ideas using topological mesh modelling to create high- genus shapes produced through advanced 3D printing. These innovations have enabled new artistic and functional applications, from intricate bronze sculptures to bespoke jewellery and architectural components.
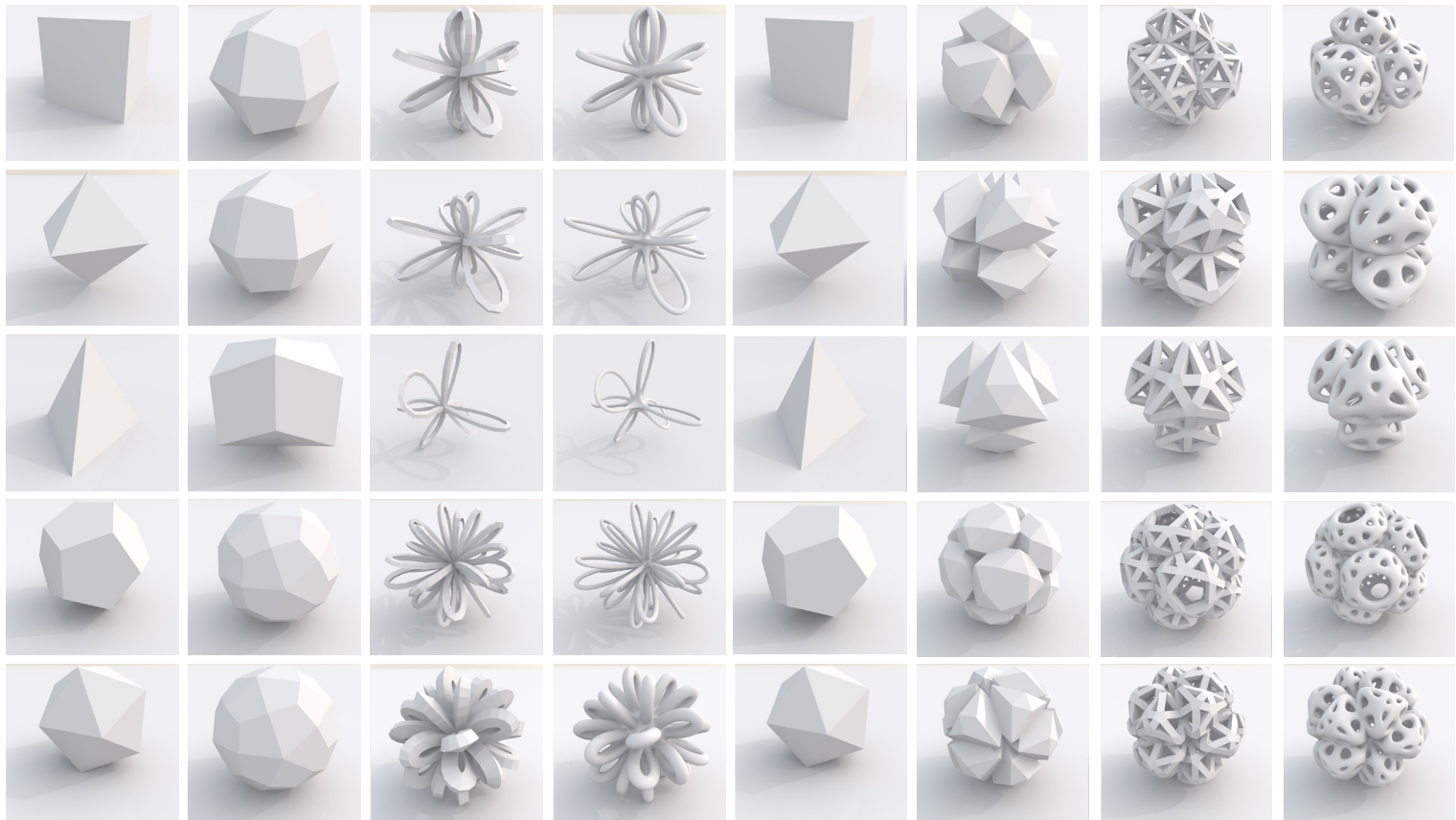
Creating a design procedure

Building upon prior experiences and advancing the discourse on the relationship between geometry and design, this study analyses a catalogue of geometric operations conceived as a spatial exploration and interpretation tool. The outcome of this process comprises a series of models analysed through digital representation, 3D printing, and holographic techniques. Diverse representational methods serve a dual purpose:

they facilitate understanding geometries and structures in their evolution while demonstrating how different media can generate distinct meanings and evolving interpretations from the same initial form[21].

The study is conducted in two distinct phases. In the first one, each regular polyhedron serves as the basis for constructing a catalogue of spatial transformations. This process employs extrusions, remeshing schemes, and high-genus modelling operators to examine the possible geometric configurations of the objects. Specifically, this phase employs seven types of extrusion of varying complexity, (figs. 7, 8) four remeshing techniques, and three topological transformations, including rind modelling and handle addition (figs. 9, 10).

After creating a catalogue of operators applied individually to each regular polyhedron, a selection of these latter is implemented in a second phase to provide

21 Cfr. Polimeni. *Producing design objects from regular polyhedra: a practical approach.*

practical design guidance for creating sculptural objects suitable for 3D printing. In this stage, two distinct design processes, each comprising four steps, are developed to produce two sets of objects perceived as part of a cohesive family.

The initial design process starts with the five regular convex polyhedra, applying the following operations in sequence: first, a remeshing process increases the number of faces in the original solids; next, handles are created to connect corresponding faces using multi-segment curved structures symmetrically; finally, the mesh is refined through the repeated application of the Catmull-Clark subdivision algorithm[22].

The second process also begins with the five regular polyhedra, following a different sequence of operations: first, icosahedral extrusion is used to increase the geometric complexity of the base forms; next, wireframe modelling converts the mesh into a three-dimensional structure by replacing each edge with a cylindrical pipe of specified thickness and volume; finally, remeshing is performed to improve surface smoothness through the iterative application of the Catmull- Clark subdivision scheme (figs. 11, 12).

The two design sequences are not rigid but serve as a flexible framework to encourage creativity within a defined set of operations. Each step can be repeated, reordered, or modified, allowing for a dynamic and iterative design process. Six graphic renderings were produced to illustrate the spatial qualities of each shape. The first set of images highlights the possible operations within the procedure, while the second set documents the transformation process based on the design guidelines. Additionally, all generated forms have been 3D printed using selective laser sintering (SLS 3D printing serves as a means of transforming physical structure for the holographic process, adding considerable value to digital models by converting them into tangible objects that can be directly interacted with). This approach enables an analysis of geometric transformations throughout the process, explores the hybridisation of physical and virtual realities, and provides alternative methods for studying architectural forms (figs. 13, 14).

11,12 | *First and second design procedures (Image by B. Polimeni).*

[22] Akleman, Ozener, Yuksel, *On a Family of Symmetric, Connected and High Genus Sculptures.*

> *"These exercises promote computational thinking and introduce design as an iterative, reflective, and exploratory process, enabling architects to engage critically with form, space, and structure through systematic experimentation.*

13 | *3D-printed object generated by the geometric transformation of the dodecahedron (Image by B. Polimeni).*

14 | *3D-printed object generated by the geometrical transformation of the icosahedron (Image by B. Polimeni).*

[23] Blanche, *Holography, and the future of 3D display.*

[24] Odoulov et al., *Interference and holography with femtosecond laser pulses of different colours.*

[25] Johnston, *Holograms: the story of a word and its cultural uses*, p. 494.

[26] Neipp, Pascual, Belendez, *Silver halide sensitized gelatin derived from BB-640 holographic emulsion.*, pp. 1348-1356.

[27] Bjelkhagen, *Denisyuk holography: From Lippmann photography to color holography.*

Optical Holography

To explore new ways of representing the object, optical holography is used as a performative medium, capturing the three-dimensional geometric qualities of the forms. This approach requires active viewer participation, encouraging them to interact with the holograms and explore the objects from multiple angles[22]. Optical holography, a branch of radiation physics, enables the storage of visual information through interference fringes created by a coherently projected laser beam. These fringes generate the three-dimensional nature of the hologram when it is replayed, making holography a perceptual phenomenon shaped by how the image is reconstructed and observed[24].

The term holography originates from Ancient Greek, where *holos* means "whole" and *graphé* means "writing", often interpreted as "total writing"[25].

However, in this context, it refers to a technique that records and reconstructs light waves to create the impression of spatial depth. Unlike conventional photography, which captures only light intensity, holography records both intensity and phase, preserving depth and spatial relationships. Holographic objects are recorded onto a holographic plate, similar to traditional photographic film but with a significantly higher information capacity due to the increased thickness of the silver halide gelatine layer[26] (fig. 15). The recorded image is reconstructed when the holographic plate is illuminated (fig. 16) by a laser beam or a standard light source at specific angles and the technique used. Two fundamental principles underpin holography, particularly in its analogue form: interference and diffraction[27]. Interference occurs when two light waves interact, either reinforcing each other (constructive interference) or cancelling each other out (destructive interference).
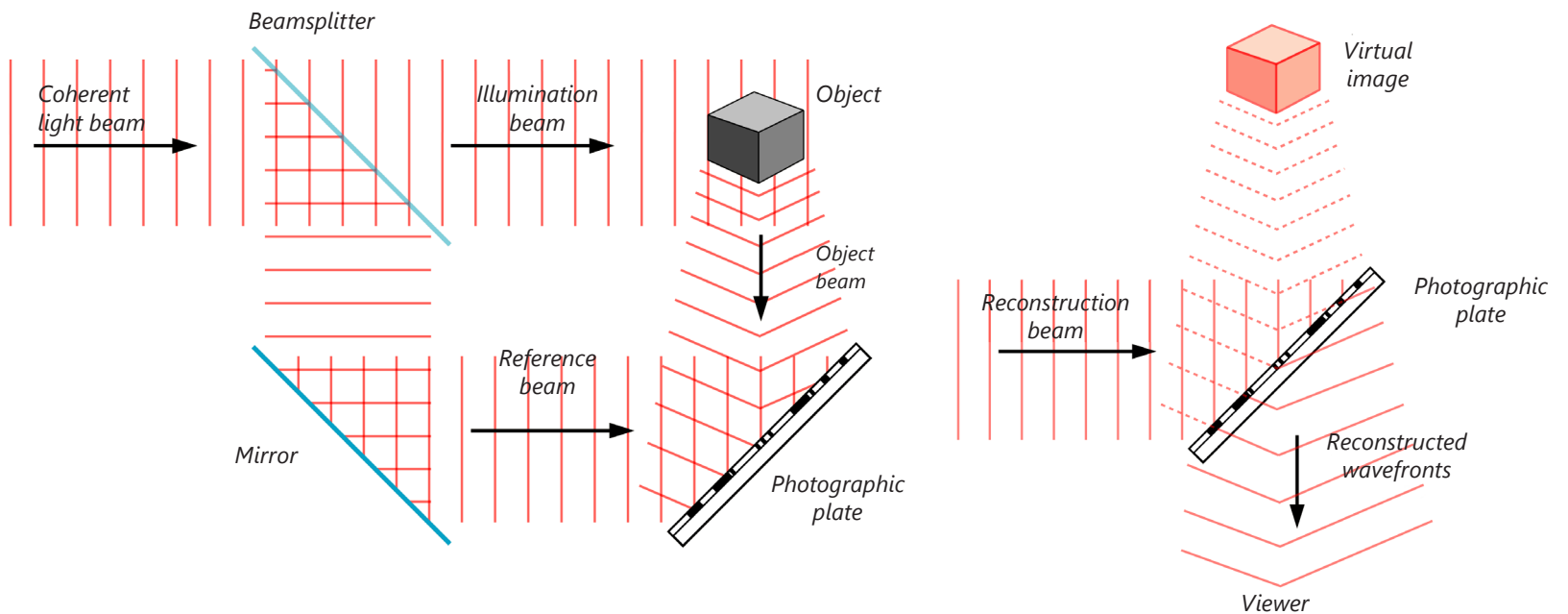
Diffraction, conversely, explains how light spreads upon encountering surfaces and subsequently reconstructs the holographic image in space.

The history of holography is complex, with numerous attempts to refine techniques for object representation. Among these, the Denisyuk hologram remains the most fundamental configuration. This study employed its modern form to explore the performative possibilities of holography in representing three-dimensional objects.

The Holographic Process

The holograms created for this research were produced in the Holography Laboratory at De Montfort University. Specifically, the holographic images presented in this study were generated from objects 3D-printed in nylon using Selective Laser Sintering (SLS) technology. This material was chosen for two key reasons: the object to be holographically recorded must be sufficiently rigid, and its surfaces must optimally reflect the laser. A laser beam incident on the printed objects created a reflection hologram. A 660 nm red cobalt laser was employed for the experiment, with an exposure time of three seconds per image. The holograms were recorded on 8" x 10" glass holographic plates coated with high-resolution, red- sensitive silver halide emulsions.

These plates were pre-treated with a water-soluble oil known as Triethanolamine (TEA) to adjust the spacing of the fringes in the silver halide structure. This treatment enhances colour uniformity and improves the brightness of the holograms. Without this phase, the holograms would appear opaque due to insufficient fringe spacing for effective red light transmission[28]. Although the holographic plates are red-sensitive, the fringes produced diffracted light at different angles, generating colour effects demonstrating how holograms act as a diffraction grating[29]. Different colours can be produced by adjusting the concentration of the TEA; this phenomenon is known as pseudo-colour holography and was widely used for making full-colour holograms before the invention of panchromatic plates[30]. When red light passes through a fringe with specific spacing, its angular deviation determines the colour the observer perceives. The holographic setup employed follows the techniques developed by Denisyuk. Creating a hologram using this method involves directing an object beam from a laser source onto the object. The light reflected from the object's irregular surface returns to the recording medium, which overlaps with the original light wave. At the plate, the direct and reflected waves interact, forming standing waves – a sequence of points exposed to the film while intermediate points (nodes) remain unexposed[31].

15,16 | *On the right, the holographic scheme shows the object and reference beams. On the left, the holographic reconstruction, where a beam of light incident on the holography plate generates the object. Images from https://commons.wikimedia.org/wiki/File:Holograph-record.svg and https://commons.wikimedia.org/wiki/File:Holography-reconstruct.svg*

[28] Nishida, *Correction of the Shrinkage of a Photographic Emulsion with Triethanolamine*. pp. 238-240.

[29] Cfr. Richardson, Wiltshire. *The hologram: principles and techniques.*

[30] Cfr. Saxby, *Practical holography.*

[31] Cfr. Richardson, Wiltshire. T*he hologram: principles and techniques,* cit.

17,18,19 | *Hologramrealised by Oliver Peacock based on a model of B. Polimeni.*

The exposed layers of the plate are recorded in the photosensitive emulsion as slightly transparent layers.

This process creates a grid of three-dimensional interference fringes that, through optical encoding, preserve all the spatial information of the object. During the reconstruction phase, where the holographically recorded object becomes visible, the light striking the hologram is reflected by the slightly transparent layers. This light must match the characteristics of the original waves reflected by the object, as rays from different layers enhance brightness only when in phase. Consequently, during image reconstruction, the hologram selectively displays different wavelengths (Figs. 16-18). A set of full-colour holographic plate was also used to represent one of the objects in the series, employing a white-light laser combining three different light beams to achieve a more accurate chromatic fidelity (Fig. 19).

The holographic technique adopted was the same, using a Denisyuk set-up, however, for these, the Cobolt 660 nm red, the Cobolt 532 nm green, and the Cobolt 457 nm blue lasers were all used in combination and aligned to create a white laser beam using dichroic mirrors before being passed through a spatial filter. This creates a grid of three-dimensional interference fringes as previously described; in the case of full colour, each of these grids is multiplexed with the three (different colour) wavelengths, which create the full-colour effect when observed[32].

Results and conclusions

This study underscores the integral relationship between geometry, design, and visualisation techniques in architectural and spatial exploration. Examining the transformation of regular polyhedra through systematic geometric op-

[32] Cfr. Saxby, *Practical Holography,* cit.

20 | *Hologram realised by Vivian Sureshkumar based on a model of B. Polimeni.*

erations demonstrates how digital and physical modelling can be effectively combined to generate and understand new sculptural forms. The iterative design process, facilitated by computational tools, allows for a structured yet flexible approach, enabling designers to explore and manipulate complex volumetric configurations with precision and creativity.

The experimental use of optical holography introduces an innovative dimension to the representation of three-dimensional objects, offering immersive and interactive perspectives that traditional visualisation techniques cannot achieve. When the light beam irradiates the holographed object, the observer can perceive its three-dimensional characteristics by altering their viewpoint. This dynamic process establishes a systematic re-elaboration of the object and its physical qualities. It suggests how optical holography can serve as a com-

plementary technique that enhances and supports other representational approaches and highlights the potential of hybrid representation methodologies in design practice and architectural pedagogy. Finally, reinforcing the importance of geometry as a foundational element in design, and bridging theoretical principles with practical applications, this research contributes to the ongoing discourse on the role of computational and material technologies in architecture. It encourages further investigation into the possibilities of digital fabrication, holography, and algorithmic modelling, paving the way for novel design methodologies that embrace the complexity and dynamism of contemporary spatial practice. Integrating these techniques presents new opportunities for creative expression and functional innovation, reaffirming the significance of geometry in shaping the built environment.

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Acknowledgments