



Exploring the Future of Soft X-Ray Polarimetry: The Capabilities of the REDSoX Instrument for X-Ray Dim Isolated Neutron Star and Magnetar Studies

Ruth M. E. Kelly^{1,2} , Herman L. Marshall² , Silvia Zane¹, Nabil Brice³, Swati Ravi² , Roberto Turolla^{1,4} , Roberto Taverna⁴ , and Denis González-Caniulef⁵

¹ Mullard Space Science Laboratory, University College London, Holmbury St Mary, Dorking, Surrey RH5 6NT, UK; ruth.kelly.22@ucl.ac.uk

² MIT Kavli Institute for Astrophysics and Space Research, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, MA 02139, USA

³ Centre for Astrophysics Research, University of Hertfordshire, College Lane, Hatfield AL10 9AB, UK

⁴ Università di Padova, Dipartimento di Fisica e Astronomia, Via Marzolo 8, I-35131 Padova, Italy

⁵ Institut de Recherche en Astrophysique et Planétologie, 9 Avenue du Colonel Roche, BP 44346 F-31028, Toulouse CEDEX 4, France

Received 2025 March 14; revised 2025 May 15; accepted 2025 May 26; published 2025 July 2

Abstract

X-ray polarimetry offers a unique window into neutron star physics and can provide answers to questions that cannot otherwise be probed. The upcoming Rocket Experiment Demonstration of a Soft X-ray Polarimeter (REDSoX) sounding rocket mission will be the first experiment equipped with a detector able to explore polarized X-rays below 1 keV, observing in the 0.2–0.4 keV range. Although REDSoX will only be capable of short, one-off observations, it will crucially test the instrument performance. In this paper we investigate how a fully fledged orbital mission with a longer lifetime, based on an instrument design similar to REDSoX, will allow us to study thermal emission from X-ray dim isolated neutron stars (XDINSs) and magnetars, probing their magnetic fields and the physics of their outer surface layers, including vacuum effects and quantum electrodynamics (QED) mode conversion at the vacuum resonance. We discuss the potentially observable features for promising values of the star's surface temperature, magnetic field, and viewing geometry. Assuming emission from the whole surface, we find that, for a source with a magnetic field $B = 5 \times 10^{15}$ G and surface temperature $T \approx 10^7$ K, the instrument can resolve a proton cyclotron absorption feature in the spectrum with high significance when collecting $\approx 25,000$ counts across a single observation. Similarly, for a source with $B = 10^{14}$ G and $T \approx 10^7$ K, a switch in the dominant polarization mode, caused by mode conversion at the vacuum resonance, can be detected by collecting $\approx 25,000$ counts, allowing for a long-sought observational test of the presence of QED effects. We then present two case studies for XDINS targets: RX J1856.5–3754 and RX J0720.4–3125.

Unified Astronomy Thesaurus concepts: Polarimetry (1278); Polarimeters (1277); Stellar atmospheres (1584); Magnetic fields (994); X-ray astronomy (1810); Neutron stars (1108)

1. Introduction

Polarimetry is a relatively young field in X-ray astronomy but is crucial to our understanding of celestial sources. Although unquestionably a powerful and useful observational technique, historically X-ray polarimetry has been underutilized in astronomy, mainly due to the lack of technology. The situation has changed very recently: the NASA-ASI Imaging X-ray Polarimetry Explorer (IXPE) space observatory was launched in 2021 December (M. C. Weisskopf et al. 2022) and it was followed in early 2024 by the Indian Space Research Organisation's X-ray Polarimeter Satellite (XPoSat; ISRO 2024). IXPE is a mission dedicated to systematically exploring polarized X-rays from the sky in the 2–8 keV energy range, and has observed over 150 targets to date. It has been the first space mission of this kind and is capable of performing spatial-, energy-, and time-resolved polarimetry. IXPE flew over 40 yr after the first pioneering attempts made in 1971, 1974, and 1975 with the launches of Aerobee-350 (R. Novick et al. 1972), Ariel-5 (J. F. Smith & G. M. Courtier 1976), and the Eighth Orbiting Solar Observatory (M. C. Weisskopf et al. 1976). Following the ongoing success of IXPE, and the scientific contributions of the PoGO+ (M. Chauvin et al. 2017) and PolarLight

(H. Feng et al. 2019) instruments, there has been an increase in interest in spaceborne polarimetric instruments. In 2024 January, XPoSat was launched with the Polarimeter Instrument in X-rays on board, capable of observing X-ray polarization in the 8–30 keV range (ISRO 2024). The Chinese Academy of Sciences is expected to launch the enhanced X-ray Timing and Polarimetry mission by 2030, with an onboard polarimeter operating in the same energy band as IXPE, 2–8 keV, but with much higher sensitivity (S. Zhang et al. 2019).

However, to fully understand the physical origin of the X-ray polarization from astrophysical sources, observations covering a broader range of energies are necessary. In particular, the soft X-ray range, below that covered by IXPE, still remains unexplored, despite potentially being key to understand fascinating phenomena ranging from vacuum birefringence (W. Heisenberg & H. Euler 1936) to the magnetic field dependence of blazar jets (R. Blandford et al. 2019).

A first attempt in this direction will be made with the Rocket Experiment Demonstration of a Soft X-ray Polarimeter (REDSoX), which is scheduled for launch in 2027 and will become the first instrument to observe below 1 keV (H. L. Marshall et al. 2018). The rocket experiment will take polarimetric measurements between 0.2 and 0.4 keV for ~ 5 minutes, with the main goal of testing the capabilities and performances of the instrument. The success of this experiment will open the door to the development of the Globe Orbiting Soft X-ray Polarimeter (GOSoX; H. L. Marshall et al.



Original content from this work may be used under the terms of the [Creative Commons Attribution 4.0 licence](https://creativecommons.org/licenses/by/4.0/). Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.

2021), a fully functioning orbital mission equipped with a similar instrument design.

The potential of an instrument capable of observing polarization in the soft X-ray range is particularly high for X-ray sources such as X-ray dim isolated neutron stars (XDINSs) and magnetars. With inferred dipolar magnetic fields in excess of $\sim 10^{13}$ G, they are ideal candidates for studying the radiation from the neutron star surface, potentially testing quantum electrodynamics (QED) effects such as vacuum birefringence (W. Heisenberg & H. Euler 1936).

Magnetars are believed to host complex magnetic fields which are the strongest among all astrophysical objects (see R. Turolla et al. 2015, for a review). They are relatively young isolated neutron stars and have recently been observed to produce highly polarized X-ray emission. To date, IXPE has observed five magnetars and has discovered some fascinating polarization properties. Not only did the highest detected polarization come from one magnetar source ($\approx 80\%$; S. Zane et al. 2023), a clear change in polarization angle (PA) of 90° , showing two different dominant polarization modes across the energy range, was also observed in another source (R. Taverna et al. 2022). The observations across all five sources show indications of both atmospheric and condensate components at the surface, as well as reprocessing of emission by resonant Compton scattering (R. Taverna et al. 2022; R. Turolla et al. 2023; S. Zane et al. 2023; J. Heyl et al. 2024; M. Rigoselli et al. 2025; R. Stewart et al. 2025). In one source, evidence of a surface magnetic loop was also discovered (D. Pizzocaro et al. 2019; J. Heyl et al. 2024). Models of particle bombardment (S. Zane et al. 2023; R. M. E. Kelly et al. 2024a; see also D. González-Caniulef et al. 2019) and partial mode conversion (D. Lai 2023) have also been suggested to explain some of the observed polarization characteristics, although the latter scenario has been challenged by R. M. E. Kelly et al. (2024b).

XDINSs, also known as the “Magnificent Seven,” are a class of seven neutron stars, which are among the closest to Earth. These neutron stars emit soft thermal radiation ($kT_{\text{BB}} \sim 40\text{--}100$ eV), with little, or no, evidence of a high-energy tail,⁶ and are therefore ideal candidates for soft X-ray polarimetry (M. H. Van Kerkwijk & D. L. Kaplan 2007; R. Turolla 2009). The surface temperature distribution is expected to be nonuniform, pointing to complex magnetic field structures (D. Viganò et al. 2014; S. B. Popov et al. 2017; D. De Grandis et al. 2021). Absorption features have been found in six of the seven XDINS sources. Their origin is not yet clear, but they are probably due to atomic transitions or cyclotron absorption. The emission from RX J1856.5–3754, the brightest of the XDINSs and the only one without an observed absorption feature (N. Sartore et al. 2012), was found to be polarized in the optical band (with a polarization degree (PD) of $16.43\% \pm 5.26\%$), which could be an indication that vacuum birefringence is at work around the star (R. P. Mignani et al. 2017). Although intriguing, it has been noticed that this measurement is significant at the $\sim 3.1\sigma$ level, and further dedicated observations of neutron star surfaces in the X-ray band hold the key to potentially providing a definite test.

The aim of this work is to explore the potential and capabilities of the REDSoX polarimeter, in view of its potential application within a space mission. The paper is laid out as follows. In Section 2 we summarize the polarization

signatures expected from atmospheric emission of magnetars and XDINSs in the energy range below 1 keV. In Section 3, we discuss the REDSoX experiment and present some predictions for potential observations of these sources that can be carried out by using the polarimeter in the context of an orbital space observatory. As case studies, we then investigate two XDINSs, RX J1856.5–3754 (RX J1856 hereafter) and RX J0720.4–3125 (RX J0720 hereafter), exploring different parameter combinations, and present our findings in Sections 4.1 and 4.2, respectively. Finally, a discussion and conclusions follow in Section 5.

2. Surface Emission from Magnetized Neutron Stars below 1 keV

Thermal emission from a strongly magnetized isolated neutron star is expected to be due to photons coming directly from its condensed surface or reprocessed by an atmosphere covering the surface (D. González Caniulef et al. 2016; R. Taverna et al. 2020). As both of these scenarios produce blackbody-like spectra, it can be difficult to disentangle one from the other without an additional observable, such as X-ray polarization.

In the presence of a strong magnetic field, radiation is expected to propagate in two orthogonal linear polarization modes, with the electric vector either parallel or perpendicular to the plane of the local magnetic field and the direction of propagation (Y. N. Gnedin & G. G. Pavlov 1974). These normal modes, called ordinary (O) and extraordinary (X), are expected to have very different opacities, with the X mode cross sections significantly reduced compared to those of the O mode photons below the electron cyclotron energy.

The physical state of the outermost layers of a highly magnetized neutron star is still poorly known, and this uncertainty impacts the polarization properties of the thermal radiation we observe. For a long time, it was assumed that neutron stars were covered by a geometrically thin plasma atmosphere, which is expected to emit highly polarized radiation (up to 80% in the X mode), as a result of the reduced X mode opacity. However, a superstrong magnetic field can dramatically affect the properties of matter. Atoms in such fields can elongate along the field lines, forming molecular chains, and this may result in a phase transition, leaving a condensate on top of a highly magnetized neutron star crust: in place of being covered by a gaseous atmosphere, the neutron star’s “condensed surface” would remain exposed (W. Brinkmann 1980; D. Lai & E. E. Salpeter 1997). Condensed surfaces can form in neutron stars when the temperature is sufficiently low and the magnetic field is strong enough, depending on the chemical composition (see, e.g., Figure 1 in R. Taverna et al. 2020). The emission properties of a condensed surface depend crucially on the assumed form of the matter (plus vacuum) dielectric tensor. Since this is still not fully understood, two limiting cases are usually considered, in which the lattice ions do not move in response to an incident electromagnetic wave or are free to respond as if the lattice is not present; the real optical properties of the medium should be intermediate between the two limits, known as the “fixed” and “free” ion models, respectively (M. Van Adelsberg et al. 2005).

The emission from a condensed surface is expected to have a modest polarization ($\lesssim 20\%$ in the 0.2–0.4 keV), and can be dominated by photons in the X or O mode (D. González Caniulef et al. 2016; R. Taverna et al. 2020). Then, it follows

⁶ A high-energy excess has been recently reported in RX J1856.5–3754 (D. De Grandis et al. 2022).

that the polarization measurement from an isolated neutron star can inform us about the physical state of the external layers of the source, indicating if the emission has a condensate or atmospheric origin.

In this and in the following section, we focus on models based on atmospheric emission. We will then add the results corresponding to the scenario of emission from a condensed surface when we deal with the two case studies (see Section 4). Our spectral atmospheric models are obtained by readapting the numerical code by D. A. Lloyd (2003). The radiation transport for the two normal modes is solved in a plane-parallel, geometrically thin slab of a fully ionized hydrogen atmosphere, along with the energy balance equation, assuming hydrostatic, radiative, and local thermal equilibrium conditions. The radiative transfer equations are solved on optical depth, frequency, and photon direction meshes, and each atmospheric slab is characterized by four parameters: the magnetic field strength and inclination with respect to the slab normal, the effective temperature, and the surface gravity (see D. A. Lloyd 2003; see also R. M. E. Kelly et al. 2024a, 2024b for details). In this section, we present some examples computed by assuming a star mass $M = 1.4 M_\odot$, a radius $R = 12$ km, and an effective temperature of $T_{\text{eff}} = 10^7$ K.

2.1. Emission from a Single Surface Patch

Under our assumption of a fully ionized H atmosphere, one prominent feature expected in the spectra of highly magnetized neutron stars is the absorption feature at the proton cyclotron resonance energy, $E_{c,p} \simeq 0.63 y_G (B/10^{14} \text{ G})$ keV, where $y_G = \sqrt{1 - (2GM/c^2R)} \approx \sqrt{1 - 2.98(M/R)}$ is the gravitational redshift factor for a neutron star with mass M and radius R (S. Zane et al. 2001). For magnetic field strengths of the order of 10^{13} – 10^{14} G, in the range of the inferred dipole field strengths of XDINSs and some magnetars, we then expect such proton cyclotron lines to appear in the spectrum at energies below 1 keV.

Figure 1 shows the degree of polarization as a function of energy, at the star surface, but redshifted to be the corresponding energy as measured by a distant observer, for some values of the magnetic field strength in this range and for the case of a pure-plasma atmosphere. Here $\text{PD} = (J_\nu^X - J_\nu^O) / (J_\nu^X + J_\nu^O)$, where J_ν^X (J_ν^O) is the mean monochromatic intensity of X (O) mode photons, therefore a positive (negative) degree of polarization indicates emission dominated by X (O) photons. The energy on the x -axis has been redshifted (by the gravitational redshift factor y_G) to allow a more immediate comparison of the results with the 0.2–0.4 keV energy range of the REDSoX instrument. The proton cyclotron line is present in the polarization spectrum, and it falls in the energy range of the detector for magnetic field strengths $3 \times 10^{13} \text{ G} \lesssim B \lesssim 10^{14} \text{ G}$. Clearly, the detection of such a polarization dip from a source would confirm the nature of the absorption feature in the spectrum and allow us to constrain the magnetic field strength.

However, QED effects on radiation transfer can induce different energy dependencies and features in the polarization signal, which can be observed in the REDSoX band. The presence of magnetic fields with strength around (or in excess of) the quantum critical field ($B_Q \simeq 4.4 \times 10^{13} \text{ G}$) affects the optical properties of the vacuum around the neutron star, which behaves like a dichroic and birefringent medium (S. L. Adler 1971). When this vacuum contribution, which is

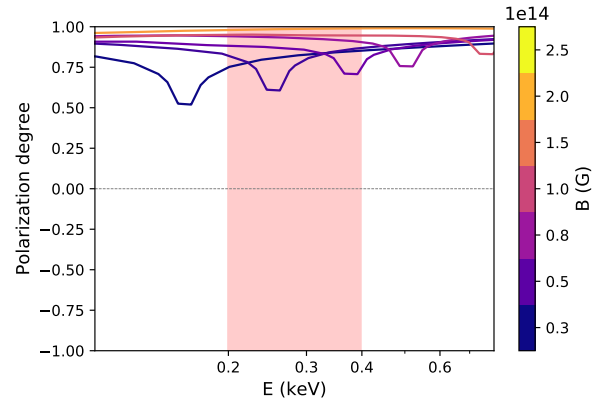


Figure 1. PD as a function of energy for a pure-plasma atmosphere with an orthogonal magnetic field for different values of the field strength in the range 3×10^{13} – 2.5×10^{14} G. Positive (negative) PDs correspond to an X (O) mode dominated emission. The shaded region highlights the 0.2–0.4 keV range in which REDSoX and GOSoX are expected to observe. The horizontal axis shows the energy as measured at infinity, i.e., redshifted by the factor y_G with respect to the photon energy at the neutron star surface. Vacuum effects in the atmosphere are neglected.

a genuine QED effect, is added to the plasma dielectric and magnetic permeability tensors, it can result in the presence of “vacuum resonances” within the neutron star atmosphere.

At the vacuum resonance, the eigenvalues of the wave equation become degenerate, and mode conversion can occur (W. C. G. Ho & D. Lai 2003; W. C. G. Ho et al. 2003; D. Lai & W. C. G. Ho 2003). Mode conversion at the vacuum resonance can dramatically affect the polarization properties of a highly magnetized atmosphere, and the detection of such properties would be an indication of the presence of vacuum birefringence. Since the physics of this effect is still poorly understood, some limiting/approximated cases are often assumed in the literature: no mode conversion, complete mode conversion, and partial mode conversion (see R. M. E. Kelly et al. 2024b, and references therein).

In the limiting case of complete mode conversion, where every photon changes the polarization mode, a change in the dominant polarization mode, from O (X) to X (O), is expected to occur, inducing a 90° swing in the PA at an energy that increases with decreasing magnetic field strength (R. M. E. Kelly et al. 2024b).

In order to illustrate this, in Figure 2 we show the PD as a function of energy for an atmospheric plasma plus vacuum model computed by assuming complete mode conversion. We find that for magnetic field strengths $\sim 10^{14}$ G, the 90° PA change (signaled in Figure 2 by the switch from a negative to a positive value of the degree of polarization) occurs within the observational energy range of REDSoX. We also find that, for larger magnetic field strengths, the effect of complete mode conversion would produce a rapid increase in the degree of polarization with energy in the REDSoX band, from a few percent up to $\sim 45\%$, while for lower magnetic field strengths the PD remains relatively constant around $\sim 30\%$, and the emission at the source is dominated by the O mode.

We then investigated the effect of partial adiabatic mode conversion at the vacuum resonance, following the formulation of D. Lai & W. C. G. Ho (2003) and W. C. G. Ho & D. Lai (2003). Specifically, we assumed a conversion

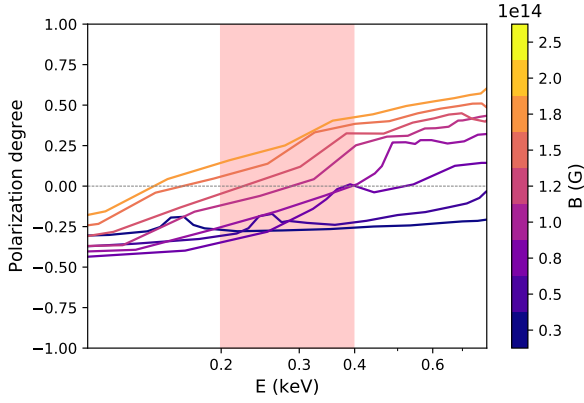


Figure 2. Same as in Figure 1 for atmospheric models computed accounting for plasma and vacuum contributions and by assuming complete mode conversion at the vacuum resonance.

probability given by

$$P_{\text{con}} = 1 - \exp \left[-\frac{\pi}{2} \left(\frac{E_s}{E_{\text{ad}}} \right)^3 \right], \quad (1)$$

where E_s is the photon energy at the neutron star surface, and E_{ad} is the limiting energy for adiabatic mode conversion, which depends on the density of the medium and on the magnetic field strength. We followed the procedure detailed in R. M. E. Kelly et al. (2024b, to which we refer for details on the numerical calculation), which triggers mode conversion when a photon has a probability larger than a fixed threshold $P_{\text{con}}^{\text{th}}$. An example of our results is shown in Figure 3, for $P_{\text{con}}^{\text{th}} = 0.1$. As can be seen, models computed by assuming partial mode conversion show, in the range below 1 keV, a PD which is systematically reduced with respect to that expected in the case of a pure-plasma atmosphere (see Figure 1) and, for instance, can be as low as $\sim 30\%$ – 40% for $B \sim 3 \times 10^{13}$ G. Interestingly, in this case radiation remains X mode dominated across the entire REDSoX energy range because, for this value of the magnetic field strength, the probability criterion is not met for these low-energy photons.

3. REDSoX

REDSoX is a NASA-funded sounding rocket mission that conducts X-ray spectropolarimetry for targets such as blazars. The instrument, if operational for a longer duration of time, also has the capability to perform spectropolarimetry for isolated neutron stars, which we explore here. REDSoX is designed to complement instruments such as IXPE and expand current X-ray polarimetry capabilities below 1 keV. It is scheduled for launch in late 2027 to observe the blazar Mrk 421. The design utilizes a focusing Wolter I optic, critical angle transmission (CAT) gratings, and laterally graded multilayer (LGML) mirrors at Brewster’s angle to perform spectropolarimetry in the energy range of 0.2–0.4 keV (see H. L. Marshall et al. 2018, 2023; A. Garner et al. 2024).

The instrument operates by passing X-rays first through full-shell grazing-incidence Wolter I focusing optics and then through one of three polarimetry channels consisting of a grating array, LGML mirror, and detector. Focused X-rays pass through high-efficiency dispersive CAT gratings, with zeroth-order light collected by a central imaging CCD

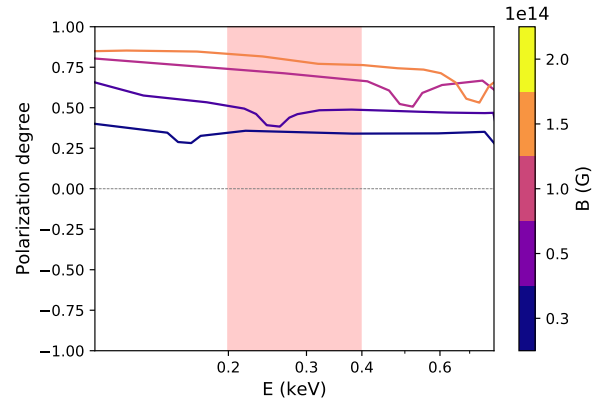


Figure 3. Same as in Figure 1 for atmospheric models computed accounting for plasma and vacuum contributions and assuming partial mode conversion (by using the method as in R. M. E. Kelly et al. 2024b, with a probability threshold $P_{\text{con}}^{\text{th}} = 0.1$; see text for details).

detector, used for target acquisition (S. N. T. Heine et al. 2024). The first-order dispersed light passes through an LGML mirror placed at Brewster’s angle (45°) to select for s -polarized light (polarized perpendicular to the plane of incidence) and is then collected by first-order polarimetry CCD detectors (H. L. Marshall et al. 2018; S. N. T. Heine et al. 2024).

3.1. Instrument Detection Capability

In this section we turn to the calculation of the signal emitted by the whole neutron star surface. We explore potential polarization measurements of surface radiation from neutron stars that can be achieved using an instrument such as that on board REDSoX in an orbital space mission.

We assumed a core-centered dipole magnetic field and divided the north hemisphere of the neutron star surface into six annular patches in magnetic colatitude, centered at $\theta = (0^\circ, 10^\circ, 30^\circ, 50^\circ, 70^\circ, 89^\circ)$. We adopted a surface temperature profile $T_s = \max(T_{\text{dip}}, T_c)$, where $T_{\text{dip}} = T_p |\cos \theta_B|^{1/2}$ represents the relation between the temperature and the local magnetic field inclination (G. Greenstein & G. J. Hartke 1983), and T_c is a minimum temperature set to avoid unrealistically low values near the equator; in the following we take $T_p = 10^7$ K and $T_c = 7 \times 10^6$ K (see R. M. E. Kelly et al. 2024b, for more details). Atmospheric models have been produced for each patch, accounting for the inclination of the magnetic field in the radiative transfer calculation (see D. A. Lloyd 2003, for details on the code) and using the assumptions discussed in Section 2. The emission from the south hemisphere is then computed using the same atmospheric models and symmetry arguments.

The observed phase-dependent, monochromatic Stokes parameters depend on the source viewing geometry through the angles χ and ξ the spin axis makes with the line of sight and the magnetic axis, respectively. Once the viewing geometry is fixed, the simulated specific Stokes parameters $\mathcal{I}_\lambda$, $\mathcal{Q}_\lambda$, and $\mathcal{U}_\lambda$ are computed by summing the contributions of the part of the surface in view at each rotational phase, accounting for general relativistic effects (see D. Page & A. Sarmiento 1996). To calculate the emission at a distant observer, we use the ray tracing code described in S. Zane & R. Turolla (2006), R. Taverna et al. (2015), and D. González

Caniulef et al. (2016; see also R. M. E. Kelly et al. 2024a, 2024b).

In this section, we present results for $\chi = 80^\circ$ and $\xi = 0.05$, which is a viewing geometry chosen among the most promising configurations, i.e., those for which the observed polarization from the source is the largest (D. González Caniulef et al. 2016).

The effect of interstellar absorption is taken into account according to the `tbabs` model,⁷ assuming the elemental abundances from J. Wilms et al. (2000) and the cross sections from D. A. Verner et al. (1996). We used a fiducial hydrogen column density $N_{\text{H}} = 1.5 \times 10^{20} \text{ cm}^{-2}$, typical of isolated neutron stars.

For the calculation of the polarization measured by the detector, the Stokes parameters $\mathcal{I}_\lambda$, $\mathcal{Q}_\lambda$, and $\mathcal{U}_\lambda$, obtained by the code, are convolved with the absorption and then rescaled with the normalization factor N , which is obtained from

$$N \int_{\lambda_0}^{\lambda_1} \mathcal{I}_\lambda d\lambda = 1 \text{ counts cm}^{-2} \text{ s}^{-1}, \quad (2)$$

where λ_0 and λ_1 are the wavelengths (in units of Å) corresponding to the energy bandpass (0.4, 0.2) keV; the rescaled quantities are denoted by I_λ , Q_λ , and U_λ . For an observation with total counts C_{obs} throughout its duration, I_λ , Q_λ , and U_λ can be rescaled again (at every wavelength) by the factor C_{obs} to obtain the specific quantities.

The expected generalized count rate $R_{\lambda,i}$ per wavelength bin for each detector ($i = 1, 2, 3$) is computed by convolving the normalized Stokes parameter values with the REDSoX instrument response such that

$$R_{\lambda,i} = A_\lambda I_\lambda + A_\lambda \mu_\lambda [Q_\lambda \cos(2\theta_i) + U_\lambda \sin(2\theta_i)]. \quad (3)$$

Here, μ_λ is a modulation factor and θ_i is the orientation of the detector i with respect to the Q and U reference frames, i.e., $-Q$ is aligned with $\theta = 0^\circ$ (the three detectors are at 120° from each other; H. L. Marshall et al. 2018). In the following, we use $\theta_1 = 0^\circ$, $\theta_2 = 120^\circ$, and $\theta_3 = 240^\circ$. The integrated area of the instrument, given by $\int A_\lambda d\lambda = 185 \text{ cm}^2 \text{ Å}$, was taken from the REDSoX design paper (H. L. Marshall et al. 2018), where the effective area A_λ was taken from their Figure 15.

The measured Stokes parameters per wavelength bin can then be obtained from the detector count rates $R_{\lambda,i}$ by

$$\begin{aligned} I_{\lambda,\text{det}} &= (R_{\lambda,1} + R_{\lambda,2} + R_{\lambda,3}) / (3A_\lambda), \\ Q_{\lambda,\text{det}} &= (2R_{\lambda,1} - R_{\lambda,2} - R_{\lambda,3}) / (3\mu A_\lambda), \\ U_{\lambda,\text{det}} &= (R_{\lambda,2} - R_{\lambda,3}) / (3^{1/2} \mu A_\lambda), \end{aligned} \quad (4)$$

from which follow $\text{PD} = \sqrt{Q_{\text{det}}^2 + U_{\text{det}}^2} / I_{\text{det}}$ and $\text{PA} = \arctan(U_{\text{det}} / Q_{\text{det}}) / 2$.

The instrument has the spectral resolution (and line response function (LRF)) of an objective grating spectrometer. The LRF of a grating spectrometer is typically described as a Gaussian with an FWHM of $\delta\lambda = P\delta x/z$, where P is the grating period, δx is the FWHM of the instrument along the dispersion, and z is the distance of the grating from the focal plane; for REDSoX $P = 2000 \text{ Å}$ (H. L. Marshall et al. 2018) and $\delta x = 0.12 \text{ mm}$ (H. Marshall et al. 2024). REDSoX has an unusual placement of gratings but most are placed at distances within 10% of the on-axis case: $z = P/(\sqrt{2}G)$, where $G = 0.88 \text{ Å mm}^{-1}$ is the

gradient of the multilayer spacing (H. Marshall et al. 2024), so z is $1600 \text{ mm} (\pm 10\%)$. Thus, $\delta\lambda$ is within 10% of 0.15 Å . The spectral bins used in this paper are all much larger than $\delta\lambda$, so the count rates $R_{\lambda,i}$ are statistically independent of each other.

The minimum detectable polarization (MDP) of the instrument at the 99% confidence level is defined as

$$\text{MDP} = \frac{4.292}{\sqrt{\sum_i (\mu_i^2 C_i^2)}}, \quad (5)$$

where the sum is extended over all the wavelength bins, and μ_i and C_i are the instrument modulation factor and source count rate in the i th wavelength bin, respectively.

We estimate that the background will be negligible. The particle background for REDSoX was estimated to be $0.001 \text{ counts s}^{-1}$ (H. L. Marshall et al. 2018, Section 3.5). We use a sky background estimate of $540 \text{ photons cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \text{ keV}^{-1}$ (H. L. Marshall et al. 2018), or $3.9 \text{ photons cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \text{ Å}^{-1}$ at 0.3 keV . For a solid angle of 2.5 radius, where the instrument sensitivity drops to 50% of peak (H. L. Marshall et al. 2018, Figure 16), and the integrated area of REDSoX ($185 \text{ cm}^2 \text{ Å}$), the sky background should contribute about $0.0012 \text{ counts s}^{-1}$. Thus, the total background is estimated to be about $0.0022 \text{ counts s}^{-1}$, which is negligible compared to the expected count rates of the targets (Section 4).

3.2. Results and Statistics

To illustrate the possible features from isolated neutron stars, we computed mock spectra for two atmospheric models: (a) a pure-plasma atmosphere (i.e., without vacuum corrections), with $B = 5 \times 10^{13} \text{ G}$, and (b) a plasma plus vacuum atmosphere with complete mode conversion and $B = 10^{14} \text{ G}$. We also computed the case of a plasma plus vacuum model with no mode conversion and obtained quantitatively similar results as in case (a); therefore case (a) can be taken as representative of both these situations. These magnetic field strengths were chosen such that a proton cyclotron feature (case (a)) or a 90° PA swing (case (b)) are present in the instrument detectable energy range.

To create a mock spectrum, we realized a Poisson-distributed count number $n_{\lambda,i} \sim \text{Poisson}(k_\lambda)$ in each detector channel ($i = 1, 2, 3$), where λ denotes the channel wavelength. The mean k_λ is the expected number of photons in each channel, which is obtained by multiplying the model spectrum count rates, given by Equation (3), with the observation time.

Figure 4 shows an example of the mock counts in each detector versus the wavelength, assuming a bin size of $\Delta\lambda = 3 \text{ Å}$ and a total of $\sim 100,000$ counts in all three detectors. The first dip, present in both cases at $\sim 37 \text{ Å}$, is a feature of the detector response. The second dip, which can be seen at $\sim 50 \text{ Å}$ ($\sim 0.25 \text{ keV}$) in detectors 2 and 3 in Figure 4(a), is due to the proton cyclotron feature. In this example, detector 1 collects a significantly reduced number of counts compared to detectors 2 and 3; this is due to the orientation of the detectors with respect to the reference frames of Q and U , together with the fact that, for the parameters of this model, there is a large difference between the expected values of Q and U . In fact, from Equation (3), when $\theta_1 = 0^\circ$, if Q is negative and $|Q| \gg |U|$ (as in this case), detector 1 will collect much fewer counts than the other two detectors. This difference in the detector count rate does not appear in the complete mode conversion scenario shown in Figure 4(b).

⁷ <https://pulsar.sternwarte.uni-erlangen.de/wilms/research/tbabs/>

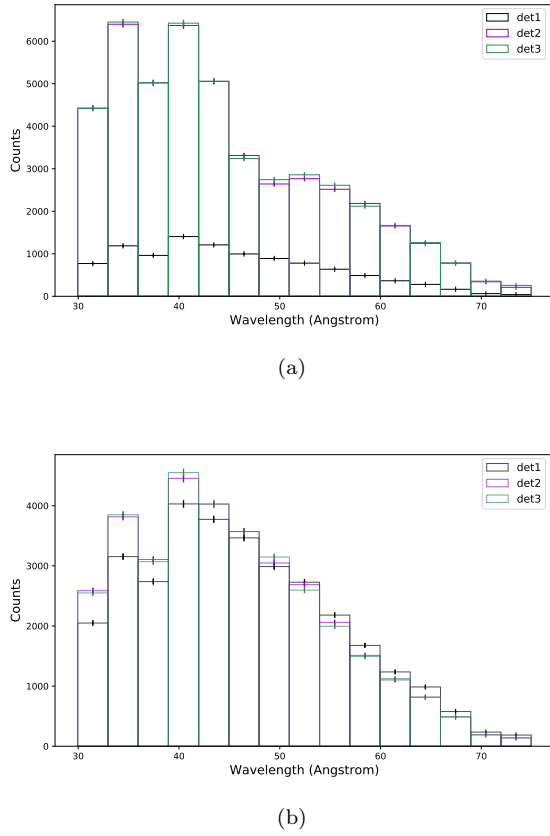


Figure 4. Counts (as a function of wavelength) for each detector channel (“det1,” “det2,” and “det3”) calculated assuming emission from an atmosphere covering the whole surface in the pure-plasma ((a); upper panel) and plasma plus vacuum with complete mode conversion ((b); lower panel) scenarios. The dipole magnetic field strengths are $B = 5 \times 10^{13}$ and 10^{14} G in the two cases, respectively. A total of 100,000 counts is assumed to be collected across all three detectors.

Additionally, as expected due to the higher magnetic field strength in case (b), no second dip (corresponding to the proton cyclotron line) is visible, and the reduction in counts due to the detector response and blackbody flux of the source is much more gradual.

Figure 5 shows the energy (wavelength)-resolved PD and PA for case (a). As can be seen, in this case high polarization can be detected, while the PA remains constant across the entire energy range of the detector. The energy-integrated values in the REDSoX band are $PD = 74.5\% \pm 0.5\%$ and $PA = 89.6 \pm 0.2^\circ$. The dip in polarization due to proton cyclotron resonance is clearly visible at ~ 50 Å (~ 0.25 keV), where the PD decreases from $\sim 80\%$ to below 60% . The polarization is well above the MDP across the entire energy range.

To investigate how many total counts are required to ensure that 99% of possible observations will give a statistically significant detection of a similar proton cyclotron feature, we simulated sets of 10,000 mock spectra for different values of the total counts. We computed the χ^2 statistic for each mock spectrum (with 5 Å binning) using a constant polarization model (i.e., fitting the mock data with a horizontal straight line). A reduced χ^2 of 2.5 (with eight degrees of freedom) was chosen as our goodness-of-fit criterion: this means that those mock data realizations that result in a higher χ^2 value statistically rule out a constant polarization model at the 98% confidence level.

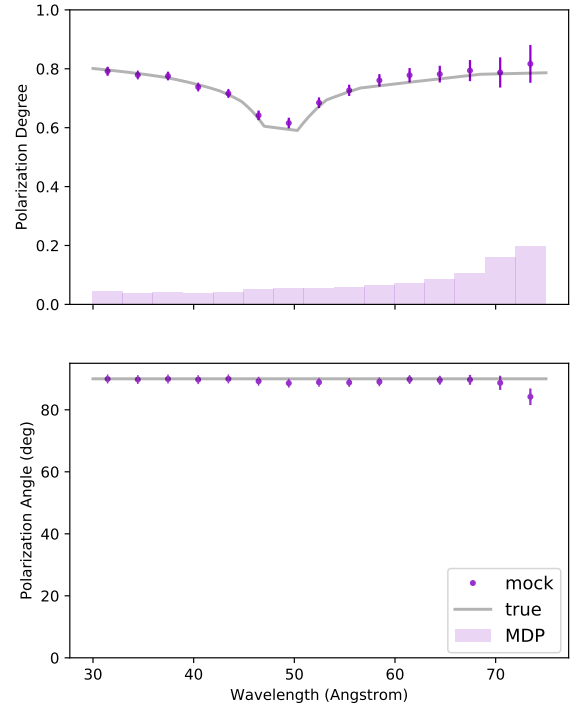


Figure 5. PD (upper panel) and PA (lower panel) as a function of the wavelength for case (a). The model from which the mock data were produced is shown by the solid line. Here, $B = 5 \times 10^{13}$ G.

We found that for the set of mock spectra with 25,000 total counts, a constant polarization model provides an unacceptable fit ($\chi^2_{\text{red}} > 2.5$) for more than 99% of the realizations in the set. With 15,000 total counts, instead, only about 77% of the mock spectra can rule out a constant polarization model at the same confidence level. However, when the total counts exceed 5000, all the mock spectra in the set are better fit by the atmospheric model with the proton cyclotron line than by a model with constant degree of polarization.

In case (b), corresponding to a plasma plus vacuum model with complete mode conversion and a larger magnetic field, a significantly lower overall polarization is expected when compared with case (a). The energy-integrated value (in the REDSoX band) is $PD = 3.5\% \pm 0.5\%$, while the corresponding MDP is 1.4%. As shown in Figure 6, the polarization changes significantly with energy in the REDSoX range: the PD reduces from almost 20% to $\approx 0\%$ at 50 Å (~ 0.25 keV) before increasing again to $\sim 20\%$ at 70 Å. At the wavelength where $PD \approx 0\%$, the PA swings by 90° , indicating a change in the dominant polarization mode induced by complete mode conversion at the QED vacuum resonance. For the majority of the bins in the REDSoX wavelength band, except for those near the dominant mode switch and for the high wavelengths, the predicted polarization is above MDP. For bins with polarization below MDP, the PA is unconstrained (these points are shown as diamonds in the figure).

We then tested whether the 90° swing in the PA could be detected, again using sets of 10,000 spectral realizations characterized by a different number of total counts. We computed the χ^2 statistic considering a model with a linear variation of the PA (i.e., a fit with a nonhorizontal straight line) across the energy range, binning the data to 5 Å for seven degrees of freedom. Our simulations showed that, for spectra with more than 15,000 total counts, a model with a constant

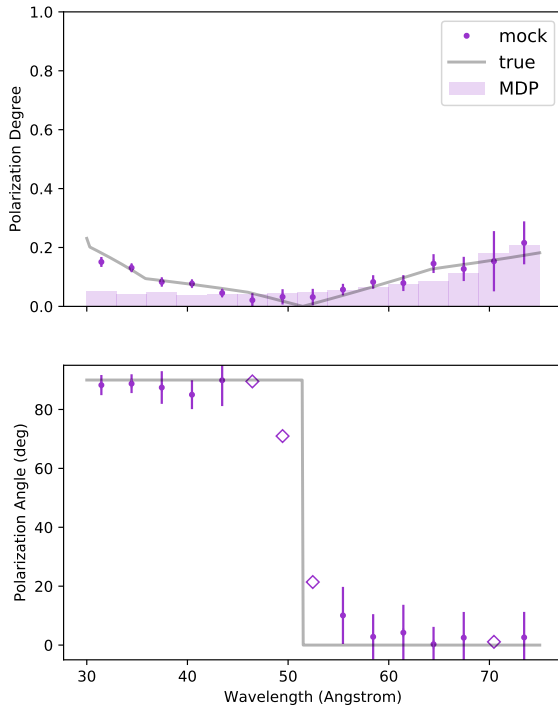


Figure 6. Same as in Figure 5 for case (b). Here, $B = 10^{14}$ G. When the PD is below the MDP, the corresponding error bars in the PD are shown at the 90% confidence level and the diamonds indicate an unconstrained PA.

rate of change in the PA was statistically ruled out ($\chi^2_{\text{red}} > 2.5$) for $>99\%$ of the set. However, for 15,000 counts, the majority of the wavelength bins are below the MDP, and as such the PAs are unconstrained. In simulations with a low number of counts, the results are less statistically significant because there are fewer data points for fitting. It is only for counts $> 25,000$ that the PD is above the MDP in the majority of the wavelength bins, when binning to 5 Å. If we increase the bin size to ~ 8 Å, the majority of the spectral bins for $\gtrsim 15,000$ counts are above the MDP.

4. Case Studies

4.1. RX J1856.5–3754

RX J1856 is the brightest and closest XDINS, with a distance from Earth of ~ 120 pc (F. M. Walter et al. 2010). It has a rotational period of $P \sim 7$ s and a very low pulsed fraction of $\sim 1.3\%$ (A. Tiengo & S. Mereghetti 2007). The X-ray spectrum (0.1–10 keV) is well fit by two blackbody components (with temperatures $T_h \approx 60$ eV and $T_c \approx 40$ eV, respectively; see N. Sartore et al. 2012). The source has also been observed in the optical band, with a sizable degree of polarization ($\approx 16\%$), strongly suggesting that QED vacuum birefringence is at work around the star (R. P. Mignani et al. 2017).

For our simulations, we follow the model described in D. González Caniulef et al. (2016), assuming a dipole magnetic field with strength $B_p = 10^{13}$ G at the pole, a polar temperature $T_p = 60$ eV and an equatorial temperature $T_e = 40$ eV, both measured at the observer, and $N_H = 0.5 \times 10^{20} \text{ cm}^{-2}$. We again divide the surface into six annular patches with magnetic colatitude centered on $\theta = (0^\circ, 10^\circ, 30^\circ, 50^\circ, 70^\circ, 89^\circ)$ in the north hemisphere. As previously, integration in the south hemisphere is then performed using symmetry properties.

We investigated the cases of the pure-plasma and plasma plus vacuum atmospheric emission models, the latter computed either assuming no mode or complete mode conversion. Again, we found that the first two cases give the same results in the REDSoX band, therefore they will not be discussed separately. We also considered the case of condensed surface emission, with the emissivity computed following A. Y. Potekhin et al. (2012; see also D. González Caniulef et al. 2016; R. Taverna et al. 2020) by assuming an iron chemical composition. As discussed in R. P. Mignani et al. (2017), in the limit of free ions the emission expected from a condensed iron layer is not compatible with the optical polarization properties of the source; therefore this limit will not be considered in our simulations.

We performed simulations by accounting for or turning off QED vacuum birefringence effects in the photon propagation in the vacuum from the star surface to the observer; these models will be correspondingly referred to as “QED on” and “QED off” in the following. The viewing geometry is set at $\chi, \xi = (14.0, 3.0)$ for all the atmospheric emission models⁸ and $\chi, \xi = (21.7, 5.5)$ for the condensed surface model in the limit of fixed ions, according to the constraints on the viewing geometry derived by R. P. Mignani et al. (2017, see their Table 1).

We assume a count rate of $0.12 \text{ counts s}^{-1}$ and an observation time of 125 ks, which give a total of $\sim 15,000$ expected counts over the three detectors, and use a bin size of 3 Å.

Figure 7 shows the detector counts over the REDSoX wavelength band for the various models. In all cases, we see no significant features in the detected counts per bin; for a field strength of 10^{13} G the proton cyclotron feature falls outside the REDSoX energy range.

The corresponding polarization features are illustrated in Figure 8. First, we considered models computed by accounting for vacuum QED effects in the photon propagation from the star to the observer (“QED on” models in the figure). As expected, in this case we found that in the case of plasma plus vacuum atmospheric emission without mode conversion radiation is highly polarized (wavelength-integrated $\text{PD} = 94.6\% \pm 1.2\%$ and $\text{PA} = 89.98^\circ \pm 0.45^\circ$ across the entire REDSoX energy band). This value of the PD is significantly higher than the energy-averaged MDP for this observation, which is 4.4%. The complete mode conversion model shows very similar features ($\text{PD} = 93.9\% \pm 1.6\%$ and $\text{PA} = 85.80^\circ \pm 2.7^\circ$). This is not unexpected since, for a field $B = 10^{13}$ G and the temperature values used here, the vacuum resonance density falls outside the X and O mode photospheres, so that mode conversion has little effect on the overall polarization properties of the emergent signal (see R. M. E. Kelly et al. 2024b). We also tested an atmospheric model computed assuming partial mode conversion for different values of the threshold probability $P_{\text{con}}^{\text{th}}$, and found a similarly high emergent polarization. In comparison, if QED effects related to photon propagation in the vacuum outside the atmosphere are not included (“QED off” cases), the polarization is significantly reduced (energy-integrated $\text{PD} = 3.7\% \pm 1.6\%$ for a pure-plasma atmosphere). A similar trend is found when considering emission from a condensed surface, in which case

⁸ We caveat that in R. P. Mignani et al. (2017) the viewing geometry has been derived by considering those models that predict a value of the optical polarization in agreement with observations. Since the assumptions in some of their atmospheric emission models are slightly different in this work, the constraint on the viewing geometry may be slightly impacted. This is not explored in this work.

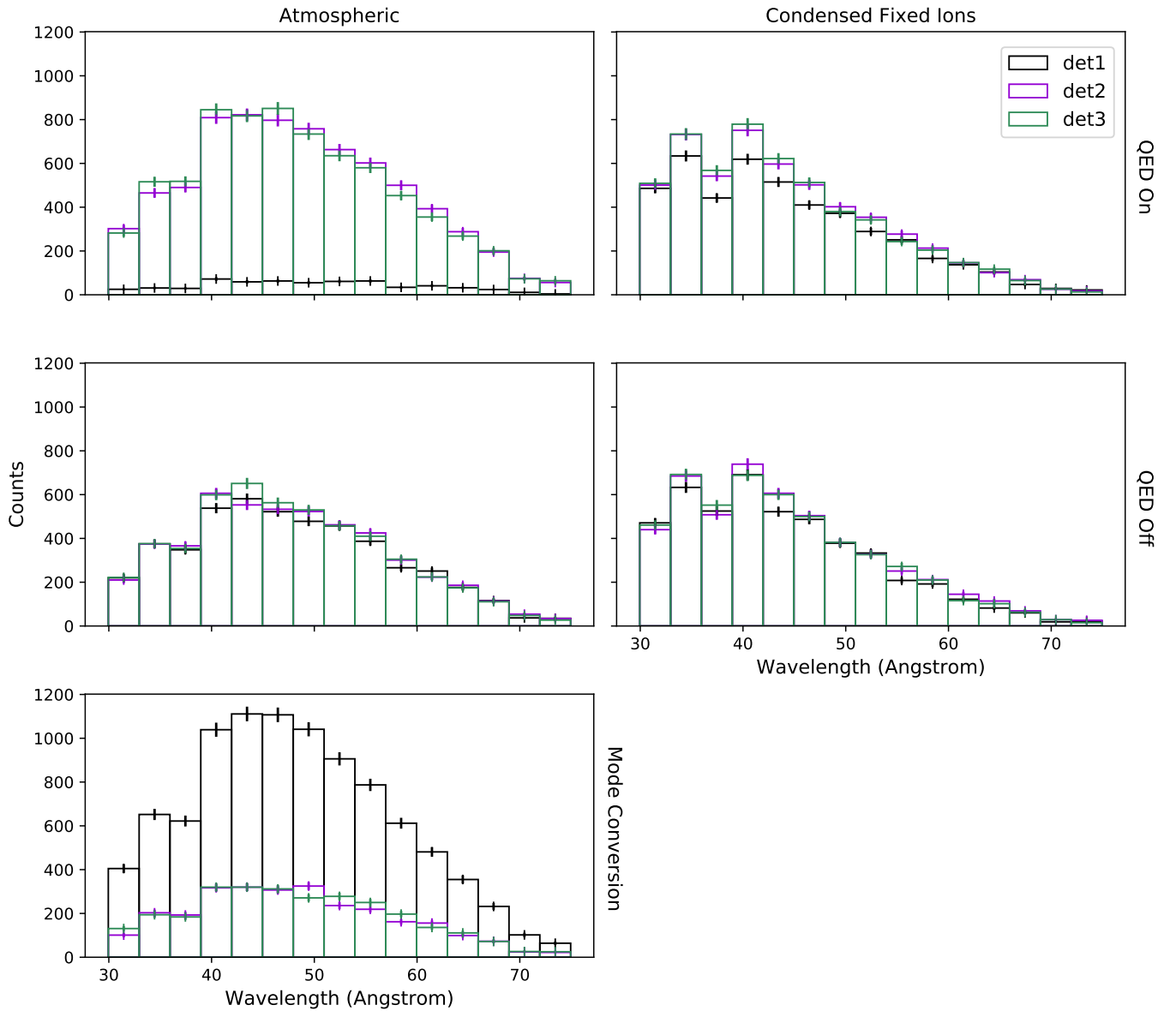


Figure 7. Expected counts for each instrument detector channel (“det1,” “det2,” and “det3”), binned at $3 \text{ \AA}$, calculated from a mock spectrum of RX J1856 assuming an observation time of 125 ks and a count rate of $0.12 \text{ counts s}^{-1}$. All models assume emission from the whole surface. The first two rows show the case of a plasma plus vacuum atmosphere (no mode conversion, left column) and a condensed iron surface (fixed-ion model, right column); both models are computed by turning on or off the effects of QED vacuum birefringence in the photon propagation in the vacuum from the star to the observer (first and second line, respectively). The third row shows the case of a plasma plus vacuum atmospheric model with complete mode conversion and QED effects in the photon propagation outside the star.

$\text{PD} = 13.6\% \pm 1.2\%$ in the QED on case and only $3.2\% \pm 1.2\%$, in the QED off one.

For both the QED on atmospheric models presented here, the observed PD turns out to be well above the MDP in all wavelength bins (see Figure 8). However, for the QED off atmospheric model and for both the QED on and QED off condensed surface models, the observed polarization is below the MDP in almost all of the wavelength bins. This results in unconstrained PAs (indicated by the diamond symbol in Figure 8; 90% error regions are shown for the corresponding detected PD). However, the QED on, condensed surface model does produce some significant detections in a few wavelength bins (around 40), and it can be observed to be different from the QED off case.

In summary, an observation of this source lasting 125 ks should, at the very least, allow us to test the structure of the outer layers, and, if the emission is atmospheric, to detect a

degree of polarization of almost 100% with high significance, confirming that QED vacuum birefringence is at work.

4.2. RX J0720.4–3125

The second brightest XDINS, RX J0720, shows two absorption features in the spectrum (at $\sim 270 \text{ eV}$ and $\sim 750 \text{ eV}$, respectively; F. Haberl et al. 2004; A. Borghese et al. 2015), and is a very interesting candidate for soft X-ray polarimetry. The first, broad ($\Delta E \sim 70 \text{ eV}$) absorption feature, is present at all rotational phases. If it is a proton cyclotron line, the feature should be related to the global field and implies a dipolar magnetic field strength $B_{\text{dip}} = 2.5 \times 10^{13} \text{ G}$ at the equator of the neutron star. Absorption has also been speculated to be associated with an atomic transition (F. Haberl et al. 2006). The second absorption feature at $\sim 750 \text{ eV}$ is instead frequency dependent and peaks in the

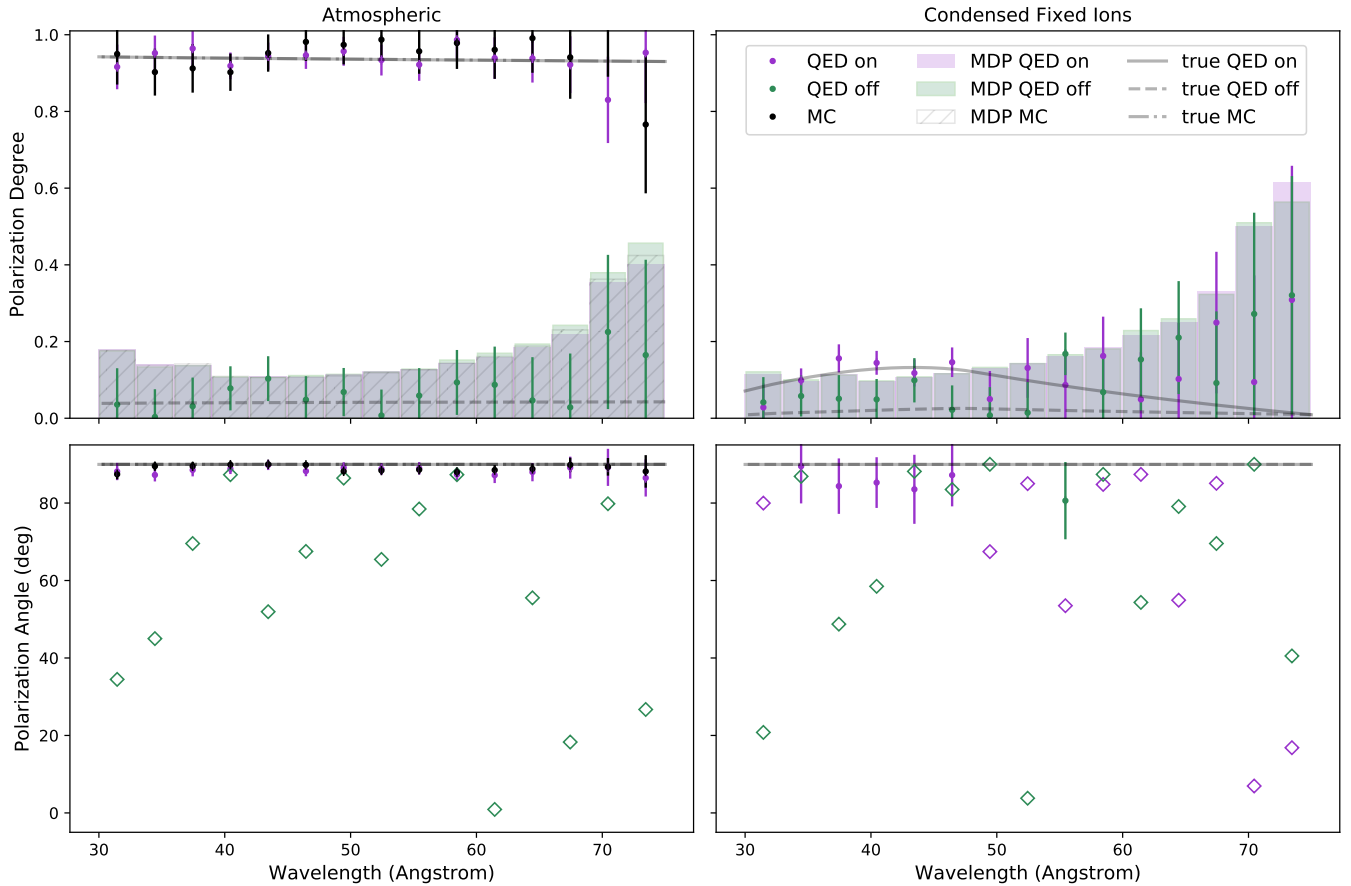


Figure 8. Expected PD (top row) and PA (bottom row) of RX J1856 as a function of the wavelength, for different models: a plasma plus vacuum atmospheric emission model with complete conversion (“MC”), a pure-plasma (no vacuum corrections in the atmosphere) atmospheric emission model (both computed for $\chi = 13^\circ$ and $\xi = 4^\circ$), and a condensed surface emission model computed in the fixed-ion limit ($\chi = 21.7^\circ$ and $\xi = 5.5^\circ$). Results were obtained with and without the inclusion of QED vacuum effects in the photon propagation outside the star (“QED on/off”). An observation time of 125 ks and a count rate of $0.12 \text{ counts s}^{-1}$ are assumed. The figure shows the mock data with their 1σ errors (points), the original synthetic polarization spectra (full lines), and the MDP for each simulated observation (shaded area). When the PD is below the MDP, the error bars are shown at the 90% confidence level. The diamonds represent an unconstrained PA.

0.1–0.3 phase interval (see Table 2 in A. Borghese et al. 2015). As discussed by these authors, it is possible that it is produced by proton cyclotron scattering/absorption in a local magnetic loop with $B \sim 2 \times 10^{14} \text{ G}$ above the emitting cap. This is particularly tantalizing. Even if the second feature is outside the REDSoX energy band, the fact that it hints at the presence of strong magnetic field components near some parts of the star surface may open the possibility of using this instrument to detect features caused by the atmospheric QED vacuum resonance (such as the 90° swing in PA discussed previously).

We assumed an observation time of 200 ks and a source count rate of $0.06 \text{ counts s}^{-1}$, for a total of 12,000 counts across the whole observation. We performed simulations mimicking the scenario by A. Borghese et al. (2015) and assumed a thermally emitting cap with radius $R_{\text{BB}} \approx 6 \text{ km}$ and temperature $kT_{\text{BB}} \approx 120 \text{ eV}$.⁹ We assume $N_{\text{H}} = 1.5 \times 10^{20} \text{ cm}^{-2}$. Once again, we consider plasma plus vacuum atmospheric models computed assuming no or complete mode conversion, and a condensed surface emission model, this time exploring both the fixed- and free-ion limits. Again, we derived the observables either with or without accounting for

QED vacuum birefringence effects in the photon propagation outside the star.

Our emission cap is centered at the pole, and extends in colatitude up to $\approx 30^\circ$. In order to account for the varying dipolar field across the cap, we divide the cap into three annular patches at $\theta = (0^\circ, 15^\circ, 30^\circ)$, and calculate the emission for each of them, assuming $B_{\text{p}} = 5 \times 10^{13} \text{ G}$ at the pole. A. Borghese et al. (2015) invoked the presence of a small-scale highly magnetic structure (a magnetic loop) to explain the phase-dependent feature observed (off peak in phase) at $\sim 720 \text{ eV}$ in terms of proton cyclotron absorption. Here this higher- B component is simulated by the addition of a fourth patch with $B = 2 \times 10^{14} \text{ G}$ and $\theta_{\text{B}} = 0^\circ$, which extends in a limited surface area, and is offset with respect to the pole. We explored different sizes for this patch, finding similar results; in the following we report an example in which it extends in the ranges $10^\circ < \theta < 30^\circ$ and $0^\circ < \phi < 40^\circ$.

Figure 9 shows the expected counts in each detector computed assuming emission from either an atmosphere or a condensate. The viewing geometry is $(\chi, \xi) = (75^\circ, 5^\circ)$, as inferred by F. Haberl et al. (2006). In the mock spectra corresponding to the atmospheric model with no mode conversion, we can see a clear dip in the counts of detectors 2 and 3 at $\sim 50 \text{ \AA}$ in both the QED on and QED off cases. This is due to the expected proton cyclotron feature at $\sim 270 \text{ eV}$ associated to the polar cap with the weakest magnetic field.

⁹ Our choice of a slightly higher temperature than that reported in A. Borghese et al. (2015), $kT_{\text{BB}} \approx 80 \text{ eV}$, is dictated by numerical convenience. Although this will affect the overall spectrum, we expect that it only marginally affects the polarization results (see R. M. E. Kelly et al. 2024b).

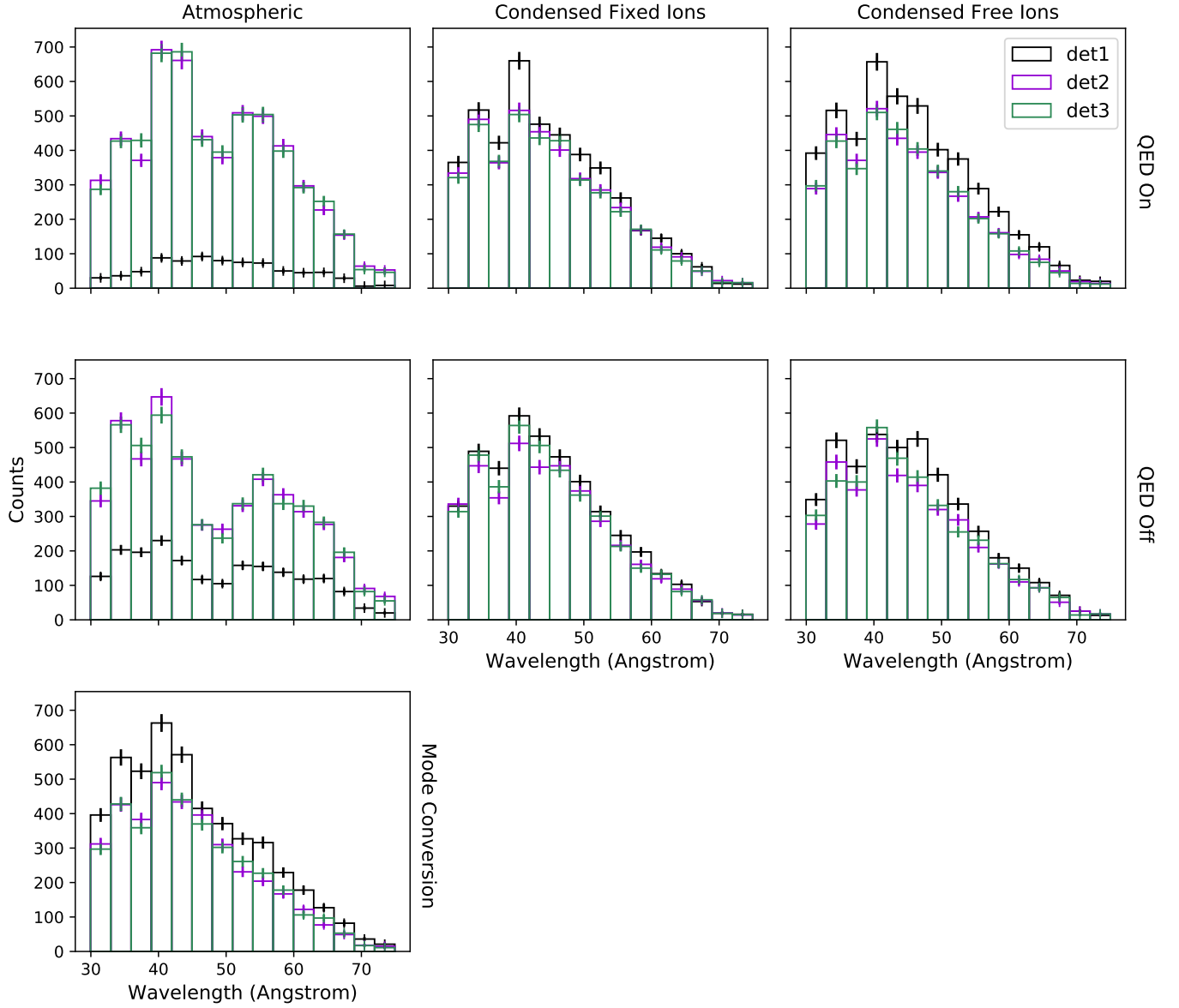


Figure 9. Counts, binned to 3 Å, for each instrument detector channel (“det1,” “det2,” and “det3”) calculated from a mock spectrum of RX J0720 assuming an observation time of 200 ks and a count rate of 0.06 counts s⁻¹. Spectra are produced for different cases. First row: a plasma plus vacuum atmosphere model with no mode conversion (left) and a condensed iron surface emission model in the limit of fixed ions (center) and free ions (right), all “QED on.” Second row: a pure-plasma atmosphere model (left) and the same condensed surface models as in the first row, but “QED off.” Third row: an atmospheric model computed assuming complete mode conversion and “QED on.” The viewing geometry is $(\chi, \xi) = (75^\circ, 5^\circ)$. See text for all details.

Instead, from the detector counts alone, for all of the condensed surface emission cases and for the atmospheric model with complete mode conversion, there is no clear indication of any feature in the detector energy band.

The expected polarization properties, computed by combining the three detectors, are shown in Figure 10. We start by discussing the expectations of atmospheric models. The model computed with no mode conversion, once again, produces very high polarization across the entire detector energy range in the QED on case, with a frequency-integrated PD = $(86.5 \pm 1.3)\%$. In this scenario, a clear dip in the PD at the proton cyclotron energy (~ 50 Å) is visible.¹⁰ This dip is less evident in

the corresponding QED off model, in which the PD remains seemingly constant with energy around a frequency-integrated value of PD = $55.2\% \pm 1.2\%$ (for this viewing geometry).

We also tested models with partial mode conversion, varying the assumption on $P_{\text{con}}^{\text{th}}$, and found qualitatively similar results to the QED on case without mode conversion, with slightly lower values for the degree of polarization.

The complete mode conversion scenario results in a relatively low polarization, PD = $(24.9 \pm 1.3)\%$ integrated over the instrument energy band, significantly reduced compared to the no mode conversion scenarios. The proton cyclotron absorption feature is clearly seen in the spectrum, although the drop in polarization causes the PD to fall below the MDP at this wavelength. However, for most of the wavelength range (excluding the highest wavelengths), the expected polarization remains above the MDP, being measurable by REDSoX.

¹⁰ In the case of pure plasma (no vacuum corrections) with QED effects accounted for in the photon propagation toward the observer, the proton cyclotron dip is even more prominent, with the PD dipping to $\sim 70\%$ at the cyclotron energy.

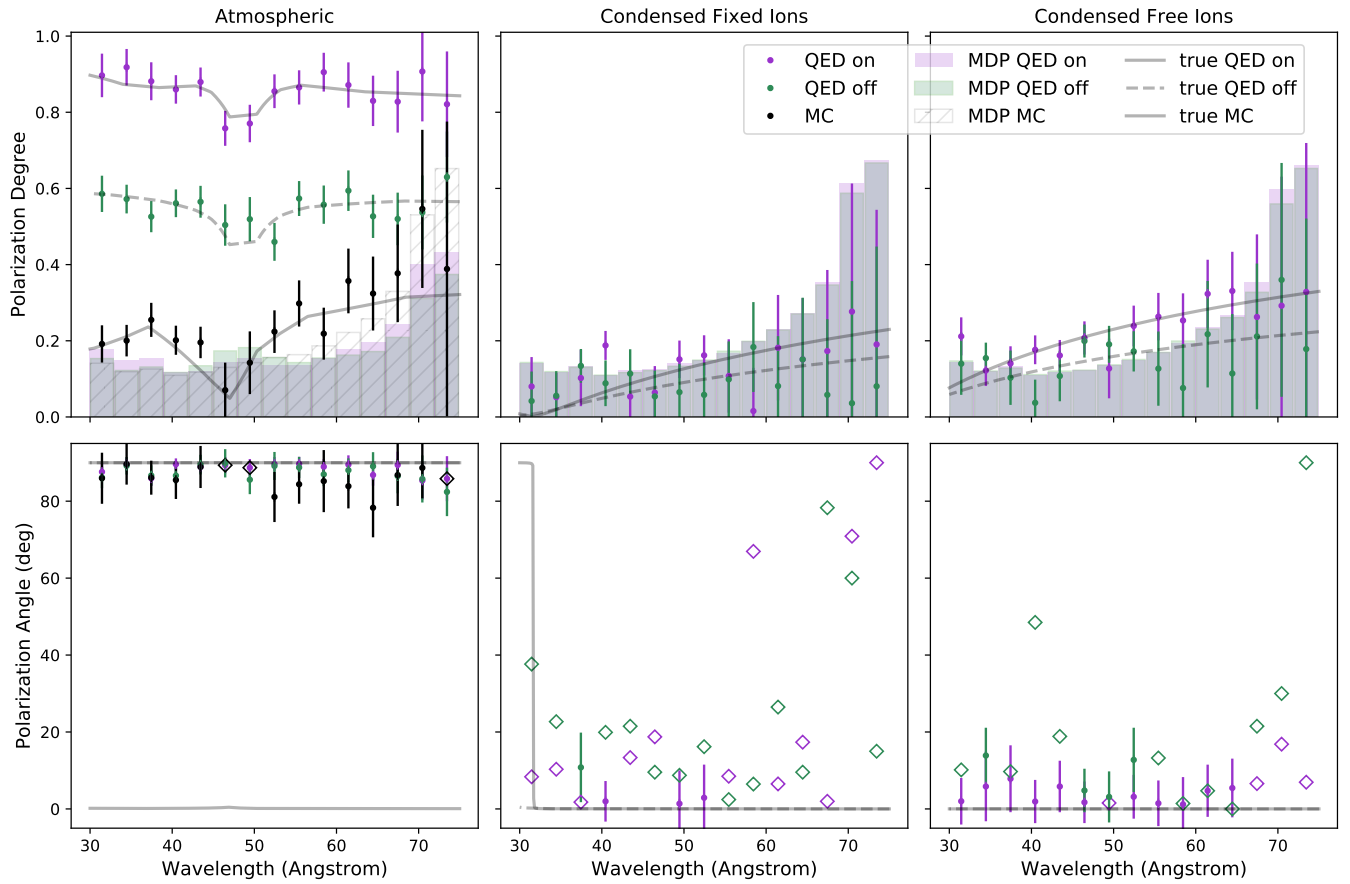


Figure 10. Expected PD and PA computed assuming the models as in Figure 9, for RX J0720. The points show the simulated data with 1σ errors, the solid lines show the original models, and the shaded areas show the MDP for each mock observation. When the PD is below the MDP, the error bars are shown at the 90% confidence level, while the diamonds represent an unconstrained PA.

In all these three cases, the PA remains constant across the whole band, around $\sim 90^\circ$, indicating that, at least for this choice of thermal map and geometry, we do not expect to detect the switch in dominant polarization mode due to mode conversion in the detector band.

The 90° swing in the PA due to mode conversion is not visible because, according to our simulations, at 2×10^{14} G, the magnetic field of the small patch is slightly too high, so the swing occurs below 0.2 keV. The emission is instead dominated by the radiation emitted by the rest of the polar cap. Interestingly, the emission from the polar cap and that from the strongly magnetized patch are dominated by opposite polarization modes (O and X, respectively) in the REDSoX band, therefore the inclusion of the patch in the simulation leads to a slightly depolarized signal.

In principle, one might expect the detectability of a 90° swing in the PA in phase-resolved observations. However, we found that the larger contribution from the region with the lower field always dominates and washes out the contribution from the magnetic loop. Only when it is assumed that the loop covers a large region (almost half) of the polar cap, then a swing is evident in the phase-resolved polarization spectrum. Noticeably, the origin of this swing is different in models with different magnetic fields strengths in the small patch; if $B \geq 10^{14}$ G it is due to the fact that two different modes dominate in the different emitting regions, while when the strength of the field in the magnetic loop is reduced to

$\lesssim 10^{14}$ G, the 90° swing in the PA due to QED mode conversion becomes visible in phase-resolved data.

On the other hand, models based on condensed surface emission produce significantly lower polarization in all cases and have no visible proton cyclotron feature in the spectra. In the limiting case of fixed ions, both QED on and QED off, the PD remains below the MDP for almost all of the wavelength bins, and the PA is therefore unconstrained. However, the energy-integrated polarization, $\text{PD} = (10.1 \pm 1.4)\%$ and $(7.3 \pm 1.2)\%$, for QED on and off, respectively, is above the energy-integrated $\text{MDP} = 4.9\%$. For the free-ion case, only in the QED off case is the expected PD below the MDP in most wavelength bins; in this case the energy-averaged $\text{PD} = (12.0 \pm 1.3)\%$. When QED effects are taken into account, the case of free ions shows a clear increase in polarization with increasing wavelength, from $\sim 10\%$ to $\sim 30\%$. In this case, the frequency-averaged $\text{PD} = (19.0 \pm 1.3)\%$.

5. Discussion and Conclusions

The observation of X-ray polarization below 1 keV is a crucial path to advancing our understanding of XDINS and magnetar phenomena. A space mission hosting an instrument with the same or similar design as the REDSoX mission, observing in the 0.2–0.4 keV range, such as the proposed global orbiting mission, GOSoX, will allow us to study the magnetic fields and external layers of isolated neutron stars, identify the emission mechanisms, and, significantly, may

even test QED vacuum birefringence. Polarimetric data in the soft X-ray band are therefore key, in themselves and also when used in combination with those from other polarimetric instruments working in different wave bands, to guiding our knowledge of the physics of these extreme sources.

Under standard assumptions of a pure-plasma, fully ionized H, passive cooler magnetized atmosphere, thermal emission is well known to be highly polarized, at $\gtrsim 80\%$, and dominated by X mode photons. However, mode conversion at the QED vacuum resonance can greatly affect the polarization signal. We found that, for dipolar magnetic field strengths $B \sim 10^{14}$ G, complete mode conversion results in the presence of a switch in dominant polarization mode (implying a 90° swing in PA) that falls in the 0.2–0.4 keV energy range. This rotation in PA can be resolved by the REDSoX instrument with $\sim 25,000$ counts. An observation of this kind from a source would therefore provide a robust observational test for the presence of QED vacuum birefringence, as well as put constraints on the physics of mode conversion and on the magnetic field strength of the source. Additionally, the nondetection of the PA swing would not rule out QED; in fact models computed assuming partial mode conversion (for the same magnetic field strength) only result, in the REDSoX range, in a reduced degree of polarization with respect to those computed turning off QED effects completely in the atmospheric model (but keeping them in the photon transport from the source to the observer). However, in all these cases, the degree of polarization is still expected to be well above that expected in the case of condensed surface emission (which is $\lesssim 20\%$).

Proton cyclotron absorption features in the polarization spectrum can also originate in the star atmosphere, and they are expected to fall in the 0.2–0.4 keV energy range for $B \sim 5 \times 10^{13}$ G. We found that observations from an instrument with the same design as REDSoX can resolve these absorption features (or, in general, spectral absorption features of similar strength) with high statistical significance when collecting $\gtrsim 25,000$ counts from a source.

In addition to the specific spectral features discussed above, the instrument will be able to detect smooth changes in the polarization signal with energy/wavelength (e.g., increases/decreases in PD across the band), which will further inform the interpretation of the source emission. In all models considered in our simulations, the signal emerging from a condensed surface exhibits a significantly lower PD (PD $\lesssim 20\%$) compared to that caused by atmospheric emission, showing that the design of the REDSoX instrument will allow us to identify the emission mechanisms and compositions of the external surface layer of the neutron star.

We also considered two case studies, focusing on two XDINS targets and producing simulations based on some examples of very simplified thermal maps, assumed in a way to mimic, at least in the first approximation, the X-ray spectral properties of the sources. Quite interestingly, in the case of RX J1856, we found that all the atmospheric models we considered (independent of the treatment of mode conversion at the vacuum resonance) concur in predicting a very high degree of polarization in the observed REDSoX emission (PD $\sim 95\%$) if, and only if, QED vacuum birefringence is at work in the vacuum around the star. An observation of such high polarization from this source will therefore allow for a long-awaited observational evidence of the existence of QED vacuum birefringence.

In the case of the second target, RX J0720, an absorption feature at ~ 270 eV has previously been observed in the spectrum, and this falls in the REDSoX energy range. Our models based on atmospheric emission exhibit this feature in the polarization spectrum, as caused by the proton cyclotron resonance, and it is particularly prominent for models computed including QED effects. We also attempted to model the presence of highly magnetized loops near the star surface, as inferred for this target. We found that the loop must be relatively large to be observable (in our simulation covering a region as large as half of the cap that extends up to $\sim 30^\circ$ in colatitude). However, would this be the case, then for some particular emission models, its presence may be detectable in the REDSoX band through phase-dependent observations of a 90° swing in PA. Although the nature of the swing depends on the magnetic field strength, its presence is always caused, directly or indirectly, by the occurrence of QED complete mode conversion at the vacuum resonance.

The geometry and magnetic field strength of an isolated neutron star is typically not well constrained. A comprehensive investigation into the affects on the resulting uncertainties by varying the pulsar geometry and magnetic field strength would be informative, but would warrant a systematic analysis and is beyond the scope of this work. Here, we aim to highlight what can be detected with an instrument such as REDSoX. To that end, for the two example cases in Section 3, the models, viewing angles, and magnetic field strengths were chosen to be most favorable for detecting the respective features. For the two case studies we used geometries and field strengths previously quoted in the literature. For our analysis of RX J1856, we rely on the χ and ξ values quoted in R. P. Mignani et al. (2017) where the pulsed fraction and polarization properties of the source are considered together. In this case the uncertainties in the measurements of χ and ξ are relatively small, and the differences in polarization only varies modestly. In contrast, in the case of RX J0720, the viewing geometry is much less constrained. Here, we have used the χ and ξ values from F. Haberl et al. (2006), which are favorable for our investigation.

Phase-resolved polarimetry is important for constraining the geometry of the surface emission, in particular because if the behavior of the PA is not well reproduced by the rotating vector model (RVM; and the magnetic field is dipolar) it may imply the presence of a nonaxisymmetric thermal map (see supplementary material from R. Taverna et al. 2020). For the models and viewing angles used in this work, a phase-resolved analysis shows only very small variations in the PD with phase, while the PAs follow a smooth sinusoidal oscillation, in agreement with the RVM.

In principle, modeling observed data with the RVM has been often proposed as an indirect test of QED vacuum birefringence. However, this test is not meaningful in the case of sources for which the magnetic field is intrinsically close to a dipole, such as the ones considered in this paper. For many cases, as highlighted above in the case of RX J1856, phase-resolved observations will be crucial for further constraining the geometries involved, and for our understanding of isolated neutron star sources more generally. A longer, orbital mission, equipped with an instrument similar to REDSoX would undoubtedly benefit from high time resolution, allowing us to make the necessary measurements to constrain emission and viewing geometries, as well as to resolve phase-dependent

absorption features (such as that observed in the magnetar IE 2259+586; J. Heyl et al. 2024).

We caveat that our atmospheric models assume a fully ionized atmosphere, and, consequently, exhibit cyclotron lines as the only absorption features in the X-ray spectrum. However, this is a simplification and the coldest atmospheric layers may be expected to be partially ionized. The inclusion of atomic transitions and resonances complicates the calculation of radiative transfer and may affect the polarization of the emitted radiation (see, e.g., A. Y. Potekhin et al. 2004; A. Y. Potekhin 2014, for details). The inclusion of partial ionization to our atmosphere models is a matter of ongoing work.

Although based on simple assumptions, our simulations of a few illustrative cases show that a fully fledged orbital mission utilizing the REDSoX instrument design will undoubtedly lend itself to important scientific advancement, specifically in the studies of neutron stars and highly magnetized environments.

Acknowledgments

R.M.E.K. would like to thank the MIT Kavli Institute for supporting and hosting her visit and the Science and Technology Facilities Council (STFC) for financial support via a Long Term Attachment (LTA) grant and a PhD studentship (grant No. ST/W507891/1). This work was supported in part by MIT Corporate Relations and its Industrial Liaison Program (ILP). H.L.M. and S.R. were supported in part by NASA grant 80NSSC23K0644 for development of the REDSoX Polarimeter. N.B. is supported by STFC grant ST/Y001060/1. R.Ta. and R.Tu. are partially supported by the PRIN grant 2022LWPEXW of the Italian Ministry of University and Research (MUR). D.G.C. acknowledges support from the Agence Nationale de la Recherche (ANR) under grant No. ANR-20-CE31-0010 (MORPHER). We would like to thank the anonymous referee for their constructive and helpful feedback.

Software: tbabs, <https://pulsar.sternwarte.uni-erlangen.de/wilms/research/tbabs/> (J. Wilms et al. 2000). Other supplementary material or numerical routines can be provided upon request.

ORCID iDs

Ruth M. E. Kelly  <https://orcid.org/0000-0002-5004-3573>
 Herman L. Marshall  <https://orcid.org/0000-0002-6492-1293>
 Swati Ravi  <https://orcid.org/0000-0002-2381-4184>
 Roberto Turolla  <https://orcid.org/0000-0003-3977-8760>
 Roberto Taverna  <https://orcid.org/0000-0002-1768-618X>

References

- Adler, S. L. 1971, *AnPhy*, **67**, 599
 Blandford, R., Meier, D., & Readhead, A. 2019, *ARA&A*, **57**, 467
 Borghese, A., Rea, N., Zelati, F. C., Tiengo, A., & Turolla, R. 2015, *ApJ*, **807**, L20
 Brinkmann, W. 1980, *A&A*, **82**, 352
 Chauvin, M., Friis, M., Jackson, M., et al. 2017, *NIMPA*, **859**, 125
 De Grandis, D., Taverna, R., Turolla, R., et al. 2021, *ApJ*, **914**, 118
 De Grandis, D., Rigoselli, M., Mereghetti, S., et al. 2022, *MNRAS*, **516**, 4932
 Feng, H., Jiang, W., Minuti, M., et al. 2019, *ExA*, **47**, 225
 Garner, A., Marshall, H. L., Trowbridge Heine, S. N., et al. 2024, *Proc. SPIE*, **13093**, 277
 Gnedin, Y. N., & Pavlov, G. G. 1974, *JETP*, **38**, 903
 González Caniulef, D., Zane, S., Taverna, R., Turolla, R., & Wu, K. 2016, *MNRAS*, **459**, 3585
 González-Caniulef, D., Zane, S., Turolla, R., & Wu, K. 2019, *MNRAS*, **483**, 599
 Greenstein, G., & Hartke, G. J. 1983, *ApJ*, **271**, 283
 Haberl, F., Turolla, R., De Vries, C. P., et al. 2006, *A&A*, **451**, L17
 Haberl, F., Zavlin, V. E., Trümper, J., & Burwitz, V. 2004, *A&A*, **419**, 1077
 Heine, S. N. T., Marshall, H. L., Schneider, B., et al. 2024, *Proc. SPIE*, **13103**, 1310316
 Heisenberg, W., & Euler, H. 1936, *ZPhy*, **98**, 714
 Heyl, J., Taverna, R., Turolla, R., et al. 2024, *MNRAS*, **527**, 12219
 Ho, W. C. G., & Lai, D. 2003, *MNRAS*, **338**, 233
 Ho, W. C. G., Lai, D., Potekhin, A. Y., & Chabrier, G. 2003, *ApJ*, **599**, 1293
 ISRO 2024, XPoSat, <https://www.isro.gov.in/XPoSat.html>
 Kelly, R. M. E., González-Caniulef, D., Zane, S., Turolla, R., & Taverna, R. 2024a, *MNRAS*, **534**, 1355
 Kelly, R. M. E., Zane, S., Turolla, R., & Taverna, R. 2024b, *MNRAS*, **528**, 3927
 Lai, D. 2023, *PNAS*, **120**, e2216534120
 Lai, D., & Ho, W. C. G. 2003, *ApJ*, **588**, 962
 Lai, D., & Salpeter, E. E. 1997, *ApJ*, **491**, 270
 Lloyd, D. A. 2003, arXiv:astro-ph/0303561
 Marshall, H., Heine, S., Garner, A., et al. 2024, in Proc. of Science, Multifrequency Behavior of High Energy Cosmic Sources XIV, **76**
 Marshall, H. L., Günther, H. M., Heilmann, R. K., et al. 2018, *JATIS*, **4**, 1
 Marshall, H. L., Heine, S., Davidson, R., et al. 2021, *Proc. SPIE*, **11822**, 1182200
 Marshall, H. L., Heine, S., Garner, A., Masterson, R., & Heilmann, R. 2023, *Proc. SPIE*, **12678**, 1267813
 Mignani, R. P., Testa, V., González Caniulef, D., et al. 2017, *MNRAS*, **465**, 492
 Novick, R., Weisskopf, M. C., Berthelsdorf, R., Linke, R., & Wolff, R. S. 1972, *ApJ*, **174**, L1
 Page, D., & Sarmiento, A. 1996, *ApJ*, **473**, 1067
 Pizzocaro, D., Tiengo, A., Mereghetti, S., et al. 2019, *A&A*, **626**, A39
 Popov, S. B., Taverna, R., & Turolla, R. 2017, *MNRAS*, **464**, 4390
 Potekhin, A. Y. 2014, *PhyU*, **57**, 735
 Potekhin, A. Y., Lai, D., Chabrier, G., & Ho, W. C. G. 2004, *ApJ*, **612**, 1034
 Potekhin, A. Y., Suleimanov, V. F., Van Adelsberg, M., & Werner, K. 2012, *A&A*, **546**, A121
 Rigoselli, M., Taverna, R., Mereghetti, S., et al. 2025, *ApJ*, **985**, 34
 Sartore, N., Tiengo, A., Mereghetti, S., et al. 2012, *A&A*, **541**, A66
 Smith, J. F., & Courtier, G. M. 1976, *RSPSA*, **350**, 421
 Stewart, R., Younes, G., Harding, A., et al. 2025, *ApJ*, **985**, 35
 Taverna, R., Turolla, R., Gonzalez Caniulef, D., et al. 2015, *MNRAS*, **454**, 3254
 Taverna, R., Turolla, R., Muleri, F., et al. 2022, *Sci*, **378**, 646
 Taverna, R., Turolla, R., Suleimanov, V., Potekhin, A. Y., & Zane, S. 2020, *MNRAS*, **492**, 5057
 Tiengo, A., & Mereghetti, S. 2007, *ApJ*, **657**, L101
 Turolla, R. 2009, Isolated Neutron Stars: The Challenge of Simplicity, Vol. 357 (Berlin: Springer), **141**
 Turolla, R., Taverna, R., Israel, G. L., et al. 2023, *ApJ*, **954**, 88
 Turolla, R., Zane, S., & Watts, A. L. 2015, *RPPH*, **78**, 116901
 Van Adelsberg, M., Lai, D., Potekhin, A. Y., & Arras, P. 2005, *ApJ*, **628**, 902
 Van Kerkwijk, M. H., & Kaplan, D. L. 2007, *Ap&SS*, **308**, 191
 Verner, D. A., Ferland, G. J., Korista, K. T., & Yakovlev, D. G. 1996, *ApJ*, **465**, 487
 Viganò, D., Perna, R., Rea, N., & Pons, J. A. 2014, *MNRAS*, **443**, 31
 Walter, F. M., Eisenbeiß, T., Lattimer, J. M., et al. 2010, *ApJ*, **724**, 669
 Weisskopf, M. C., Cohen, G. G., Kestenbaum, H. L., et al. 1976, *NASSP*, **389**, 81
 Weisskopf, M. C., Soffitta, P., Baldini, L., et al. 2022, *JATIS*, **8**, 2026002
 Wilms, J., Allen, A., & McCray, R. 2000, *ApJ*, **542**, 914
 Zane, S., Taverna, R., González-Caniulef, D., et al. 2023, *ApJL*, **944**, L27
 Zane, S., & Turolla, R. 2006, *MNRAS*, **366**, 727
 Zane, S., Turolla, R., Stella, L., & Treves, A. 2001, *ApJ*, **560**, 384
 Zhang, S., Santangelo, A., Feroci, M., et al. 2019, *SCPM*, **62**, 29502