

Dynamics of asteroidal Hopfield neural network under electromagnetic radiation and its application in the mechanical optimization design

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Abstract

When the neurons of the human brain transmit information through electrical and chemical signals, they will show a complex chaotic phenomenon if they are stimulated externally. Unlike the circular or fully connected Hopfield neural network models that are currently widely studied, this paper introduces a novel four-dimensional asteroidal Hopfield neural network (AHNN) model. The AHNN model incorporates a significant number of intentionally omitted synaptic weights in its architecture, making it more closely resemble the phenomenon of partial neuronal disconnection observed in real biological networks. This design enables the model to more accurately simulate the adaptive capacity of the brain's neural architecture, aiding in understanding the AHNN's ability to retain functionality amid partial structural damage and exploring how such changes affect the network's overall dynamics and stability. Besides, a memristor is used to simulate the effect of electromagnetic radiation on the network. The AHNN exhibits typical steady state or periodic state without external stimulus. When the electromagnetic radiation stimulation simulated by the memristor is added, the network exhibits various nonlinear dynamic behaviors, including but not limited to single-period, multi-period, chaos and transient chaos, which shows that electromagnetic radiation plays a significant role in regulating the dynamic behavior of neural networks. At this point, due to the need to transfer information between neurons, the synaptic weights between neurons change, which will also affect the dynamic behavior of AHNN. Based on this AHNN, a chaotic optimization algorithm was designed and applied to the mechanical optimization design problem of tension and compression springs and pressure vessels. Through the simulation of this problem and the comparison of the experimental results with some famous algorithms, the superiority of AHNN in solution quality was proved, providing a new biologically inspired idea for engineering optimization.

Keywords: Chaotic dynamics, Hopfield neural networks, Electromagnetic radiation, Chaotic optimization algorithm

1 Introduction

As the largest and most complicated structure in the central nervous system, the brain regulates body function through the vast network of around 86 billion neurons, and achieves advanced cognitive activities through synaptic plasticity [1–3], such as consciousness and memory. This information-processing mechanism has the characteristics of nonlinearity, distributed storage and self-organization. The electrochemical signals generated by a single neuron propagate in the neural network through the action potential, forming a cascade reaction. The disruption of this dynamic balance often leads to complex chaotic phenomena. Research about neuroscience indicates that the feedback loop and dynamic regulatory mechanism of synaptic weights in the cerebral cortex are the physical basis for memory formation and pattern recognition, providing biological inspiration for the design of artificial neural networks.

Neural networks, as a computational model that simulates the structure and function of the human brain, achieve the processing and output for the input data through the interconnection of a large number of nonlinear processing units [4]. Therefore, by constructing an artificial neural network similar to the human brain and simulating the activity mode of the human brain, we can research generalization and differentiation under emotion transfer in associative memory [5], and also further deepen understanding of the activity mechanism of the human brain which plays an important role in the analysis of certain brain diseases [6–8]. In addition, the issue of secure synchronization control in piecewise-homogeneous Markovian jump delay neural networks affected by denial-of-service attacks can be solved [9].

Li et al. proposed a biophysical neuron model with a double-layer membrane structure. The research found that this model could exhibit different discharge patterns under external stimuli and noise interference, and predicted mode selection and conversion through the energy function. This model provides a new perspective for understanding the adaptive regulation of biological neurons [10]. Ma explored how to define energy functions for discrete mappings and nonlinear oscillators to predict the changes in oscillation states, providing a certain theoretical basis for the design of digital circuits and logic control [11]. Ma also emphasized the importance of considering the physical properties of the cell membrane itself when constructing neural circuits and proposed a method for defining energy functions based on physical descriptions. By introducing specific electrical components (such as memristors, thermistors, etc.), the functional complexity of neural circuits has been enhanced, and their applications in controlling electromechanical equipment have been discussed [12]. The above research provides a feasible path for further constructing more realistic neural networks, which can even be added to the Hopfield neural network studied in this paper.

Hopfield neural network (HNN) is a well-known neural network proposed by Professor J. Hopfield of the California Institute of Technology in the United States in 1982 [13]. This kind of network has a unique loop structure and realizes the feedback connection from output to input. Memristors, with their nonlinearity, small size, low energy consumption and other characteristics, perfectly match the structural and functional requirements of biological devices. The integration of memristors into HNNs holds significance since memristors can simulate dynamic synaptic weight changes, improving self-organization and learning capabilities of the network. In addition, their nonlinear properties introduce chaotic behavior, which enriches the overall dynamical characteristics of the network. This enhanced functionality enables memristors to support more complex computational tasks within HNNs, such as chaotic encryption and pattern recognition, thus offering new directions for the future development and application of neural network technologies.

Up to now, the research on dynamics based on HNN has become more in-depth. Yao et al. constructed a four-dimensional HNN model, and introduced one or more memristors to simulate the influence of electromagnetic radiation, focusing on the chaotic dynamic behavior of HNN under the influence of electromagnetic radiation. This means that the dynamic behavior of the HNN system can be controlled by changing the number of external stimulus or the number of affected neurons in the HNN [14]. Lin et al. constructed a new HNN by simultaneously introducing two memristors into the HNN to act on the same neuron, in which one memristor was used to simulate the autosynapse of the neuron and the other memristor is used to describe the electromagnetic induction effect, and the dynamic behaviors of these two memory systems have been deeply studied [15]. Yu et al. proposed a novel memristor model, verified its characteristics through hysteresis curve, and based on this, constructed a four-dimensional fractional memristor HNN. The system is solved by using the Caputo fractional definition and the Adomian decomposition method. By adjusting the parameters and changing the order of HNN, combined with the bifurcation diagram and Lyapunov exponent analysis, rich dynamic behaviors such as transient chaos, chaos and coexisting attractors were observed [16].

Due to its outstanding performance in dealing with chaotic systems and nonlinear problems, HNN has been widely applied in various fields. In terms of image processing, HNN can be applied to image encryption applications, ensuring the security of sensitive images during storage and transmission, and enabling encrypted images to resist various attacks. Huang et al. simulated the external electromagnetic radiation effect through memristors and proposed a four-neuron memristive recurrent topology Hopfield neural network with dual self-loops. The medical image encryption algorithm based on this network achieve simultaneous permutation and diffusion at the same time, thereby enhancing overall computational efficiency. It has good encryption performance and effectively meets the security requirements for the storage and transmission of medical images[17]. Ding et al. integrated two memristors into a classical HNN to construct a chaotic memristive neural network capable of generating hidden multi-wing chaotic attractors. Based on this structure, the authors further designed the hidden multiwing memristive neural network encryption algorithm and applied it to the remote sensing system to achieve encryption and decryption of remote sensing images, providing a highly secure solution for the transmission and storage of sensitive spatial data[18].

In terms of associative memory, the HNN can store multiple patterns in a distributed manner and dynamically reconstruct the complete memory through partial inputs, giving it unique advantages in memory storage and dynamic association. Zhang et al. replaced the nodes of the traditional Hopfield network with underdamped stochastic resonance nodes. This stochastic Hopfield network not only retains the associative memory function of the traditional HNN, but also introduces the stochastic resonance mechanism, which enhances the sensitivity and robustness of weak signal processing. Moreover, it can adaptively adapt to the detection of bearing faults at variable speeds, providing a reliable solution for industrial bearing health monitoring[19].

HNNs also perform well in the field of solving optimization problems, among which the most representative is the solution of the Traveling Salesman Problem (TSP). TSP requires finding the shortest path to visit all cities and return to the starting point, which is an NP-hard problem. Hopfield and Tank first proposed using the continuous HNN to solve TSP in 1985 [20], opening up a new way for neural networks to solve combinatorial optimization problems.

In biological neural networks, not all synaptic connections are consistently active or structurally present. The intentional omission or reduction of synaptic weights can simulate the natural variability of biological neural connections as well as the disruptions caused by external factors. The loss of synaptic weights can also represent the effects of synaptic pruning, a process where the brain eliminates weak or unused synaptic connections to optimize neural efficiency. This design allows the model to more accurately simulate the adaptive capacity of the brain's neural architecture, it can help us understand the ability of the HNN to maintain its functionality when facing partial structural damage and explore how such structural changes can influence the overall dynamics and stability of the network.

However, most of the existing studies focus on annular HNN or fully connected HNN and their dynamic behaviors [14–16, 21–23], HNN models with mostly missing synaptic weights and its dynamic behaviors have rarely been studied, leading to limitations in his practical application scope and performance possibly. For instance, the application scenarios of HNN show a distinct concentration: most studies focus on its application in the field of image encryption, relying on its nonlinear dynamic characteristics and associative memory function to provide technical support for image data security. In contrast, research on the application of HNN in industrial scenarios such as bearing quality inspection[19] is relatively rare. Moreover, among the existing related explorations, the HNN used for bearing quality inspection has undergone structural optimization rather than a traditional infrastructure. This phenomenon indicates that making targeted adjustments to the structure of HNN is the key path to breaking through its application boundaries and expanding its practical application fields.

Therefore, this paper innovatively proposes a brand-new asteroidal HNN (AHNN) without some synaptic weights under the influence of electromagnetic radiation, analyzes its dynamic behavior, and applies it to two types of mechanical design optimization problems [24, 25]. By comparing with many algorithms [24–38], the optimization algorithm based on this model has higher accuracy and provides a more feasible solution for engineering optimization design.

The remaining part of this article is organized as follows. Section 2 establishes the mathematical model of AHNN and analyzes the stability of its equilibrium point. In Section 3, the dynamic analysis of the AHNN model is conducted using MATLAB R2023a. In Section 4, the AHNN model is applied to mechanical optimization design. In Section 5, Multisim is used for circuit simulation to further verify the experimental results. Finally, we summarize this article in Section 6.

2 The AHNN model establishment

2.1 The mathematical model

The AHNN model has a grid structure similar to the real biological nervous system and can generate dynamic phenomena similar to brain chaos, making it a simplified biological nervous system model.

This paper uses the electromagnetic radiation (EMR) simulated by the memristor model as an external stimulus to construct an AHNN consisting of four neurons. Based on the characteristics of the mathematical model of the memristor and Ohm's law, we can derive the flux-controlled memristor model as shown in **Eq. 1**.

$$\begin{cases} I = W(\varphi)V \\ W(\varphi) = \rho(\alpha + 3\beta\varphi^2) \\ \frac{d\varphi}{dt} = \mu V - \varepsilon\varphi, \end{cases} \quad (1)$$

where $W(\varphi)$ denotes the memconductance of the memristor with coupling strength ρ and parameters α and β . The term μV is the changes of magnetic flux caused by the membrane potential, and the term $\varepsilon\varphi$ is the leakage of magnetic flux, μ and ε are used as the regulatory parameters for the variation of magnetic flux and the leakage of magnetic flux. Applying sine excitation to the memristor model to obtain **Eq. 2**:

$$\begin{cases} V = A \sin(F \cdot t) \\ I = \rho(\alpha + 3\beta\varphi^2)V \\ \dot{\varphi} = \mu V - \varepsilon\varphi, \end{cases} \quad (2)$$

where V , I and φ respectively represent the membrane potential, terminal current, and internal magnetic flux flowing through the memristor. A and F denote the amplitude and frequency of the sinusoidal stimulus, respectively. When the parameters are fixed as $\alpha=3.1$, $\beta=1$, $\rho=1.5$, $\mu=1$, $\varepsilon=1$ and $A=1$, $F=1$; 3; 6; respectively. The unique nonlinear feature of memristors is the presence of a tilted "8"-shaped hysteresis loop, as shown in Fig. 1. The area of the hysteresis loop decreases with the increase of voltage frequency F . When $F \rightarrow \infty$, the hysteresis loop is close to straight line.

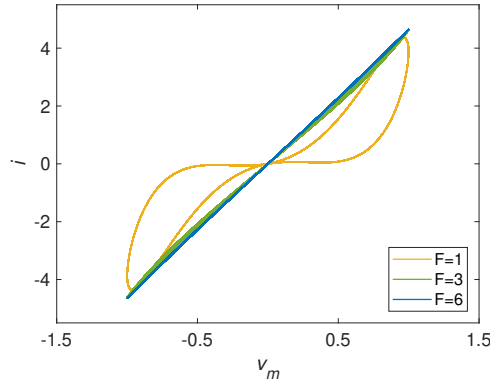


Fig. 1 Hysteresis loops of the memristor model at $A=1$, $F=1$; $F=3$; $F=6$

The expression for the universal HNN model[39] is:

$$C_i \dot{x}_i = -x_i/R_i + \sum_{j=1}^n W_{ij} \tanh(x_j) + I_{Bi} + I_{Ei} (i, j \in N^*), \quad (3)$$

where C_i and R_i represent the capacitance and resistance inside and outside the cell membrane of neuron N_i , x_i and I_{Bi} are the membrane-voltage and bias current of neuron N_i , W_{ij} is defined as the synaptic weight between neuron N_i and neuron N_j , and I_{Ei} represents the electromagnetic induced current caused by external EMR.

Setting $C_i = 1$, $R_i=1$, $I_{Bi}=0$, the following model has been designed:

$$\begin{cases} \dot{x}_1 = -x_1 + W_{21} \tanh(x_2) - 8 \tanh(x_3) \\ \quad + 1.5 \tanh(x_4) + 1.5(c - a\varphi^2)x_1 \\ \dot{x}_2 = -x_2 + 2.65 \tanh(x_1) + 1.2 \tanh(x_2) \\ \dot{x}_3 = -x_3 - 0.7 \tanh(x_1) + W_{33} \tanh(x_3) \\ \dot{x}_4 = -x_4 - 2.3 \tanh(x_1) + \tanh(x_4) \\ \dot{\varphi} = 1.91x_1 - 0.45\varphi. \end{cases} \quad (4)$$

Neuron N_1 is connected to the other three neurons which are not connected with each other. In section 3.1, none of the four neurons are stimulated by EMR. In section 3.2, the EMR model affects the neuron N_1 . In order to more intuitively represent the structure of the four-dimensional, the physical model of AHNN is shown in Fig. 2.

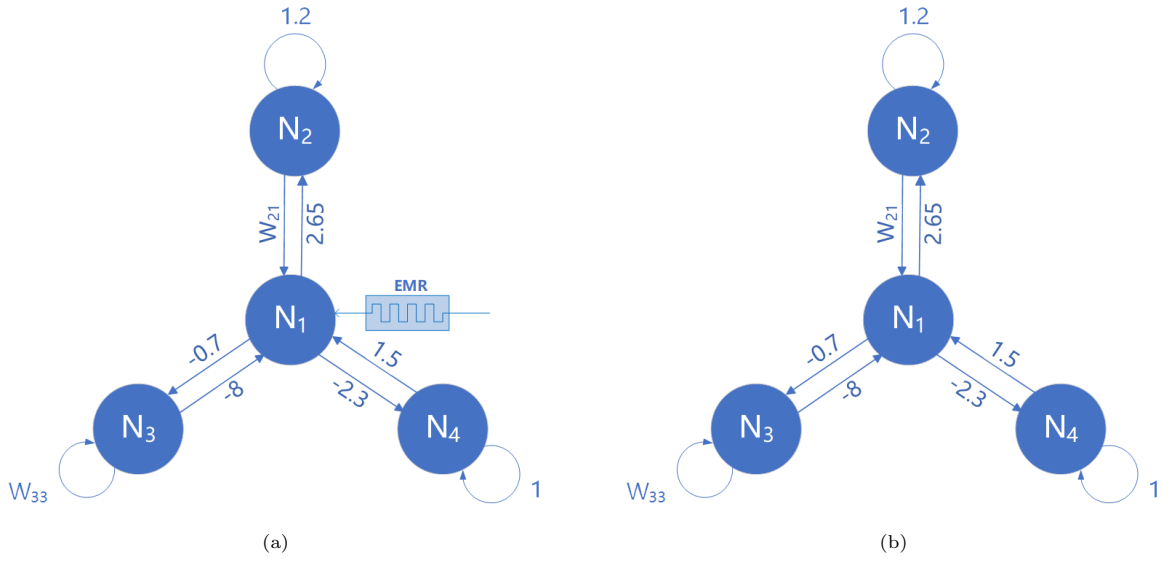


Fig. 2 AHNN model. (a) with EMR. (b) without EMR

2.2 Stability analysis

This article uses a combination of numerical and graphical methods to analyze the stability of the AHNN model. Letting the left-hand side of **Eq. 4** be 0, $W_{33} = -20$, $W_{21} = -1.7$ and $c=3.1$, we have

$$\begin{cases} 0 = -x_1 - 1.7 \tanh(x_2) - 8 \tanh(x_3) \\ \quad + 1.5 \tanh(x_4) + 1.5(3.1 - a\varphi^2)x_1 \\ 0 = -x_2 + 2.65 \tanh(x_1) + 1.2 \tanh(x_2) \\ 0 = -x_3 - 0.7 \tanh(x_1) - 20 \tanh(x_3) \\ 0 = -x_4 - 2.3 \tanh(x_1) + \tanh(x_4) \\ 0 = 1.91x_1 - 0.45\varphi. \end{cases} \quad (5)$$

It can be inferred that:

$$\begin{cases} \tanh(x_1) = (x_2 - 1.2 \tanh(x_2))/2.65 \\ \tanh(x_1) = (-x_3 - 20 \tanh(x_3))/0.7 \\ \tanh(x_1) = (-x_4 + \tanh(x_4))/2.3. \end{cases} \quad (6)$$

We define:

$$\begin{cases} y_1 &= (x_2 - 1.2 \tanh(x_2))/2.65 \\ y_2 &= (-x_3 - 20 \tanh(x_3))/0.7 \\ y_3 &= (-x_4 + \tanh(x_4))/2.3. \end{cases} \quad (7)$$

The image of the above equation set can be drawn as shown in Fig. 3.

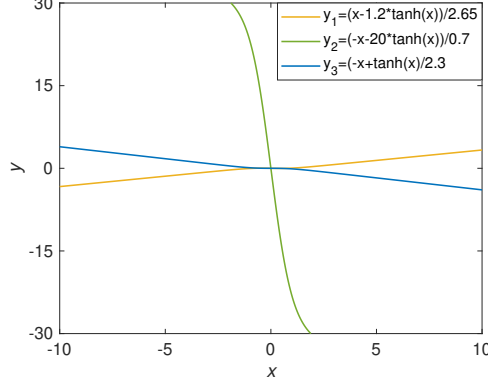


Fig. 3 Stabilization point of AHNN

Obviously, y_1 , y_2 , and y_3 are all monotonic functions, so y_1 , y_2 , and y_3 have only one intersection point, which is $(0, 0, 0)$. Substituting $x_1 = 0$, $x_2 = 0$, and $x_3 = 0$ to **Eq. 5**, we can know that when the system of **Eq. 5** takes values $(0, 0, 0, 0, 0)$, the AHNN is in a stable state.

3 Dynamic analysis

This section aims to analyze the rich dynamic behavior exhibited by the four-dimensional AHNN model without and with an external stimulus. In the research process, we used various mathematical methods and graphs such as Lyapunov exponent, bifurcation diagram, phase diagram and time-domain diagram to perform dynamic analysis on the AHNN.

In bifurcation diagrams, step size is 0.01. The amount of iterations is 2,000. To prevent potential interference from initial transients that may affect the accurate characterization of dynamics, the initial 1,000 iterations were excluded from the analysis. In Lyapunov exponent diagrams, step size of a , c , W_{21} and W_{33} is 0.05. In phase diagrams and time-domain diagrams, step size is 0.01, the amount of iterations is 100,000. And the first 10,000 iterations are omitted in phase diagrams. We simulated each of the above features using MATLAB R2023a.

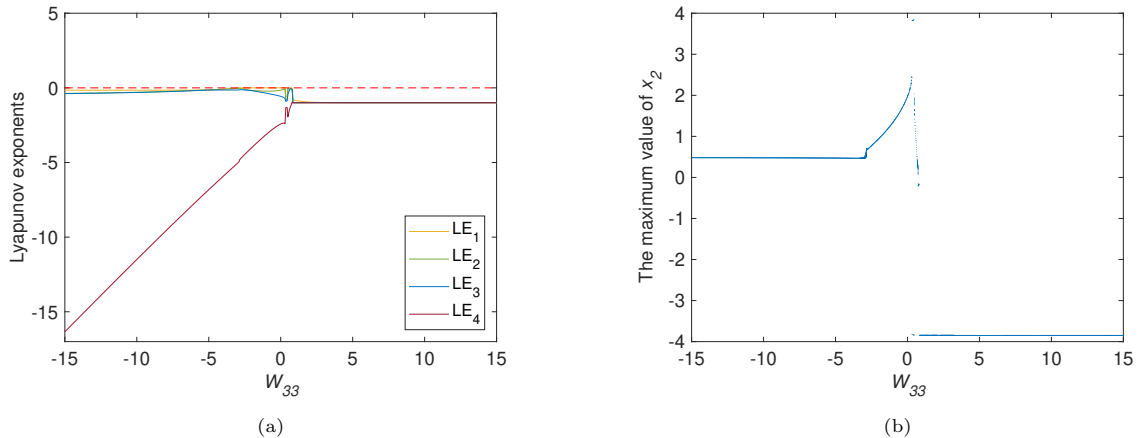


Fig. 4 The W_{33} -dependent dynamics without EMR. (a) Lyapunov exponent diagram. (b) Bifurcation diagram

3.1 AHNN without EMR

Without external stimulation, there are periodic and stable phenomena. When no external stimulus are added, the topology of the AHNN model is shown in Fig. 2(b).

Taking the self synaptic weight W_{33} of neuron N_3 as a parameter and setting $W_{21}=-1.7$, a Lyapunov exponent diagram is drawn using W_{33} as the parameter in Fig. 4(a). Combined with bifurcation diagram in Fig. 4(b), we explore the dynamic behavior of AHNN without EMR.

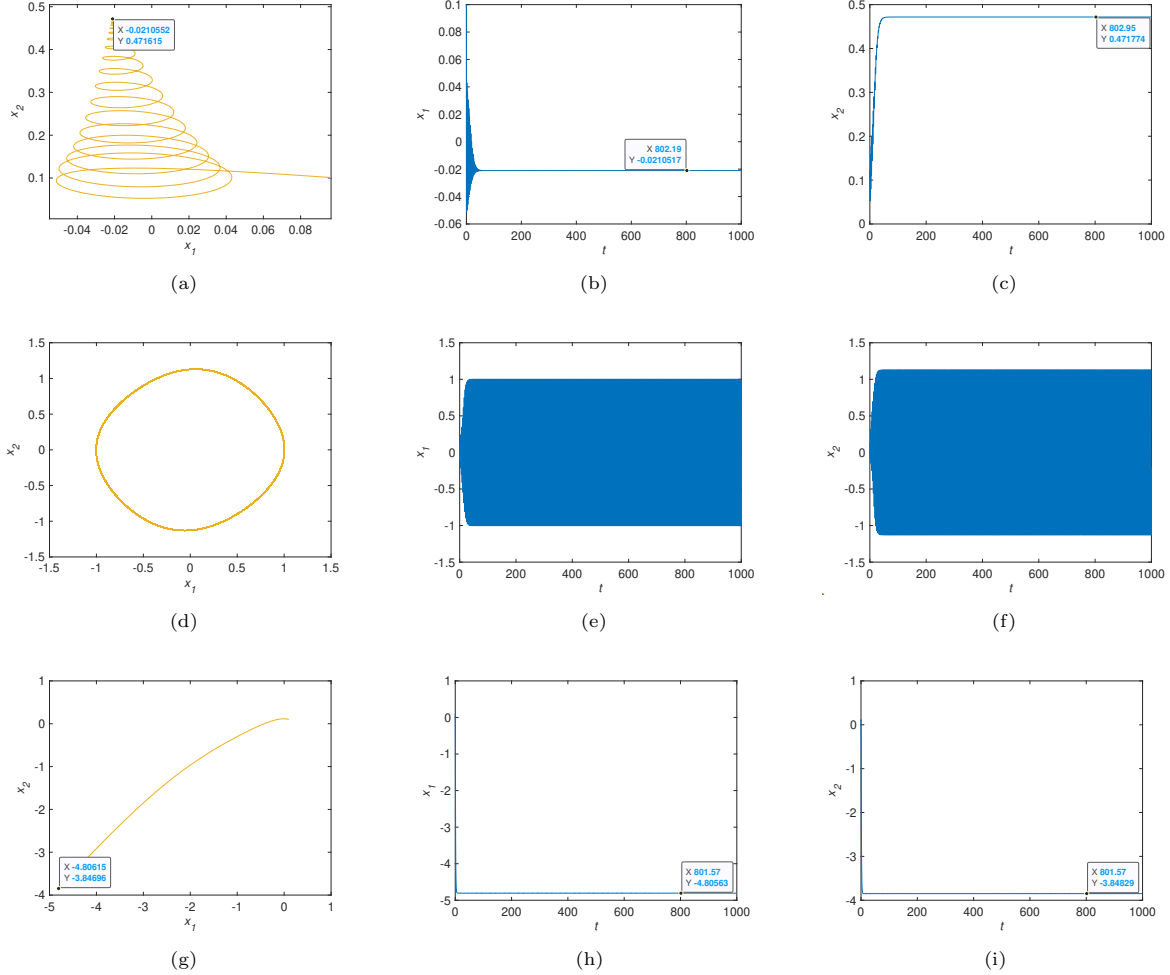


Fig. 5 Phase diagram of x_1 - x_2 and time-domain diagrams of x_1 and x_2 when W_{33} takes different value. When $W_{33}=-5$, (a) Phase diagram, (b) Time-domain diagram of x_1 , (c) Time-domain diagram of x_2 . When $W_{33}=-1.5$, (d) Phase diagram, (e) Time-domain diagram of x_1 , (f) Time-domain diagram of x_2 . When $W_{33}=5$, (g) Phase diagram, (h) Time-domain diagram of x_1 , (i) Time-domain diagram of x_2 .

When $W_{33}=-5$, the spiral shape can be clearly observed from the x_1 - x_2 phase diagram of Fig. 5(a). This spiral morphology indicates that the system exhibits a convergent trend in the phase space. Further observation of the time-domain graphs of x_1 and x_2 in Fig. 5(b) and Fig. 5(c) can provide a more intuitive understanding of the process of the system over time. In the time-domain graph, the values of x_1 and x_2 in the initial stage fluctuate greatly and show an unstable state. This might be due to the system being disturbed by some kind of disturbance or being affected by its own initial conditions at the initial moment. However, as time goes by, the fluctuation amplitude gradually decreases and eventually tends to a stable value. This indicates that after going through a transient process, the AHNN model can transition from the initial unstable state to a stable state, demonstrating the system's ability to self-regulate and tend towards stability.

When $W_{33}=-1.5$, a comprehensive observation of the images in Fig. 5(d), Fig. 5(e), and Fig. 5(f) reveals that the x_1 - x_2 phase diagram exhibits the characteristics of period-1. This periodic behavior indicates that the system is in a regular and stable oscillation state, and its dynamic characteristics have good repeatability and predictability.

When $W_{33}=5$, the x_1 - x_2 phase diagram in Fig. 5(g) is shown as a curve, and this curve eventually tends towards a point. This phenomenon suggests that during the evolution of the system, the changes of state variables x_1 and x_2 gradually converge and eventually stabilize at a specific state point. By combining the time-domain graphs of x_1 and x_2 in Fig. 5(h) and Fig. 5(i), it can be further inferred that within a certain error range, the AHNN model is in a balanced state. In a balanced state, the state variables of the system no longer undergo significant changes over time and remain relatively stable.

Furthermore, all the dynamic behaviors mentioned above are highly consistent with the phenomena presented by the Lyapunov exponent diagram and the bifurcation diagram. This consistency not only verifies the reliability of the observed dynamic phenomena, but also indicates that the AHNN model has strong stability in the absence of external stimulus. This stability enables the model to maintain its basic dynamic characteristics when confronted with minor fluctuations in internal parameters, and it is less likely to undergo drastic state changes, providing a solid foundation for the practical application of the model.

3.2 AHNN with EMR

When external stimulus are added, the topological structure of the AHNN model is shown in Fig. 2(a). The dynamic behavior of AHNN is very rich when stimulated by EMR. In this section, we respectively investigate the influence of synaptic interaction and EMR intensity on the dynamic behaviors of AHNN. In section 3.2.1, the synaptic weight W_{21} from neuron N_2 to neuron N_1 is selected as the variable. In section 3.2.2, parameters a and c that regulate the intensity of EMR are respectively selected as variables.

3.2.1 The influence of synaptic weights on AHNN

In the human brain, neurons are interconnected through synapses, and the synaptic weights between neurons reflect the intensity of their mutual influence. When neurons need to exchange information with each other, synaptic weights will change accordingly, thereby affecting the dynamic behavior of whole neural networks. Therefore, delving deeply into their regulatory mechanisms on neural network functions is a key research direction to reveal the principles of brain information processing and the construction of bionic neural network models, which holds significant theoretical and applicable value. In order to research

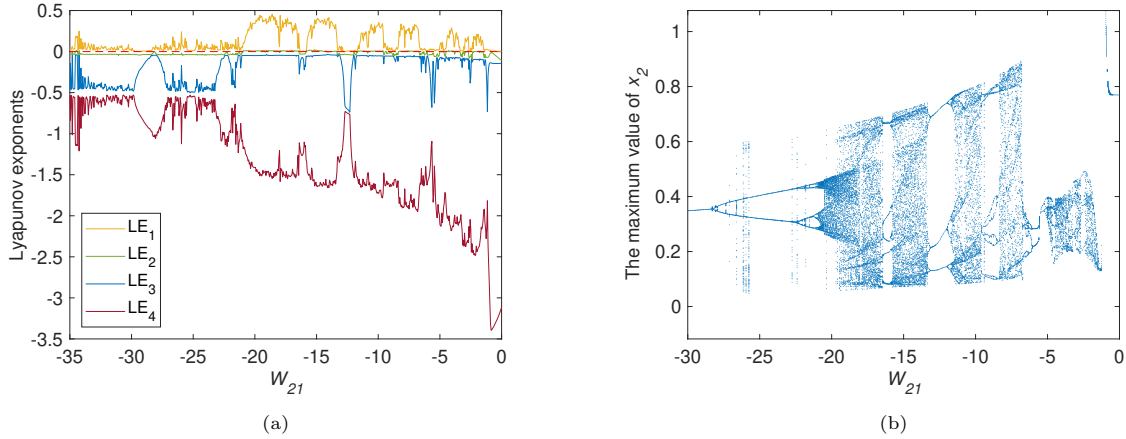


Fig. 6 The W_{21} -dependent dynamics when neuron N_1 is stimulated by EMR. (a) Lyapunov exponent diagram. (b) Bifurcation diagram

conveniently, a and c are set to 15 and 3.1, respectively. Taking the synaptic weight W_{21} as the variable, the Lyapunov exponent diagram and bifurcation diagram of the system are plotted in Fig. 6. It can be seen from the figures that as W_{21} increases, the system shows an evolutionary trend of gradually transitioning from single-period behavior to a chaotic state. What is particularly crucial is that this transition process is not a simple linear transformation: within the periodic state interval, there are local interweaving of chaotic phenomena. After entering the chaotic state, it is accompanied by the intermittent occurrence of periodic behavior, forming a complex dynamic feature of alternating and nested "periodic - chaos". This phenomenon fully demonstrates that the subtle changes in synaptic weights have a significant regulatory effect on the dynamic behavior of neural networks, and also indirectly confirms: When neurons conduct

signal transmission and information interaction, the brain's processing of information is not in a single mode but presents complex characteristics of multi-stability and nonlinearity, reflecting the flexibility and complexity of dynamic regulation in biological neural networks.

Further analysis was conducted by taking different values of W_{21} to draw phase diagrams of x_1 - x_2 in Fig. 7. It can be seen that there are six attractors in AHNN, namely period-1, period-2, period-3, period-4, and two different types of chaos. Moreover, this is consistent with the previous mention of the existence of chaos within periods and the existence of periods within chaos, further confirming the complexity of the brain's overall response to information processing.

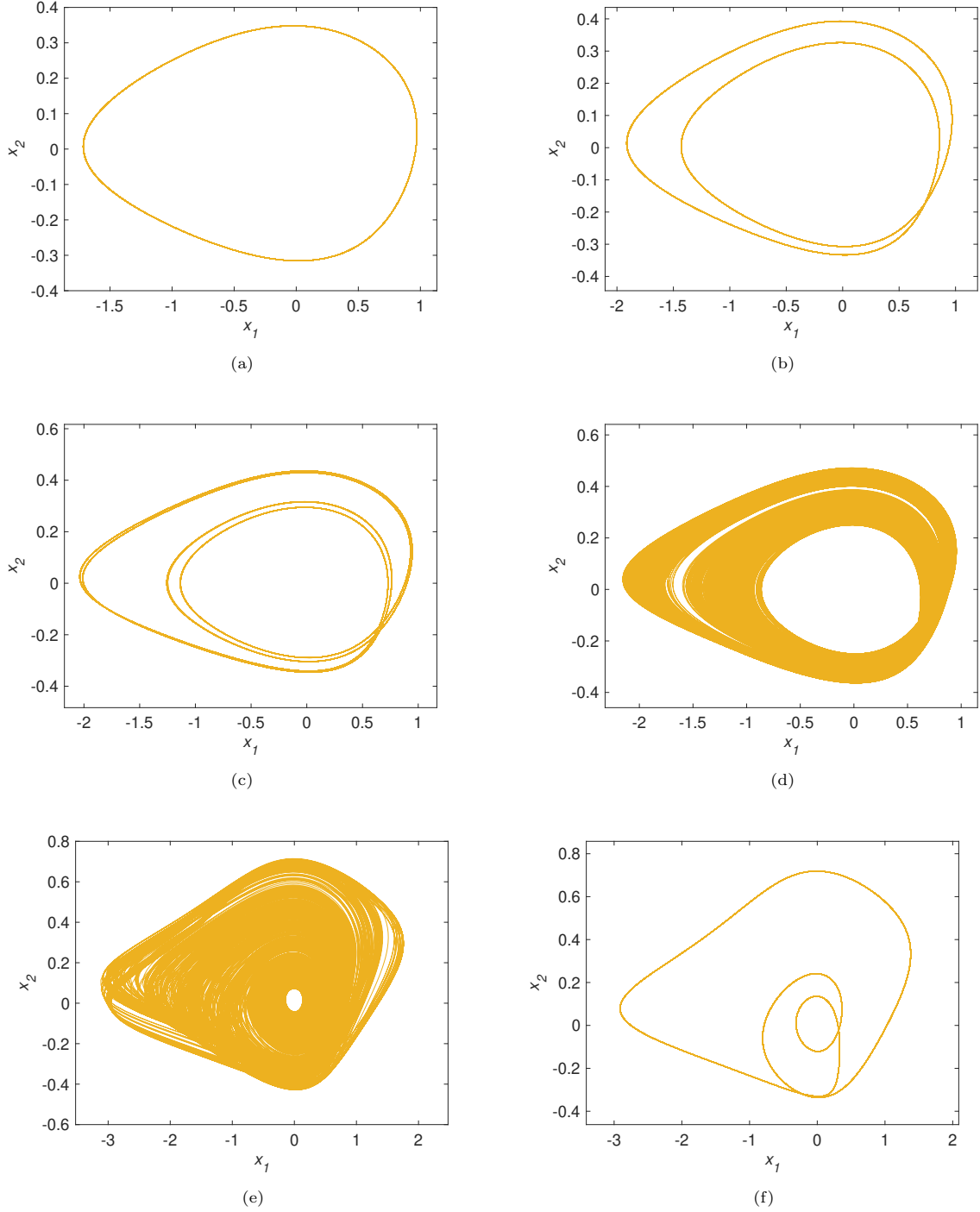


Fig. 7 Phase diagrams of AHNN under different value of W_{21} when $a=15$ and $c=3.1$. (a) $W_{21}=-30$. (b) $W_{21}=-26$. (c) $W_{21}=-22$. (d) $W_{21}=-20$. (e) $W_{21}=-15$. (f) $W_{21}=-12.5$

3.2.2 The influence of EMR on AHNN

Since the memristor model plays a significant role in simulating EMR, the values of its various parameters will affect the specific performance of EMR. Therefore, it is important to discuss how nonlinear dynamics vary with the parameters of the memristor. With the addition of external stimulus caused by EMR, this AHNN exhibits a rich array of dynamic phenomena.

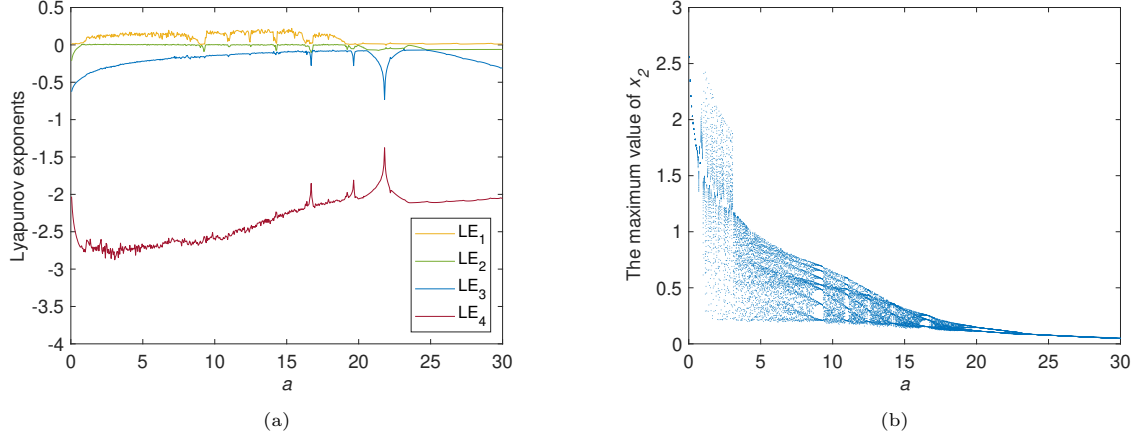


Fig. 8 The a -dependent dynamics when neuron N_1 is stimulated by EMR. (a) Lyapunov exponent diagram. (b) Bifurcation diagram

Case 1. a is chosen as the variate. W_{21} and c are set to -1.7 and 3.1. Since a does not exhibit dynamic behavior when taking a negative value, we mainly focus on the dynamic behavior of the network when a is positive. Let the memristor parameter a in **Eq. 4** be a variable parameter with a value range of $[0, 30]$. The initial value for $(x_1, x_2, x_3, x_4, \varphi) = (0.1, 0.1, 0.1, 0.1, 0.1)$ and $W_{33} = -20$. Four Lyapunov exponents and bifurcation diagram of AHNN are obtained, as shown in Fig. 8(a) and Fig. 8(b).

From the Lyapunov exponent diagram and bifurcation diagram in Fig. 8(a) and Fig. 8(b), it can be seen that when the network is subjected to external stimulus, it generates quite rich dynamic behaviors. When a gradually increases from 0, the system will briefly experience period-1 and transient chaos, and then enter a distinct chaotic state in the range $[0.85, 19.1)$. Within this range, there are a great number of points in the bifurcation graph and the Lyapunov exponent satisfies $(+, 0, -, -)$, proving that they are precisely matched. At this time, the system is indeed chaotic. When $a \in [19.1, 20.1)$, the system is in period-4; when $a \in [20.1, 23.6)$, it enters period-2; and after a is more than 23.6, the system is in period-1 and remains so.

To gain a more direct understanding of the dynamic behavior of the network, phase diagrams of x_1 - x_2 of the network based on different values of a are presented respectively in Fig. 9. The results show that there are four different attractors in AHNN: when $a = 25, 22$, and 19.5 , the AHNN is in period-1, period-2, and period-4, respectively. When $a = 15$, there is a significant chaotic phenomenon.

Case 2. Given that the intensity of EMR is regulated by two parameters in coordination, to independently explore the influence of parameter c on AHNN, this study fixed $W_{21} = -1.7$, $a = 15$ and conducted analysis by drawing Lyapunov exponential diagram and bifurcation diagram in Fig. 10.

The results show that compared with the situation where parameter a regulates the intensity of EMR, the influence of parameter c on AHNN is significantly weaker, and the induced dynamic behavior is also more simple: as c increases, AHNN only briefly enters the chaotic state from the stable state and then rapidly returns to stability.

The cause of this difference can be explained from the physical mechanism level: Although c directly affects the memristive characteristics of the memristor, a directly determines the flow intensity of the internal magnetic flux and the magnetic flux itself is in a dynamic state of change in the network. The regulation of a will increase the complexity of this change, thereby making the influence of EMR on the network present more abundant nonlinear characteristics. In contrast, c mainly exerts its effect by directly altering the EMR intensity, with a more single regulatory path, and thus its regulatory effect on network dynamics is more limited.

It can be seen that the intensity of EMR has a significant regulatory effect on the dynamic behaviors of the AHNN. In addition, there are significant differences on the dynamic behaviors of AHNN when

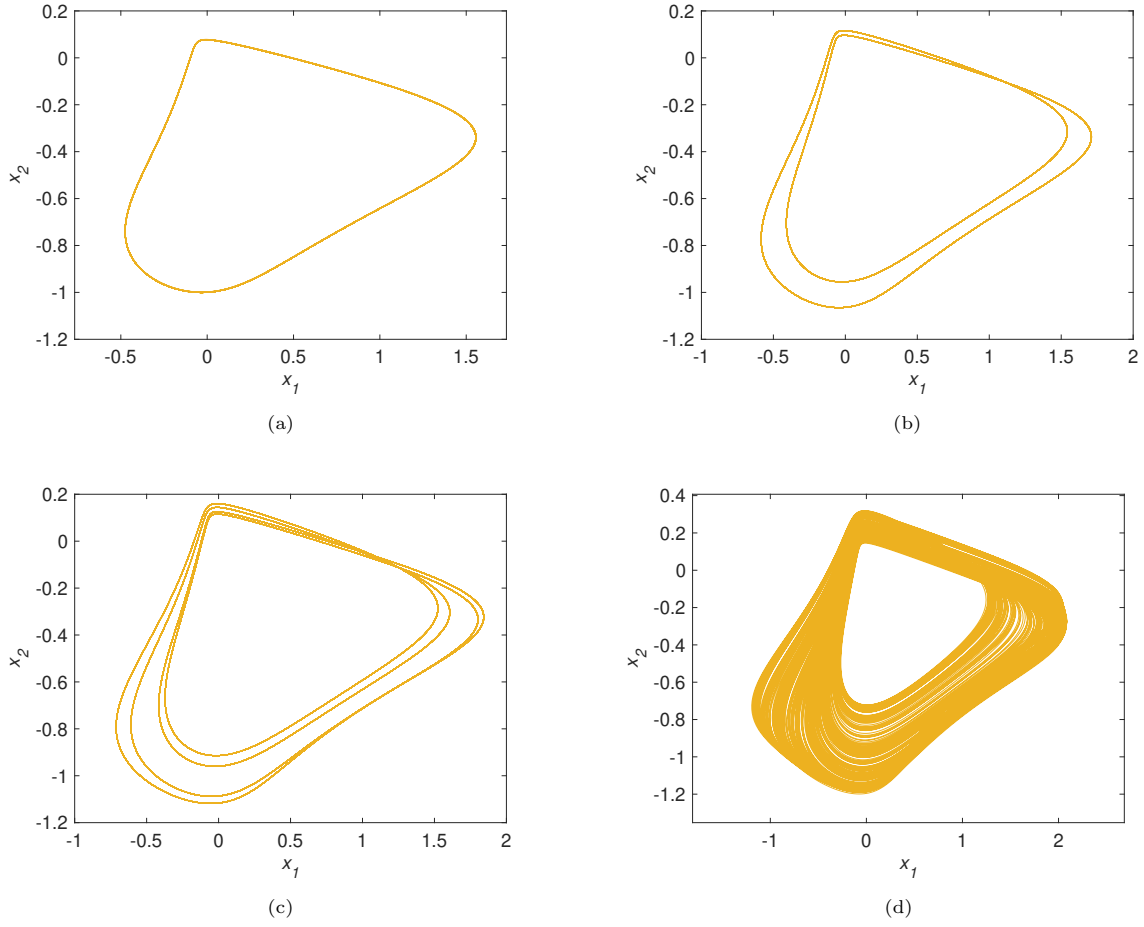


Fig. 9 Phase diagrams of AHNN under different value of a when $W_{33}=-20$. (a) $a=25$. (b) $a=22$. (c) $a=19.5$. (d) $a=15$

choosing different parameters in EMR. Specifically, different EMR-related parameters will induce AHNN to exhibit different dynamic evolution patterns, resulting in significant differences in the richness of its dynamic behavior and transition rules.

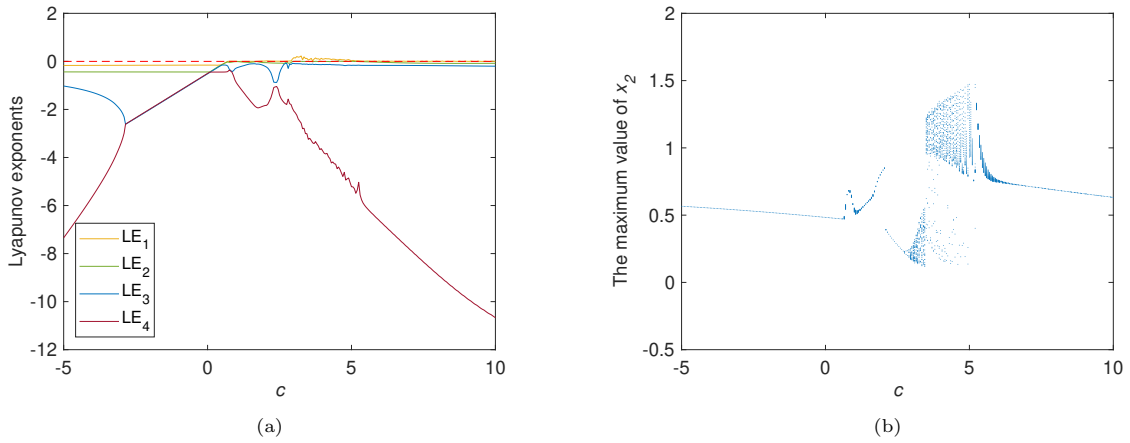


Fig. 10 The c -dependent dynamics when neuron N_1 is stimulated by EMR. (a) Lyapunov exponent diagram. (b) Bifurcation diagram

4 Circuit verification

We implement AHNN mathematical models through discrete electronic components [40–42], enabling theoretical models to be constructed in real life and demonstrating the feasibility of the proposed solutions. It is noted that in the following v_1, v_2, v_3, v_4, v_5 are used to replace $x_1, x_2, x_3, x_4, \varphi$ in **Eq. 4** for the ease of circuit design. The circuit model for AHNN is given in **Eq. 8**.

$$\begin{cases} RC \frac{dv_1}{dt} = -v_1 - \frac{R}{R_1} \tanh(v_2) - \frac{R}{R_2} \tanh(v_3) \\ \quad + \frac{R}{R_3} \tanh(v_4) + R(\frac{1}{R_4} - \frac{v_5^2}{R_5})v_1 \\ RC \frac{dv_2}{dt} = -v_1 + \frac{R}{R_6} \tanh(v_1) + \frac{R}{R_7} \tanh(v_2) \\ RC \frac{dv_3}{dt} = -v_1 - \frac{R}{R_8} \tanh(v_1) - \frac{R}{R_9} \tanh(v_3) \\ RC \frac{dv_4}{dt} = -v_1 - \frac{R}{R_{10}} \tanh(v_2) + \frac{R}{R_{11}} \tanh(v_4) \\ RC \frac{dv_5}{dt} = \frac{R}{R_{12}}v_1 - \frac{R}{R_{13}} \tanh(v_5). \end{cases} \quad (8)$$

Assuming $RC=1\text{ms}$ and $R=10\text{k}\Omega$, the capacitance $C=100\text{nF}$. We can calculate the resistance in **Eq. 8**: $R_1=5.8824\Omega$, $R_2=1.2500\text{k}\Omega$, $R_3=6.6667\text{k}\Omega$, $R_4=2.1505\text{k}\Omega$, $R_6=3.7736\text{k}\Omega$, $R_7=8.3333\text{k}\Omega$, $R_8=14.2857\text{k}\Omega$, $R_9=0.5000\text{k}\Omega$, $R_{10}=4.3478\text{k}\Omega$, $R_{11}=10\text{k}\Omega$, $R_{12}=5.2356\text{k}\Omega$, $R_{13}=22.2222\text{k}\Omega$, while R_5 serves as a variable parameter.

The circuit diagrams are drawn by Multisim as shown in Fig. 11 and Fig. 12. The voltages $-V_a, -V_b, -V_c, -V_d$ in the figure are the outputs of the respective circuit module. Within a certain margin of error, the results obtained from simulating the dynamic behaviors of AHNN using Multisim shown in Fig. 13 is consistent with those obtained from MATLAB simulation, which verifies the reliability of the AHNN model and also indicates that the model has good hardware feasibility.

This consistency indicates that the complex dynamic behavior of AHNN can be simulated not only theoretically through mathematical models, but also through actual electronic circuits, which is of great significance for the fields of neuroscience and bioelectronics.

5 Application in mechanical engineering

5.1 Complexity of chaotic sequence generated from AHNN affected by EMR

Complexity algorithms include sample entropy (SamEn) [43], permutation entropy (PE) [44], and spectral entropy (SE) [45, 46]. In this paper, the SE algorithm is used to calculate the complexity of initial control chaotic sequences, which can estimate the complexity of sequences quickly and accurately.

Similarly, take a in the external stimulus as the variable parameter in the region $[0,30]$, with the initial conditions being $(x_1, x_2, x_3, x_4, \varphi)=(0.1, 0.1, 0.1, 0.1, 0.1)$, and $W_{33}=-20$. The SE value in Fig. 14 varies with the variation of parameter a . It is worth mentioning that the complexity of the model when it is in a chaotic state is higher than that in a periodic state. Moreover, the complexity spectrum obtained by the SE algorithm is consistent with the bifurcation diagram and the Lyapunov exponent shown in Fig. 8(a) and Fig. 8(b), further demonstrating the effectiveness of this algorithm in the analysis of nonlinear dynamics.

5.2 A novel chaos optimization algorithm

Chaotic systems have typical characteristics such as ergodicity, regularity, and randomness. Within a certain range, chaotic behavior can pass through all possible state points without repetition. This characteristic enables chaos-based optimization methods to systematically explore the entire solution space, thereby increasing the probability of obtaining high-quality solutions. Meanwhile, due to the unpredictability of chaotic motion, this type of algorithm can achieve global search of the solution space and effectively avoid the local optimum trap.

The AHNN model adopted in this study is capable of generating rich chaotic sequences. It is confirmed through the analysis of the SE algorithm that these sequences have significant nonlinear characteristics and high complexity, fully meeting the basic conditions for constructing chaotic optimization algorithms. We can use the framework of the chaotic optimization algorithm proposed by Zhang et al. [38] to solve the problem of mechanical optimization design.

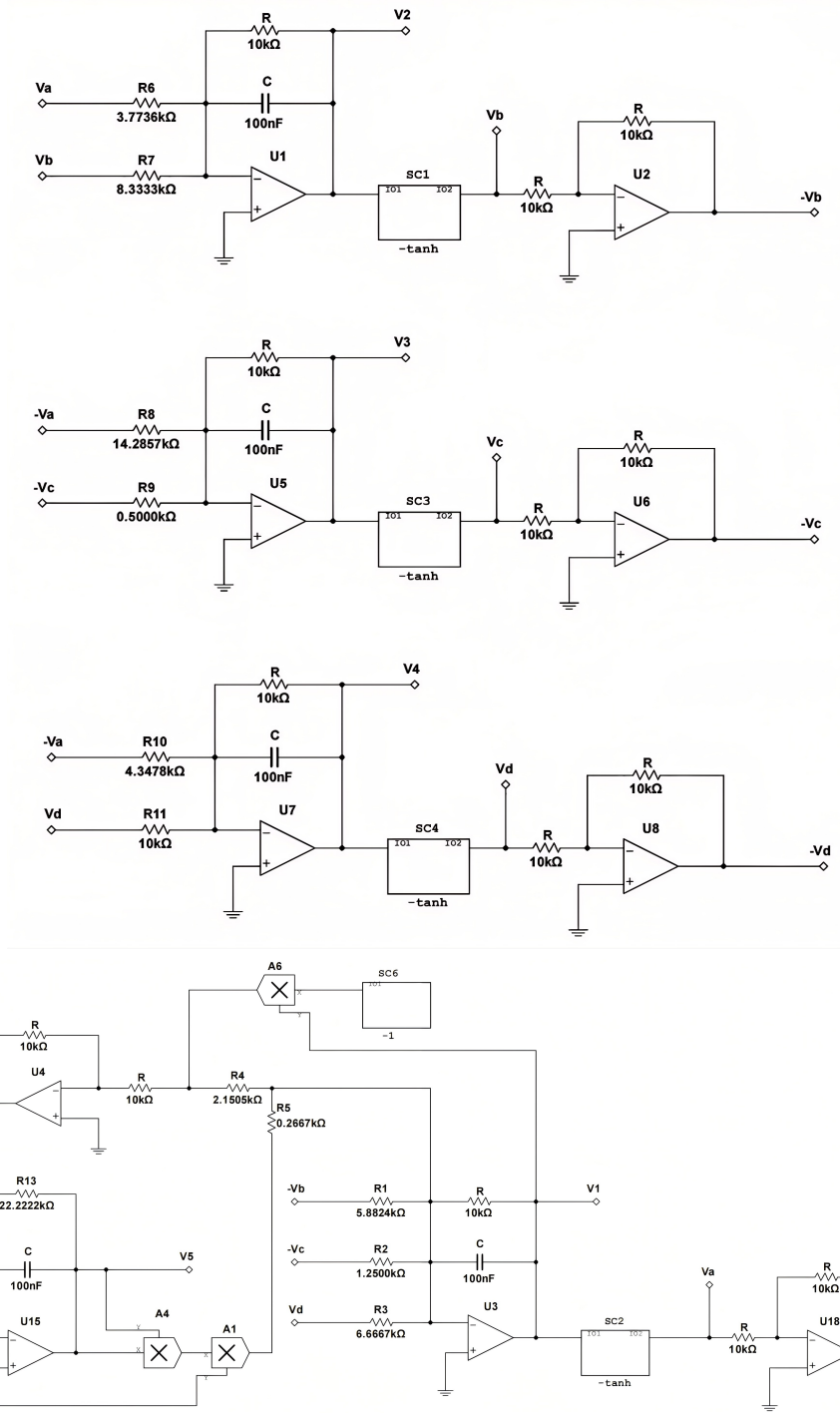


Fig. 11 The proposed AHNN circuit schematic

5.3 Examples of the application of mechanical optimization design

5.3.1 The design of tension/compression coil springs

This paper examines an optimization design problem related to tension/compression coil springs. The problem was first introduced by Belegundu [26], and the corresponding schematic diagram was provided by Arora [24]. The optimization process involves three continuous design variables: the number of spring coils N , the winding diameter D , and the wire diameter d . The design variable vector x can be expressed as follows:

$$x = (x_1, x_2, x_3)^T = (N, D, d)^T, \quad (9)$$

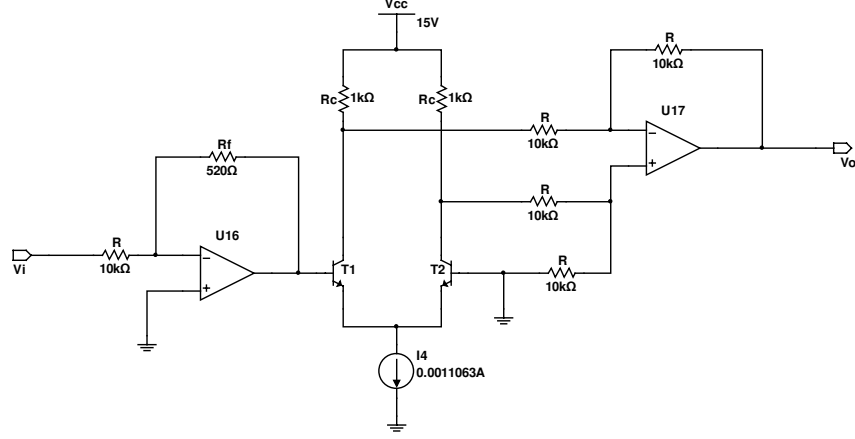


Fig. 12 Circuit implementation of hyperbolic sinusoidal function

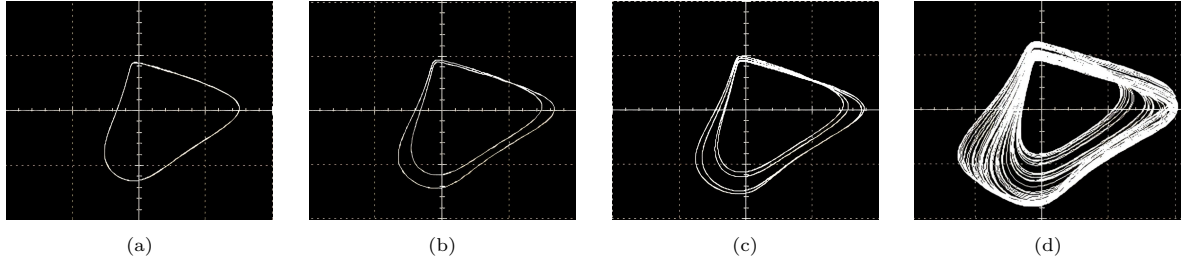


Fig. 13 Dynamic behaviors of AHNN simulated by Multisim. (a) When $R_5=0.2667\text{k}\Omega$, exhibits period-1. (b) When $R_5=0.3030\text{k}\Omega$, exhibits period-2. (c) When $R_5=0.3350\Omega$, exhibits period-4. (d) When $R_5=0.4444\text{k}\Omega$, exhibits chaotic phenomenon

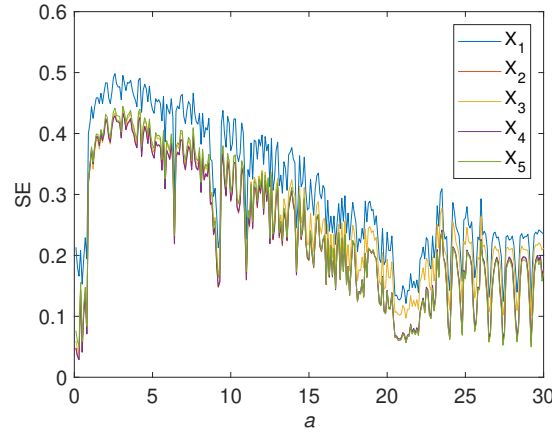


Fig. 14 The SE complexity spectrum for $a \in [0, 30]$ in the AHNN

where x_1 , x_2 , and x_3 correspond to N , D , and d respectively. The goal of the optimization is to reduce the spring's weight, subject to constraints on minimum deflection, shear stress, surge frequency, and outside diameter, which are constrained in the inequality $h(\mathbf{x})$. The mathematical formulations for the objective function $f(\mathbf{x})$ and the constraint inequality $h(\mathbf{x})$ are detailed in **Eqs. 10-11** [24, 35].

$$\min f(x) = (x_1 + 2)x_2x_3^2. \quad (10)$$

Subject to:

$$\begin{cases} h_1(x) = 1 - \frac{x_1 x_2^3}{71785 x_3^4} \leq 0 \\ h_2(x) = \frac{4x_2^2 - x_2 x_3}{12566(x_2 x_3^3 - x_3^4)} + \frac{1}{5108 x_3^2} - 1 \leq 0 \\ h_3(x) = 1 - \frac{140.45 x_3}{x_1 x_2^2} \leq 0 \\ h_4(x) = \frac{x_2 + x_3}{1.5} - 1 \leq 0 \\ H = \{(x_1, x_2, x_3)^T | (2.0, 0.25, 0.05)^T \leq x \\ \leq (15.0, 1.3, 2.0)^T\}. \end{cases} \quad (11)$$

To evaluate the performance of the algorithm, we compared the results with the previously published algorithms. If the $f(best)$ in this paper is smaller than that obtained by other algorithms, it proves that the optimization quality of the algorithm in this paper is higher than that of other algorithms. As can be seen from **Table 1**, the $f(best)$ in this paper is lower than the results of most papers, which proves that the algorithm in this paper has achieved better results in finding the optimal solution.

Table 1 Comparison with the reported results for the tension/compression coil spring design problem.

Ref.	$x_1(best)$	$x_2(best)$	$x_3(best)$	$f(best)$
Tomassetti (2010)[25]	11.35304	0.355632	0.051644	0.012665453
Kim, Maruta, and Sugie (2010)[35]	11.2889652	0.356717749	0.051689062	0.012665233
Ozsoydan and Gölcük (2021)[36]	11.29969473	0.356534948	0.051681468	0.012665241
Ozsoydan and Baykasoglu (2021)[37]	11.24027071	0.357551224	0.051723685	0.012665258
Zhang et al. (2024)[38]	11.30566513	0.356433231	0.051677232	0.012665235
This paper	11.29627975	0.356593055	0.051683878	0.012665234

5.3.2 The design of pressure vessels

Kannan and Kame proposed this problem in 1994 [47], and Kim drew the corresponding graph in 2010 [35]. The design variables for this optimization problem include the shell thickness T_s , head thickness T_h , inner radius R , and the length of the cylindrical section of the vessel L (excluding the heads). The thicknesses T_s and T_h are required to be multiples of 0.0625 to match the available rolled steel plate thicknesses, whereas R and L are continuous variables. The design variable vector x , objective function f , and constraint inequality $h(x)$ are formulated mathematically in **Eqs. 12-14**.

$$x = (x_1, x_2, x_3, x_4)^T = (T_s, T_h, R, L)^T, \quad (12)$$

where x_1 , x_2 , x_3 , and x_4 correspond to T_s , T_h , R , and L respectively.

$$\begin{aligned} \min f(x) = & 0.6224x_1x_3x_4 + 1.7781x_2x_3^2 \\ & + 3.1661x_1^2x_4 + 19.84x_1^2x_3. \end{aligned} \quad (13)$$

Subject to:

$$\begin{cases} h_1(x) = 0.0193x_3 - x_1 \leq 0 \\ h_2(x) = 0.00954x_3 - x_2 \leq 0 \\ h_3(x) = 1296000 - \pi x_3^2 x_4 - \frac{4}{3}\pi x_3^3 \leq 0 \\ h_4(x) = x_4 - 240 \leq 0 \\ D = \{(x_1, x_2, x_3, x_4)^T | (0, 0, 10, 10)^T \leq x \\ \leq (99, 99, 200, 200)^T\}. \end{cases} \quad (14)$$

We compared the performance of the algorithm presented in this paper with that of previously published algorithms. As shown in **Table 2**, the $f(best)$ obtained in this study is lower than or equal to that of most existing algorithms, thereby demonstrating the promising effectiveness of our proposed algorithm in achieving optimal solutions.

From the aforementioned mechanical optimization design results based on AHNN, it can be concluded that, in comparison with other previously published algorithms, AHNN-based mechanical optimization design is relatively good and is especially prominent when addressing pressure vessels design problems. This outcome clearly substantiates the significant feasibility of AHNN in the domain of mechanical optimization design, warranting further in-depth investigation.

Table 2 The best solutions reported for the pressure vessel design problem

Ref.	$x_1(best)$	$x_2(best)$	$x_3(best)$	$x_4(best)$	$f(best)$
Tomassetti (2010)[25]	0.8125	0.4375	42.098446	176.636596	6059.714337
Kim, Maruta, and Sugie (2010)[35]	0.8125	0.4375	42.0984456	176.6365958	6059.714355
Gandomi et al. (2013)[48]	0.8125	0.4375	42.0984456	176.6365958	6059.714335
Brajevic and Tuba (2013)[49]	0.8125	0.4375	42.098446	176.636596	6059.714335
Ozsoydan and Gölcük (2021)[36]	0.8125	0.4375	42.09844495	176.6366112	6059.7145
Ozsoydan and Baykasoglu (2021)[37]	0.8125	0.4375	42.09844611	176.6365895	6059.714273
Zhang et al. (2024)[38]	0.8125	0.4375	42.09844559	176.6365959	6059.714335
This paper	0.8125	0.4375	42.09844559	176.6365959	6059.714335

6 Conclusion

In this study, a four-dimensional AHNN was constructed by significantly reducing the synaptic connections between neurons to simulate the partial disconnection commonly observed in real biological neural networks. This design enables the model to maintain basic functional stability when simulating localized structural damage and to exhibit complex dynamic behaviors upon the introduction of external stimulus. Furthermore, it was confirmed that EMR and the interaction between neurons exerts a regulatory influence on their dynamic behavior, thereby offering a reliable biological simulation platform for investigating the impact of structural variability on neural network dynamics. Based on the AHNN framework, this paper innovatively proposes a novel chaotic optimization algorithm, which is applied to two classical mechanical optimization design problems. The proposed algorithm demonstrates superior performance compared to existing methods, effectively reducing production costs and expanding the application scope of HNNs, thus providing technical support for their implementation in the field of mechanical engineering optimization. The feasibility of AHNN in physical implementations was verified through Multisim circuit simulations, which demonstrated that discrete electronic components can accurately replicate the core dynamic behaviors of the network. This confirms its potential for deployment in bioelectronic and industrial control systems, laying a solid foundation for future engineering applications. However, due to the current AHNN configuration consisting of only four neurons with a relatively simple architecture, it is insufficient to fully capture the complexity of human brain activity. Future research will focus on increasing the number of neurons to better approximate real neural networks. Further efforts will be also made to expand the application of AHNN to broader engineering problems, thereby narrowing the gap between bio-inspired neural networks and practical engineering implementations.

Funding

This work was supported in part by the National Natural Science Foundation of China (Grant Nos. 62202198 and 62461022), the Hunan Natural Science Foundation of China (Grant No. 2024JJ7372), the Scientific Research Project of Hunan Provincial Department of Education (Grant Nos. 24A0550 and 23B0318).

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