

An improved model for the effect of correlated Si III absorption on the one-dimensional Lyman- α forest power spectrum

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ABSTRACT

We present an analysis of Si III absorption and its effect on the one-dimensional Ly forest power spectrum using the Sherwood–Relics hydrodynamical simulation suite. In addition to oscillations from the Ly α – Si III cross-correlation that are damped towards smaller scales, we find an enhancement in small-scale power that has been ignored in previous studies. We therefore develop a new analytical fitting function that captures two critical effects that have previously been neglected: distinct Ly α and Si III line profiles, and a variable ratio for coeval Ly α and Si III optical depths. In contrast to earlier work, we also predict amplitudes for the Si III power spectrum and Ly α – Si III cross power spectrum that decrease towards lower redshift due to the hardening metagalactic UV background spectrum at $z \lesssim 3.5$. The fitting function is validated by comparison against multiple simulated data sets at redshifts $2.2 \leq z \leq 5.0$ and wavenumbers $k < 0.2 \text{ s km}^{-1}$. Our model has little effect on existing warm dark matter constraints from the Ly α forest when adopting a physically motivated prior on the silicon abundance. It will, however, be an essential consideration for future, high precision Ly α forest power spectrum measurements.

Key words: methods: numerical – intergalactic medium – quasars: absorption lines.

1 INTRODUCTION

The Ly α forest, characterized by absorption features imprinted on the spectra of distant quasars, is produced when light from background quasars is absorbed by neutral hydrogen in the intergalactic medium (IGM). It stands as one of the most informative probes for studying the structure and evolution of the Universe (A. A. Meiksin 2009; M. McQuinn 2016). As the light traverses regions of varying hydrogen gas density along the line of sight, a complex pattern of absorption emerges. This pattern reflects both the physical state of the IGM and the underlying dark matter distribution, making the Ly α forest an essential tool for investigating a range of astrophysical and cosmological phenomena.

One of the most widely used summary statistics used for extracting information from the Ly α forest is the one-dimensional Ly α power spectrum, which quantifies the statistical distribution of absorption features as a function of spatial scale. Its study remains an important area of research. In particular, it plays a crucial role in constraining cosmological parameters (e.g. R. A.

C. Croft et al. 2002; M. Viel, M. G. Haehnelt & V. Springel 2004; U. Seljak et al. 2005; P. McDonald et al. 2006), dark matter models (e.g. V. K. Narayanan et al. 2000; M. Viel et al. 2013; K. K. Rogers & H. V. Peiris 2021; D. C. Hooper et al. 2022; B. Villaseñor et al. 2023; V. Iršič et al. 2024; O. Garcia-Gallego et al. 2025), and primordial magnetic fields (M. Pavičević et al. 2025), complementing observations from the cosmic microwave background, galaxy clustering, and weak gravitational lensing.

Current measurements of the one-dimensional power spectrum are divided into two regimes. High-resolution spectroscopic surveys, such as the High-Resolution Echelle Spectrometer on the Keck telescope (Keck/HIRES) or the Ultraviolet and Visual Echelle Spectrograph from the Very Large Telescope (VLT/UVES) with $R \gtrsim 20\,000$ and signal-to-noise ratio (S/N) > 10 , resolve small-scale features but have limited sample sizes with $\lesssim 100$ Quasi-Stellar Objects (QSOs), leading to large statistical uncertainties (e.g. V. Iršič et al. 2017; E. Boera et al. 2019; N. G. Karaçaylı et al. 2022). Representing the first regime, E. Boera et al. (2019) measured the power spectrum at $z = 4.2, 4.6,$ and 5.0 using 15 high-resolution QSO spectra from Keck/HIRES and VLT/UVES, reaching scales down to $k \sim 0.2 \text{ s km}^{-1}$ with ~ 15

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per cent precision. Conversely, surveys like the extended Baryon Oscillation Spectroscopic Survey of the Sloan Digital Sky Survey (SDSS/eBOSS) and the Dark Energy Spectroscopic Instrument (DESI) achieve larger samples ($\sim 10^5$ QSO spectra) at the cost of spectral resolution, limiting their sensitivity to wavenumbers beyond $k \sim 0.03 \text{ s km}^{-1}$. For the second regime, the most recent public results come from SDSS/eBOSS (S. Chabanier et al. 2019), measuring the power spectrum from $z = 2.2$ to 4.6 and extending to $k \sim 0.02 \text{ s km}^{-1}$, and DESI DR1 (N. G. Karaçaylı et al. 2025; C. Ravoux et al. 2025), measuring from $z = 2.2$ to 4.2 and extending to $k \sim 0.026 - 0.039 \text{ s km}^{-1}$ depending on redshift.

Accurate modelling of the Ly α forest power spectrum, however, faces challenges due to contamination from metal absorption lines. These lines are broadly categorized as either correlated or uncorrelated with Ly α absorption. Correlated metal lines trace the Ly α forest due to their spectral and redshift proximity within the same large-scale structures. The most significant contributor is Si III ($\lambda 1207$), arising from doubly ionized silicon. Uncorrelated metal lines, conversely, do not align spatially or in redshift with the Ly α forest and often trace distinct astrophysical environments, resulting in a stochastic relationship with Ly α absorption. Prominent examples include C IV ($\lambda\lambda 1548, 1551$), Si IV ($\lambda\lambda 1394, 1403$), and Mg II ($\lambda\lambda 2796, 2804$). These metal lines introduce systematic distortions that complicate cosmological measurements and IGM modelling using the Ly α forest power spectrum. Consequently, improving models of their impact on the Ly α forest power spectrum is essential.

In practice, the effect of uncorrelated metals on the one-dimensional power spectrum can be reduced to negligible levels by estimating their power spectrum using sidebands – spectral regions devoid of Ly α absorption that lie redward of the QSO Ly α emission lines (e.g. N. Palanque-Delabrouille et al. 2013; E. Boera et al. 2019; S. Chabanier et al. 2019; N. G. Karaçaylı et al. 2022, 2024; C. Ravoux et al. 2023). Contamination from correlated metals, however, is harder to isolate due to the fact they are blended in the Ly α forest. The canonical modelling approach, first introduced by P. McDonald et al. 2006 (hereafter M06), assumes that the transmitted flux contrast of Si III absorption can be obtained by a linear rescaling of the Ly α flux contrast. This ansatz yields an expression for the Si III power spectrum in terms of the Ly α power spectrum. At large scales, it is well-established that this model accurately describes the oscillatory behaviour in the power spectrum induced by Si III absorption. Furthermore, the resulting systematic errors from the Si III oscillations are small compared to other uncertainties.

However, the M06 ansatz will break down towards smaller scales as differences in individual absorption line profiles become significant. Furthermore, the redshift evolution of the Si III power spectrum is also uncertain. These limitations pose problems for high-resolution measurements of the power spectrum, particularly as statistical errors become smaller. To address this issue, this paper presents a revised approach for the impact of correlated Si III absorption on the Ly α forest spectrum. We develop a physically motivated fitting function based on the Sherwood–Relics simulation suite (J. S. Bolton et al. 2017; E. Puchwein et al. 2023), from which we directly compute Ly α and Si III optical depths and construct mock QSO spectra.

The paper is organized as follows: Section 2 outlines our methodology for creating the mock QSO spectra and quantifying the power spectrum. Section 3 discusses the limitations of previous work and an analytical argument that helps build insight into our revised model. We test and validate our model against simulation data sets with varying redshifts, temperatures, and metallicities in Section 4. Readers who are primarily interested in how to implement our model

within parameter inference frameworks may wish to skip directly to Section 4.4 and equation (20). The implications of our model for warm dark matter constraints are discussed briefly in Section 5. Finally, Section 6 summarizes our results and conclusions.

2 NUMERICAL METHOD

2.1 Hydrodynamical simulation

We use a simulation from the Sherwood–Relics project (E. Puchwein et al. 2023) to construct Ly α ($\lambda 1216$) and Si III ($\lambda 1207$) absorption spectra. Sherwood–Relics is an extensive series of high-resolution cosmological hydrodynamical simulations performed using a customized version of P-GADGET-3 (V. Springel 2005). In this work, we use a cosmological box of size $40h^{-1} \text{ cMpc}$ with 2×2048^3 dark matter and gas particles. The dark matter particle mass is $M_{\text{dm}} = 5.37 \times 10^5 h^{-1} M_{\odot}$ and the gas particle mass is $M_{\text{gas}} = 9.97 \times 10^4 h^{-1} M_{\odot}$. The box size and mass resolution have been chosen to resolve the small-scale structure of the Ly α forest (see e.g. J. S. Bolton & G. D. Becker 2009).

The initial conditions for the simulation are identical to those used in the earlier Sherwood simulation project (J. S. Bolton et al. 2017). We assume a flat Lambda cold dark matter (Λ CDM) cosmology with $\Omega_{\Lambda} = 0.692$, $\Omega_{\text{M}} = 0.308$, $\Omega_{\text{b}} = 0.0482$, $\sigma_8 = 0.829$, $n_s = 0.961$, $h = 0.678$, and a primordial helium mass abundance of $Y = 0.24$. A computationally efficient star-formation scheme is used, where gas particles are converted into collisionless star particles if they have gas overdensities $\Delta > 10^3$ and gas temperatures $T < 10^5 \text{ K}$ (M. Viel et al. 2004). The simulation uses a spatially uniform ultraviolet (UV) background (E. Puchwein et al. 2019) and the thermal and ionization state of the hydrogen and helium gas is computed using a non-equilibrium thermochemistry solver (E. Puchwein et al. 2015).

Periodic sightlines are drawn through the simulation to extract the gas densities, temperatures, peculiar velocities, and H I fractions needed to compute mock Ly α absorption spectra. We use the interpolation scheme described by T. Theuns et al. (1998) for this purpose. The sightlines are extracted at a redshift interval of $\Delta z = 0.1$, and for each redshift we draw a total of 5000 such sightlines with 2048 pixels, giving a pixel size of $19.5h^{-1} \text{ ckpc}$. The Ly α optical depths along each sightline are then computed by performing a convolution with the Voigt line profile from T. Tepper-García (2006).

We use a similar approach to construct the Si III absorption spectra, although with some additional steps. First, the Sherwood–Relics simulations do not use a sub-grid model for metal production and dispersal. Instead, we assume a fiducial silicon abundance model that follows the gas overdensity, $\Delta = \rho/\langle\rho\rangle$, such that

$$[\text{Si}/\text{H}] = -2.70 + 0.08(z - 3) + 0.65(\log_{10} \Delta - 0.5). \quad (1)$$

Equation (1) follows from the pixel optical depth silicon abundance constraints presented by J. Schaye et al. (2003) and A. Aguirre et al. (2004) using data at $z = 1.8\text{--}4.1$.¹ The Si III optical depth, $\tau_{\text{Si III}}$, is then proportional to

$$\tau_{\text{Si III}} \propto x_{\text{Si III}}(\Delta, T) 10^{[\text{Si}/\text{H}] + (\text{Si}/\text{H})_{\odot}}, \quad (2)$$

¹In more detail, we use $[\text{Si}/\text{H}] = [\text{C}/\text{H}] + [\text{Si}/\text{C}]$, with $[\text{C}/\text{H}]$ from equation (8) in J. Schaye et al. (2003) and $[\text{Si}/\text{C}] = 0.77$ from A. Aguirre et al. (2004). These metallicity constraints were derived using ionization corrections based on the F. Haardt & P. Madau (2001) UV background model.

where $x_{\text{Si III}}$ is the Si III fraction (which is, in general, density, and temperature dependent given some UV background model) and $(\text{Si}/\text{H})_{\odot} = -4.49$ is the solar abundance for silicon (M. Asplund et al. 2009). We note, however, that the precise form of the silicon distribution is not critical for our analysis. We have verified that adopting a constant silicon abundance instead of equation (1) yields a qualitatively similar effect on the shape of the (Si III contaminated) Ly α forest power spectrum (see also Appendix A).

The equilibrium ionization balance of the silicon is calculated using the photoionization code CLOUDY (G. J. Ferland et al. 1998; M. Chatzikos et al. 2023). We use CLOUDY in combination with the E. Puchwein et al. (2019) UV background spectrum to create a three-dimensional look-up table listing the Si III fraction, $x_{\text{Si III}}$, as a function of the logarithm of the gas temperature, $\log_{10}(T/\text{K})$, the logarithm of the hydrogen number density, $\log_{10}(n_{\text{H}}/\text{cm}^{-3})$, and redshift, z . The table data are linearly interpolated to obtain the $x_{\text{Si III}}$ for each pixel in our simulated sightlines.

An example of the predicted Si III fraction as a function of temperature and density are shown in the left panel of Fig. 1. The black contours display the volume-weighted temperature-density plane for gas in the hydrodynamical simulation at $z = 3$, with a colour scale displaying the logarithm of the Si III fraction. The right panel of Fig. 1 shows the redshift evolution of various silicon ion fractions for gas with a density and temperature typical of the Si III absorption, $\log_{10}(n_{\text{H}}/\text{cm}^{-3}) = -3.4$ and $\log_{10}(T/\text{K}) = 4.5$. The Si III fraction is largest at $z = 5$, and the decline in the Si II, Si III, and Si IV fractions at $z \lesssim 3.5$ is driven by the hardening of the UV background spectrum due to ionizing photons emitted by quasars (i.e. the same spectral hardening that drives He II reionization, e.g. P. Gaikwad, F. B. Davies & M. G. Haehnelt 2025). Note also that for the UV background model and redshift range considered in this study, Si III (with ionization potential 33.5 eV) is more abundant than Si II (with ionization potential 16.3 eV), where the latter will also correlate with Ly α for the transitions Si II ($\lambda\lambda 1190, 1193$) and Si II ($\lambda 1260$). We initially included these Si II absorption lines in our analysis, but neglected them after establishing their contribution to the one-dimensional power remains small compared to Si III in our model. We note, however, that if the Si II absorption is typically associated with dense circumgalactic gas that self-shields to hydrogen ionizing photons, it will be underestimated in our model due to the lack of self-shielded gas with $\Delta > 10^3$ in Sherwood–Relics.

Finally, before performing the Gaussian line profile convolution to obtain the Si III optical depths, we resample the sightline quantities on to a finer grid with pixel size $9.8h^{-1}$ ckpc. This resampling is needed to resolve the thermal Doppler widths of the silicon absorbers, $b_{\text{Si III}} = 2.4 \text{ km s}^{-1} (T/10^4 \text{ K})^{1/2}$, and thus ensure the Si III optical depths are converged. Note also that the Ly α and Si III optical depths are all extracted from the same randomly selected sightlines, ensuring the absorption traces the same physical structures in the simulation.

2.2 Mock quasar sightlines

Once the optical depths are extracted from the hydrodynamical simulations, we combine the individual sightlines drawn from the hydrodynamical simulation into mock quasar spectra. First, for a given quasar redshift z_{QSO} , we construct a full-length Ly α forest spectrum [i.e. from $\lambda_{\alpha}(1 + z_{\text{QSO}})$ to $\lambda_{\beta}(1 + z_{\text{QSO}})$, where λ_{α} and λ_{β} are the Ly α and Ly β rest-frame wavelengths] by combining the simulated sightlines at the required redshifts. These are stitched together so that the optical depth remains continuous; the periodic nature of the sightlines allows us to shift each segment to maintain

continuous intersections. The same process is used to extract the coeval Si III absorption spectra over a matching length scale. The Ly α absorption is furthermore normalized to match the effective optical depth measured by G. D. Becker et al. (2013) at the appropriate redshifts. Finally, the Si III optical depths are shifted to align at the correct observed wavelengths and then added to the Ly α optical depths, where we assume the rest-frame wavelengths of the transitions are Ly α ($\lambda 1216$) and Si III ($\lambda 1207$). An example of the predicted absorption in a noiseless spectrum at $z \sim 3$ is displayed in the upper panel of Fig. 2.

2.3 Computing the power spectrum

We shall use the noiseless mock spectra to compute the one-dimensional Ly α forest power spectrum, both including and excluding the correlated Si III absorption. Following the approach commonly adopted in the literature (e.g. N. Palanque-Delabrouille et al. 2013; S. Chabanier et al. 2019; C. Ravoux et al. 2023; N. G. Karaçaylı et al. 2024), we compute the power spectrum, $P(k)$, using the Fast Fourier Transform (FFT) approach, where

$$P(k) \propto |\tilde{\delta}(k)|^2. \quad (3)$$

Here $\tilde{\delta}(k)$ is the Fourier transform of the flux contrast, $\delta(v)$, which is given by

$$\delta(v) = \frac{F(v)}{\overline{F(v)}} - 1, \quad (4)$$

where $F(v)$ is the transmitted flux in a pixel at velocity v , and $\overline{F(v)}$ is the mean transmission of the intervening Ly α forest. For the latter we assume the effective optical depth, τ_{eff} , reported by G. D. Becker et al. (2013), where $\overline{F(v)} = e^{-\tau_{\text{eff}}}$.

3 MODELLING THE LY α –SI III CORRELATION

3.1 The M06 analytical model

It is well known that the correlations between metal line transitions with wavelengths similar to the Ly α transition are an important systematic in analyses of the one-dimensional Ly α forest power spectrum. For Si III, which is one of the most prominent correlated metals, a feature at $\Delta v = 2271 \text{ km s}^{-1}$ is produced in the Ly α forest auto-correlation. This also manifests as an oscillatory feature in the one-dimensional power spectrum with peak separations of $\Delta k = 2\pi/\Delta v$.

Hence, in order to obtain a good match between observations and models, the contribution of Si III to the power spectrum is usually treated as a nuisance parameter. M06 first outlined the model framework adopted in the literature. These authors made the ansatz that the Si III transmitted flux contrast, $\delta_{\text{Si III}}$, will have the same shape as the Ly α forest transmitted flux contrast, $\delta_{\text{Ly}\alpha}$, up to some normalization factor a , where for a pixel at velocity v ,

$$\begin{aligned} \delta_{\text{tot}}(v) &= \delta_{\text{Ly}\alpha}(v) + \delta_{\text{Si III}}(v), \\ &= \delta_{\text{Ly}\alpha}(v) + a \delta_{\text{Ly}\alpha}(v + \Delta v). \end{aligned} \quad (5)$$

Here, $\Delta v = 2271 \text{ km s}^{-1}$ is the difference in velocity between the rest-frame wavelengths of the Ly α and Si III transitions, $\Delta\lambda = 9.17 \text{ \AA}$. Within this framework the total power spectrum, $P_{\text{tot}}(k)$, including both the Ly α and Si III absorption is given by

$$\begin{aligned} P_{\text{tot}}(k) &\propto |\tilde{\delta}_{\text{Ly}\alpha}(k) + \tilde{\delta}_{\text{Si III}}(k)|^2, \\ &= P_{\text{Ly}\alpha}(k) + P_{\text{Si III}}(k) + P_{\text{Ly}\alpha\text{–Si III}}(k), \end{aligned} \quad (6)$$

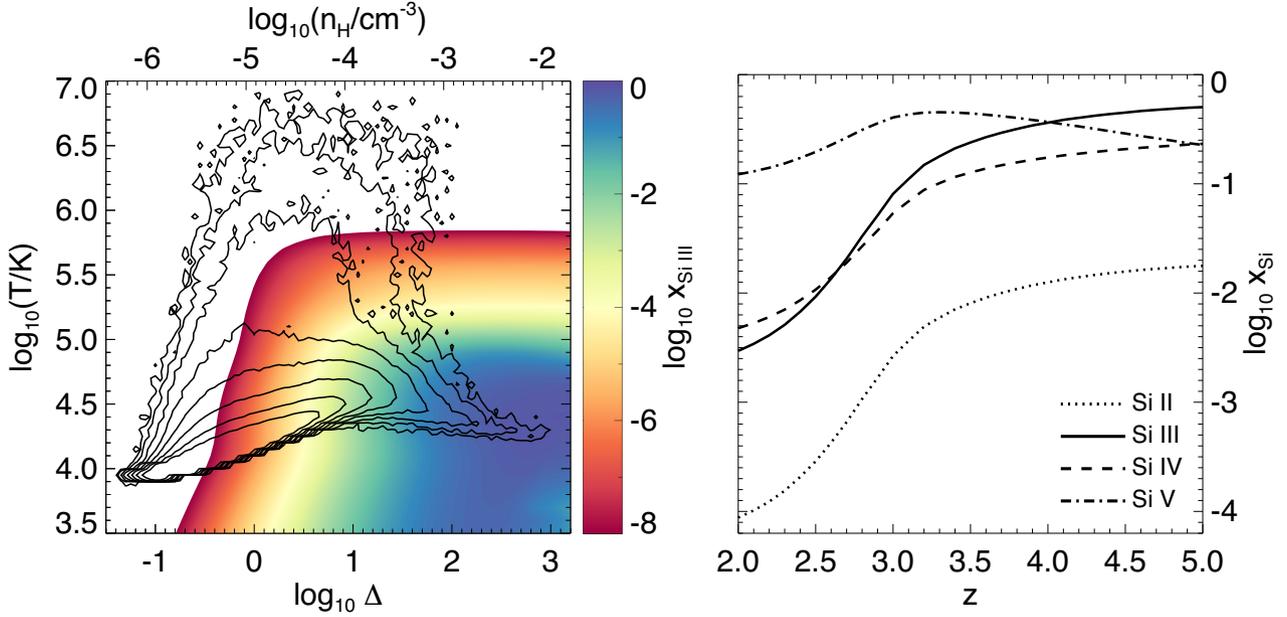


Figure 1. Left panel: the solid black contours show the volume weighted temperature-density plane at $z = 3$ for the Sherwood–Relics hydrodynamical simulation used in this work. The number density of points increases by 0.5 dex within each contour level. Note the horizontal axis is given in terms of both the logarithm of the gas overdensity, $\log_{10} \Delta$, and the logarithm of the hydrogen number density, $\log_{10}(n_{\text{H}}/\text{cm}^{-3})$. The overlaid colour scale shows the logarithm of the Si III fraction, $\log_{10} x_{\text{Si III}}$, predicted by the photoionization code CLOUDY (G. J. Ferland et al. 1998; M. Chatzikos et al. 2023) using the UV background synthesis model from E. Puchwein et al. (2019). Right panel: the redshift evolution of silicon ion fractions for gas with density $\log_{10}(n_{\text{H}}/\text{cm}^{-3}) = -3.4$ and temperature $\log_{10}(T/\text{K}) = 4.5$. The decline in the Si II, Si III, and Si IV fractions at $z \lesssim 3.5$ is driven by the hardening of the UV background spectrum due to ionizing photons emitted by quasars.

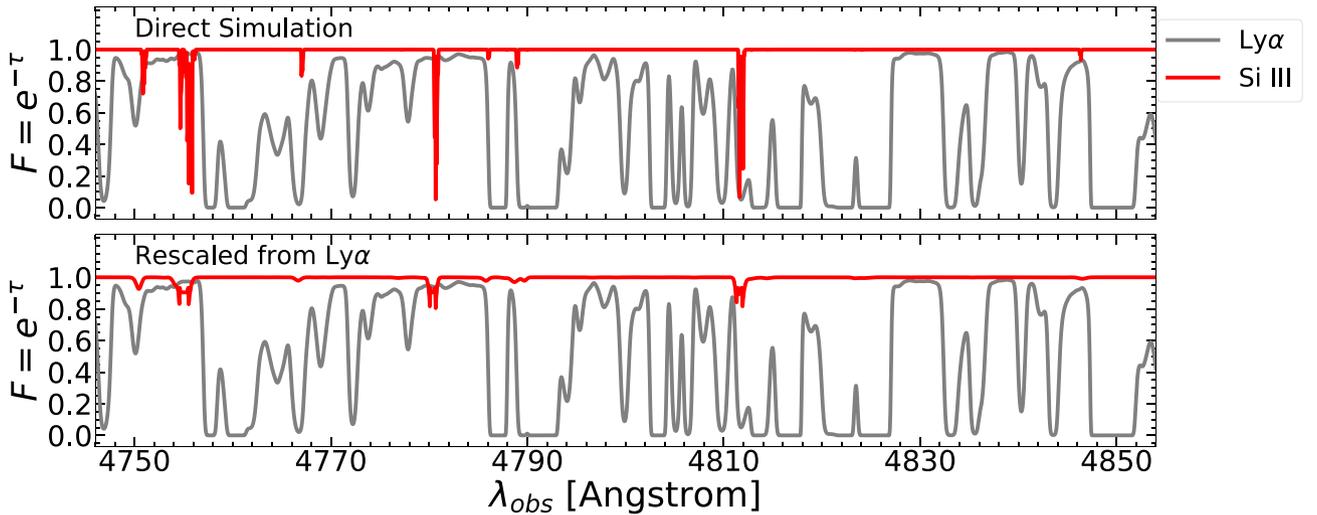


Figure 2. The transmitted flux for the Ly α forest (grey) and Si III absorption (red) for a random simulated sightline at $z \sim 3$. The upper panel shows the results from our direct calculation of the Si III spectrum from the hydrodynamical simulation described in Section 2. The lower panel compares this to the Si III absorption obtained by a rescaling of the Ly α optical depths. This is similar to the approach often used in the literature when modelling the contribution of correlated metal absorption to the one-dimensional Ly α forest power spectrum. In this example the scaling is chosen so that the mean Si III transmission in both panels is equal. Note the narrower widths for the Si III absorption in the direct calculation. The Ly α absorption is identical in both panels.

where the first term, $P_{\text{Ly}\alpha}(k)$, is the Ly α power spectrum, the second term,

$$P_{\text{Si III}}(k) = a^2 P_{\text{Ly}\alpha}(k), \quad (7)$$

is the Si III power spectrum, and the third term,

$$P_{\text{Ly}\alpha-\text{Si III}}(k) = 2a \cos(\Delta\nu k) P_{\text{Ly}\alpha}(k), \quad (8)$$

represents the cross term between Ly α and Si III that gives the oscillatory behaviour in the total power spectrum. In the existing literature the normalization a is often given a redshift dependence $a(z) = f/(1 - \bar{F}(z))$, where $\bar{F}(z)$ is the mean Ly α forest transmission, and f is treated as the free parameter.

We may compare this approach to the Si III absorption predicted by our hydrodynamical simulation. Fig. 2 shows the directly

simulated Si III transmission (red curve, upper panel) and the Si III transmission obtained by rescaling the Ly α optical depths (lower panel). More specifically, in the lower panel of Fig. 2 we assume $\tau_{\text{Si III}}(v) = A\tau_{\text{Ly}\alpha}(v + \Delta v)$, where we select A so that $\bar{F}_{\text{Si III}} = (e^{-\tau_{\text{Si III}}})$ is identical to the hydrodynamical simulation result. Although this does not follow exactly from equation (5) (which relies on rescaling $\delta_{\text{Ly}\alpha}$, rather than $\tau_{\text{Ly}\alpha}$ as we do here),² it serves to visually illustrate how the Si III line widths, $b_{\text{Si III}}$, and hence also $\delta_{\text{Si III}}$ will be too broad if scaled from the Ly α absorption. Note that for a thermally broadened line

$$b_{\text{Si III}} = \left(\frac{2k_{\text{B}}T}{m_{\text{Si}}}\right)^{1/2} = \left(\frac{m_{\text{H}}}{m_{\text{Si}}}\right)^{1/2} b_{\text{Ly}\alpha} \simeq 0.19b_{\text{Ly}\alpha}. \quad (9)$$

We may gain further insight by calculating each of the terms in equation (6) for the two cases displayed in Fig. 2. The results are displayed in Fig. 3, where the left column shows the power spectrum terms obtained on performing a linear rescaling of the Ly α optical depths, while the right column shows the direct simulation of the Si III absorption. The value for the linear rescaling parameter A used for the results displayed in the left column is chosen so the amplitude of $P_{\text{Si III}}(k)$ matches the direct simulation on large scales (i.e. at small k).

There are several key points to highlight in Fig. 3. First, the agreement between $P_{\text{Si III}}(k)$ in the rescaled and directly simulated models is reasonable on large scales, $k \lesssim 0.04 \text{ s km}^{-1}$. However, the ratio $P_{\text{Si III}}(k)/P_{\text{Ly}\alpha}(k)$ strongly increases towards small scales in the direct simulation. This contrasts with the scale independent behaviour observed for the linearly rescaled model, and suggests that applying equation (5) to one-dimensional Ly α forest power spectrum constraints at wavenumbers larger than $k \sim 10^{-2} \text{ s km}^{-1}$ will result in an underestimate of the contribution of $P_{\text{Si III}}(k)$ to the total small-scale power. A second difference is the oscillatory cross term, $P_{\text{Ly}\alpha-\text{Si III}}(k)$. Although this retains the same oscillation period in both models, in the direct simulation it has an amplitude that declines towards small scales and makes a negligible contribution to the total power spectrum once $P_{\text{Si III}}(k) > P_{\text{Ly}\alpha-\text{Si III}}(k)$. In contrast, the amplitude is constant in the rescaled model [i.e. from equation (6), we expect the ratio of the amplitudes for $P_{\text{Ly}\alpha-\text{Si III}}(k)$ and $P_{\text{Ly}\alpha}(k)$ to be $a/2$].

The physical origin of these differences arise from several effects missed in the ansatz made by equation (5). The first – as already discussed earlier – is that the Ly α and Si III line profiles are different. In general, unless the gas is highly turbulent, the Si III line widths will be narrower than Ly α (cf. equation 9) resulting in a concomitant increase in small scale power in the Si III power spectrum. Second, the ratio of the coeval Si III and Ly α optical depths is not constant. If ignoring redshift space effects by assuming the line profiles are Dirac delta functions, the ratio of the Si III to Ly α optical depths is

$$\beta(v) = \frac{\tau_{\text{Si III}}(v)}{\tau_{\text{Ly}\alpha}(v + \Delta v)}, \quad (10)$$

$$= \frac{f_{\text{Si III}}\lambda_{\text{Si III}}}{f_{\text{Ly}\alpha}\lambda_{\text{Ly}\alpha}} \frac{x_{\text{Si III}}(\Delta, T)}{x_{\text{H I}}(\Delta, T)} 10^{[\text{Si/H}] + (\text{Si/H})_{\odot}},$$

where $f_{\text{Si III}}\lambda_{\text{Si III}}/f_{\text{Ly}\alpha}\lambda_{\text{Ly}\alpha} = 3.88$ is the constant ratio of the transition oscillator strengths and rest-frame wavelengths (D. C. Morton 2003). At any given redshift we therefore expect $\beta(v)$ to have a pixel dependency arising from variations in the silicon abundance, $[\text{Si/H}]$,

and Si III fraction, $x_{\text{Si III}}$, where the latter will depend on density, Δ , and temperature, T , given some UV background model. This will impact on the shape and amplitude of both $P_{\text{Si III}}(k)$ and $P_{\text{Ly}\alpha-\text{Si III}}(k)$. Thirdly, equation (5) assumes the Ly α and Si III optical depths coherently trace the same underlying structures. While this is true for the Si III and Ly α optical depths in our models, this does not hold for the observable, $F = e^{-\tau}$. This is because in practice $\beta(v) \ll 1$ in equation (10); intergalactic Si III absorption lines with $\tau_{\text{Si III}} \sim 1$ typically arise from much higher density gas compared to Ly α lines with $\tau_{\text{Ly}\alpha} \sim 1$. Therefore, even relatively weak Si III absorbers in our model are coeval with Ly α absorption lines that are saturated.

Fig. 4 illustrates this behaviour by comparing coeval Si III and Ly α absorption profiles in different Si III column density bins, $N_{\text{Si III}}$, in our simulated spectra. Here, we have estimated $N_{\text{Si III}}$ using the apparent optical depth method (B. D. Savage & K. R. Sembach 1991). For Si III systems with $\log_{10}(N_{\text{Si III}}/\text{cm}^{-2}) \gtrsim 10$, the corresponding Ly α forest absorbers are saturated. This saturation-induced decorrelation between the two tracers manifests as a suppression of the oscillatory amplitude in the cross-power spectrum towards smaller scales, as shown in Fig. 3.

3.2 Towards a more accurate model for Si III contamination in the one-dimensional Ly α forest power spectrum at small scales

It is instructive to consider a simple model that approximately captures the behaviour in Fig. 3. This must take into account two factors: (i) the different line profiles for Ly α and Si III absorption and (ii) the fact that the ratio between the Ly α and Si III optical depths will in general depend on temperature and gas density, making the scaling factor a in equation (5) pixel dependent, instead of a constant. To incorporate these effects, we modify the original ansatz in equation (5) to

$$\delta_{\text{Si III}}(v) \otimes \phi(v) = [a\bar{\beta}(1 + \delta_{\beta}(v))\delta_{\text{Ly}\alpha}(v + \Delta v)]. \quad (11)$$

Here, $\otimes \phi(v)$ corresponds to a Gaussian kernel convolution that describes the line profile, where for some line-broadening scale b

$$\phi(v) = \frac{1}{\sqrt{\pi}b} \exp\left(-\frac{v^2}{b^2}\right). \quad (12)$$

The perturbative term $\delta_{\beta}(v)$ represents the pixel dependency implied by equation (10), where

$$\delta_{\beta}(v) \simeq \frac{\beta(v)}{\bar{\beta}} - 1, \quad (13)$$

if the optical depth is small. Next, using the standard result

$$\tilde{\phi}(k) = \exp\left(-\frac{b^2k^2}{4}\right), \quad (14)$$

the Fourier transform of $\delta_{\text{Si III}}(v)$ can be obtained by applying a Fourier transform to both sides of equation (11) and rearranging, yielding

$$\tilde{\delta}_{\text{Si III}}(k) = a\bar{\beta} \exp\left(\frac{b^2k^2}{4} + ik\Delta v\right) [\tilde{\delta}_{\text{Ly}\alpha}(k) + \tilde{\delta}_{\epsilon}(k)]. \quad (15)$$

For notational convenience we have written the convolution arising from the Fourier transform of $\delta_{\beta}(v)\delta_{\text{Ly}\alpha}(v)$ as $\tilde{\delta}_{\epsilon}(k) \equiv \tilde{\delta}_{\beta}(k) \otimes \tilde{\delta}_{\text{Ly}\alpha}(k)$. The one-dimensional Ly α forest power spectrum once again follows from equation (6), but the Si III power spectrum and cross-power spectrum now have some additional terms. For the Si III power spectrum, we obtain

$$P_{\text{Si III}}(k) = a^2\bar{\beta}^2 \exp\left(\frac{b^2k^2}{2}\right) [P_{\text{Ly}\alpha}(k) + P_{\text{Ly}\alpha-\epsilon}(k) + P_{\epsilon}(k)], \quad (16)$$

²Strictly speaking, the flux contrast is proportional to the optical depth, $\delta_{\text{Ly}\alpha} \propto \tau_{\text{Ly}\alpha}$, only when $\tau_{\text{Ly}\alpha}$ is small, such that the first-order expansion $F = e^{-\tau_{\text{Ly}\alpha}} \simeq 1 - \tau_{\text{Ly}\alpha}$ is valid.

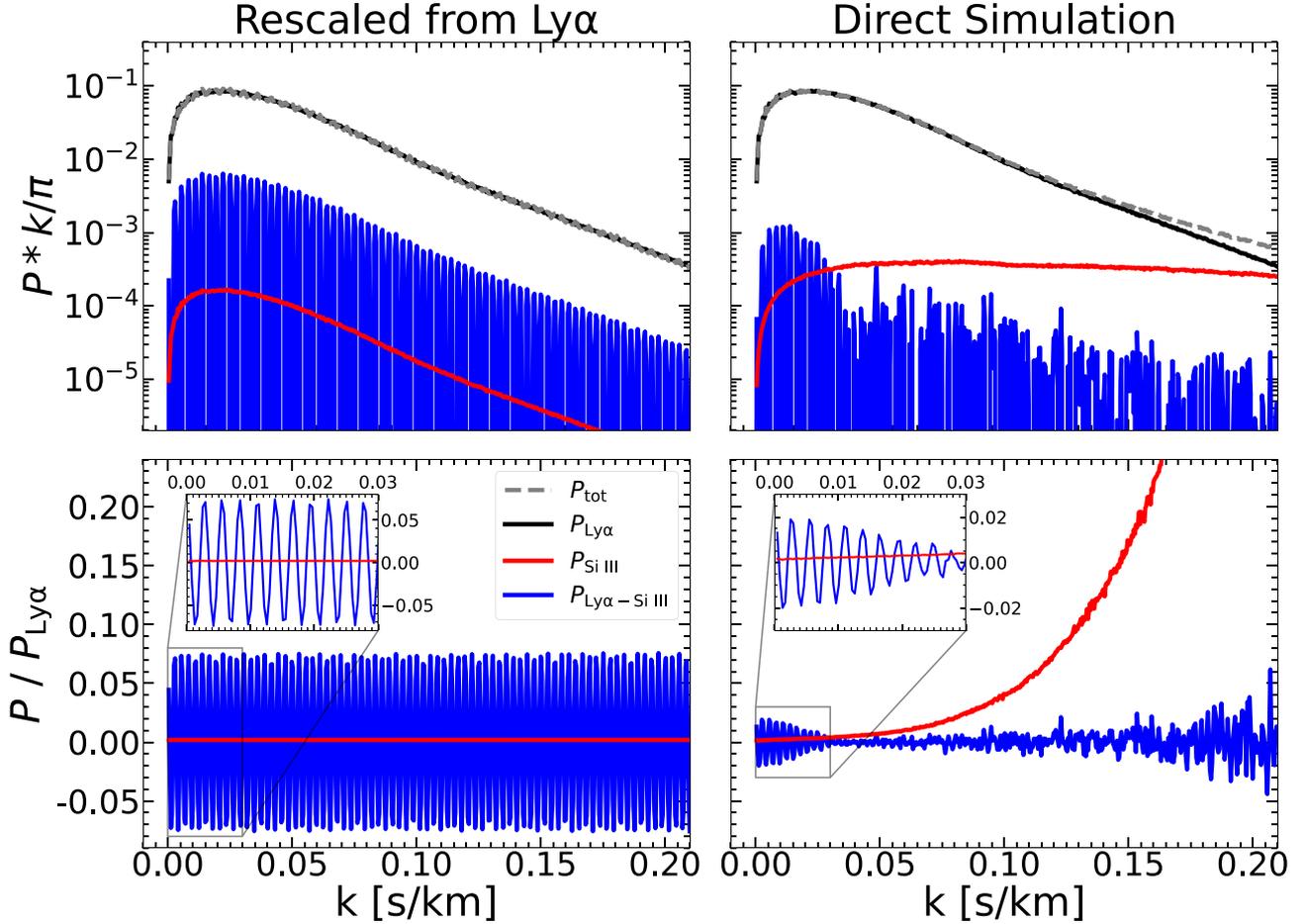


Figure 3. A comparison of the total Ly α forest power spectrum including the contamination by correlated Si III absorption [$P_{\text{tot}}(k)$, grey dashed curves] and its constituent terms (see equation 6) at $z = 3$, where the Si III absorption has been computed either by a linear rescaling of the Ly α optical depths (left column) or from a direct simulation following the procedure described in Section 2 (right column). Additional curves show the power spectrum for Ly α absorption only [$P_{\text{Ly}\alpha}(k)$, black solid curve], Si III absorption only [$P_{\text{Si III}}(k)$, red solid curve], and the cross term between Ly α and Si III [$P_{\text{Ly}\alpha - \text{Si III}}(k)$, blue solid curve]. The bottom panels show the ratio of $P_{\text{Si III}}(k)$ and $P_{\text{Ly}\alpha - \text{Si III}}(k)$ to $P_{\text{Ly}\alpha}(k)$. In the left column, the value for the linear rescaling parameter A , where $\tau_{\text{Si III}}(v) = A\tau_{\text{Ly}\alpha}(v + \Delta v)$, was chosen so that the amplitude of $P_{\text{Si III}}(k)$ is similar to the direct simulation on large scales.

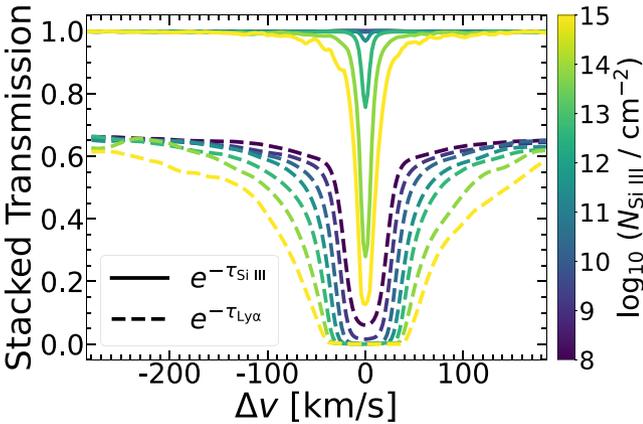


Figure 4. Stacked absorption line profiles for different Si III absorbers as a function of column density, $N_{\text{Si III}}$, at $z = 3.0$. Solid curves represent the stacked Si III absorption profiles, while dashed curves show the coeval Ly α absorption profiles. The line colours correspond to the $N_{\text{Si III}}$ in each stack. For most strong Si III systems, the Ly α absorption is saturated. This decorrelates the two tracers and leads to a suppression of the oscillation amplitude in the Ly α - Si III cross-power spectrum towards smaller scales.

where $P_{\text{Ly}\alpha - \epsilon}(k)$ is the power spectrum of the cross-correlation between $\delta_{\text{Ly}\alpha}(v)$ and $\delta_{\beta}(v)\delta_{\text{Ly}\alpha}(v)$, and $P_{\epsilon}(k)$ is the power spectrum of $\delta_{\beta}(v)\delta_{\text{Ly}\alpha}(v)$. The oscillatory cross-term is instead given by

$$P_{\text{Ly}\alpha - \text{Si III}}(k) = 2a\bar{\beta} \cos(\Delta vk) \exp\left(\frac{b^2 k^2}{4}\right) [P_{\text{Ly}\alpha}(k) + P_{\text{Ly}\alpha - \epsilon}(k)], \quad (17)$$

where we have used the identity $\cos(\Delta vk) = (e^{i\Delta vk} + e^{-i\Delta vk})/2$. Comparing equations (16)–(17) to equations (7)–(8), we therefore expect (i) an additional growing exponential factor that arises from the narrower Si III line widths,³ and (ii) some extra, scale dependent terms that originate from the spatial variations in the silicon abundance and Si III fraction relative to hydrogen (i.e. equation 10).

Overall, these results suggest that an improved model for Si III contamination in the one-dimensional Ly α forest power spectrum at

³Note that we assume constant b in this illustrative model. In practice, the temperature will vary spatially such that b is also pixel dependent. There may also be a turbulent contribution to the Si III line widths, which would increase the line-broadening scale.

small scales can be provided by a function with four free parameters,

$$\begin{aligned}
 P_{\text{tot}}(k) &= P_{\text{Ly}\alpha}(k) + P_{\text{Si III}}(k) + P_{\text{Ly}\alpha\text{-Si III}}(k), \\
 P_{\text{Si III}}(k) &= a_{\text{auto}} \exp\left(\frac{k}{k_{\text{auto}}}\right) P_{\text{Ly}\alpha}(k), \\
 P_{\text{Ly}\alpha\text{-Si III}}(k) &= a_{\text{cross}} \cos(\Delta v k) \exp\left(\frac{k}{k_{\text{cross}}}\right) P_{\text{Ly}\alpha}(k), \quad (18)
 \end{aligned}$$

where a_{auto} , a_{cross} , k_{auto} , and k_{cross} are the free parameters. The exponential terms capture the scale dependence of the line broadening and the spatial variations in the silicon abundance and Si III fraction, and also allow for decorrelation of the Si III and Ly α absorption in the cross-power spectrum oscillations (we will show later that k_{cross} is always negative in our models). In the next section, we shall demonstrate equation (18) provides good approximation to our numerical simulations.

4 VALIDATION OF THE IMPROVED MODEL

4.1 Fit to the fiducial hydrodynamical simulation

We now test equation (18) by performing a direct comparison to the power spectrum computed from the Ly α and Si III absorption in the Sherwood–Relics hydrodynamical simulation described in Section 2.

Our first task is to assess how our proposed model fares relative to the approach used in the existing literature, given by equations (6)–(8) (P. McDonald et al. 2006). The results of this test are summarized in Fig. 5, where we compare our model to simulation data (black circles with error bars) binned to match the scales probed by one-dimensional power spectrum measurements from eBOSS (S. Chabanier et al. 2019, top row) and high resolution data from Keck/HIRES and VLT/UVES (E. Boera et al. 2019, bottom row). We consider six different redshift bins spanning $2.2 \leq z \leq 5.0$. The error bars on $P_{\text{tot}}(k)/P_{\text{Ly}\alpha}(k)$ show the 68 per cent confidence interval obtained by bootstrapping the simulated sightlines. Note these error bars represent the variance in the simulation and do not correspond to expected uncertainties on observational data (for the latter, see Section 5).

The top three panels in Fig. 5 use the eBOSS binning. This only extends to $k \sim 0.02 \text{ s km}^{-1}$ due to the low resolution and signal-to-noise of these data. The blue and red curves show the best fits for equations (6)–(8) M06 and our proposed modification in equation (18), respectively. The minimum χ^2 and degrees of freedom are displayed in the inset; the fits are extremely good. The χ^2 values for the model proposed here are slightly smaller than those of M06. This is because our model reproduces the modest decline in the oscillation amplitude, which the M06 model fails to capture. Hence, at the scales typically probed by low to moderate resolution Ly α forest spectra, the small-scale enhancement to power expected from the Si III absorption is negligible. Reassuringly, this also confirms that the M06 ansatz remains a reasonable approximation for modelling the Si III oscillations in the power spectrum at $k \lesssim 0.02 \text{ s km}^{-1}$. Furthermore, the best fit to our hydrodynamical simulation requires $a = 0.008$ at $z = 3$ which is consistent with the value inferred from eBOSS, $a = 0.018 \pm 0.012$ (S. Chabanier et al. 2019).⁴

However, when including smaller scales using the high-resolution binning in the bottom three panels of Fig. 5, the enhancement in small-scale power arising from the Si III absorption becomes relevant. In this case equation (18) continues to provide a very good approximation to $P_{\text{tot}}(k)/P_{\text{Ly}\alpha}(k)$, but the M06 model fails to capture this behaviour and the fit to the simulation becomes increasingly poor towards smaller scales/larger wavenumbers. Our revised model, given by equation (18), thus provides a more accurate template for modelling the correlated Si III absorption.

The best-fitting parameters in equation (18) to the power spectrum from our fiducial simulation are summarized in Table 1. Here, k_{auto} and a_{auto} are obtained by fitting $P_{\text{Si III}}(k)/P_{\text{Ly}\alpha}(k)$ at $k < 0.2 \text{ s km}^{-1}$. For k_{cross} and a_{cross} , however, we restrict the fit to $P_{\text{Ly}\alpha\text{-Si III}}(k)/P_{\text{Ly}\alpha}(k)$ to smaller wavenumbers, $k < 0.06 \text{ s km}^{-1}$. This upper limit is chosen to ensure the fit to $P_{\text{Ly}\alpha\text{-Si III}}(k)$ is sensitive to the oscillation amplitude in the cross term, rather than noise at small scales where we expect $P_{\text{Si III}}(k) > P_{\text{Ly}\alpha\text{-Si III}}(k)$ (see Fig. 3). At each redshift, we also report the absolute value of the fractional deviation, $\langle |r(k)| \rangle$, between the simulation and best fit, where

$$r(k) = \frac{[P_{\text{tot}}(k)/P_{\text{Ly}\alpha}(k)]_{\text{sim}}}{[P_{\text{tot}}(k)/P_{\text{Ly}\alpha}(k)]_{\text{fit}}} - 1. \quad (19)$$

For $k < 0.2 \text{ s km}^{-1}$, $\langle |r(k)| \rangle$ is always within 1 per cent of $P_{\text{tot}}(k)/P_{\text{Ly}\alpha}(k)$ obtained from the hydrodynamical simulation.

4.2 The effect of varying silicon abundance and temperature

Our second task is to test the generality of equation (18); how well does our revised analytical fit perform when confronted with simulations constructed using different silicon abundances and temperatures (i.e. the parameters that primarily set the column densities and line widths of the Si III absorption lines)? We test this by contrasting our fiducial model with spectra constructed assuming a different IGM silicon abundance, [Si/H], and/or IGM temperatures that have been rescaled in post-processing following the approach used by V. Iršič et al. (2024).

It is straightforward to change the [Si/H] used in our fiducial model. We will consider four alternative values for [Si/H] here. The first two bracket the [Si/H] constraints from J. Schaye et al. (2003) and A. Aguirre et al. (2004) as given by equation (1) and correspond to the 1σ lower and upper bounds on that measurement. Note, however, this does require the uncertain extrapolation of this measurement from $z = 4.1$ to $z = 5$. The other two models adopt constant values⁵ of [Si/H] = -4.0 and -3.5 . The temperature scaling is instead performed by rotating and translating the temperature–density plane in our fiducial simulation⁶ around the power-law temperature–density relation, $T = T_0 \Delta^{\gamma-1}$, which holds at $\Delta \leq 10$ (L. Hui & N. Y. Gnedin 1997; M. McQuinn & P. R. Upton Sanderbeck 2016). We vary this by $\Delta \log_{10} T_0 = \pm 0.2$ and $\Delta \gamma = \pm 0.2$, giving a range similar to the 1σ uncertainties on measurements of the IGM thermal state at $2 < z < 6$ (P. Gaikwad et al. 2020, 2021). By cross-varying [Si/H], T_0 , and γ , we therefore generate 25 sets of simulated spectra per redshift bin. These variations are explored for 15 redshift bins, ranging from $z = 2.2$ to 5.0 in steps of 0.2, yielding a total of 375 simulated data sets encompassing different redshifts, silicon abundances, and gas temperatures.

⁴We compute this from $f_{\text{Si III}} = 0.006 \pm 0.004$ reported by S. Chabanier et al. (2019), where they impose a redshift dependence $a = f_{\text{Si III}}/[1 - \bar{F}(z)]$ in their parametrization. Here, we have assumed the mean transmission from G. D. Becker et al. (2013) at $z = 3$, given by $\bar{F} = 0.669$.

⁵We show in Appendix A that our fiducial silicon abundance model gives very similar results to a model with constant [Si/H] = -3.0 .

⁶Recall that our fiducial thermal history is provided by the E. Puchwein et al. (2019) UV background model (see their fig. 7).

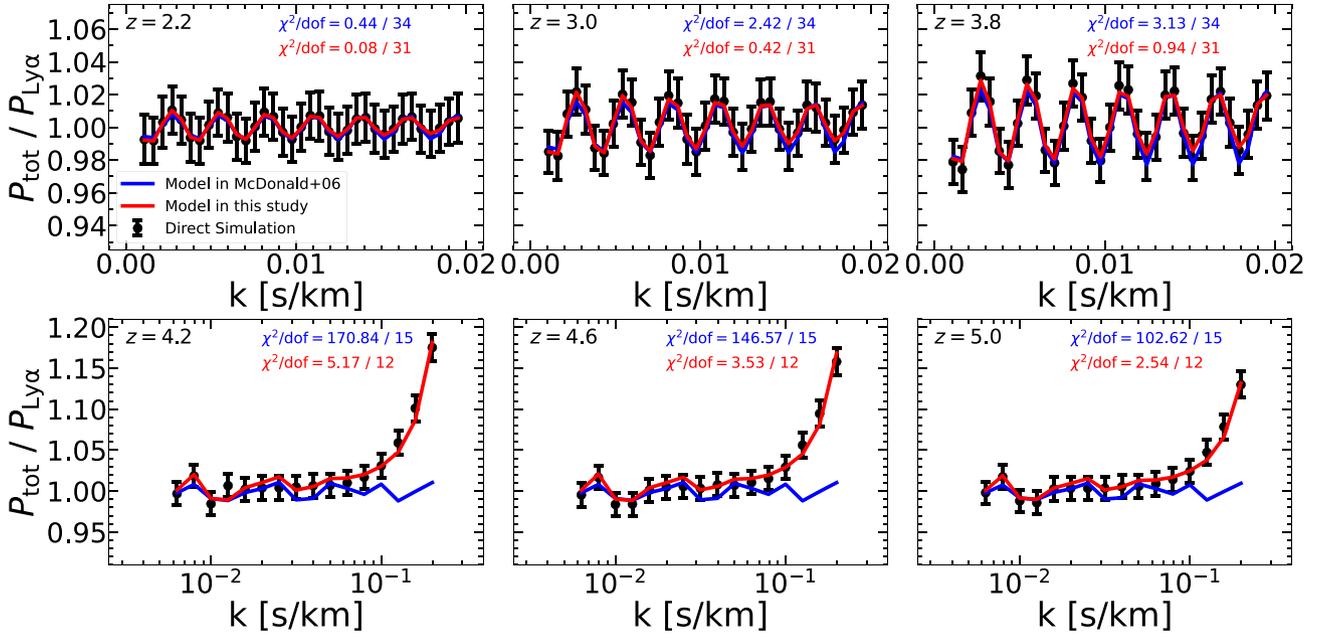


Figure 5. Comparison of analytical fits for $P_{\text{tot}}(k)/P_{\text{Ly}\alpha}(k)$ to the results from a hydrodynamical simulation drawn from the Sherwood–Relics suite (black circles with error bars). Each panel shows a different redshift, spanning the range $2.2 \leq z \leq 5.0$. The binning with wavenumber in the top row follows the eBOSS one-dimensional power spectrum (S. Chabanier et al. 2019, linear binning), while the bottom row follows the one-dimensional power spectrum measurement from E. Boera et al. (2019, logarithmic binning) obtained from high resolution Keck/HIRES and VLT/UVES spectra. The error bars give the 68 per cent confidence interval and represent the variance in the 5000 sightlines of length $40 h^{-1}$ cMpc used to construct the Ly α and Si III absorption spectra. The red and blue curves, respectively show the best fit using the function proposed in this work, equation (18), and the original M06 ansatz. At large scales, $k \lesssim 0.02 \text{ s km}^{-1}$, both models provide an excellent fit to the simulation. However, at smaller scales equation (18) performs much better because the contribution from the Si III power spectrum, $P_{\text{SiIII}}(k)$, becomes significant.

Table 1. Best-fitting values of equation (18) to the one-dimensional power spectrum predicted by our fiducial hydrodynamical simulation data at redshifts $2.2 \leq z \leq 5.0$. The parameters k_{auto} and a_{auto} are fit simultaneously to $P_{\text{SiIII}}(k)/P_{\text{Ly}\alpha}(k)$ within the range $k < 0.2 \text{ s km}^{-1}$, while k_{cross} and a_{cross} are fit simultaneously to $P_{\text{Ly}\alpha-\text{SiIII}}(k)/P_{\text{Ly}\alpha}(k)$ within the range $k < 0.06 \text{ s km}^{-1}$ (see the text for details). The last column reports the absolute value of the fractional deviation between the simulation and the fit for wavenumbers $k < 0.2 \text{ s km}^{-1}$.

z	$k_{\text{auto}}/10^{-2} \text{ s km}^{-1}$	$k_{\text{cross}}/10^{-2} \text{ s km}^{-1}$	$a_{\text{auto}}/10^{-3}$	$a_{\text{cross}}/10^{-2}$	$\langle r(k) \rangle$
2.2	2.96	−1.52	0.99	1.30	0.0063
2.4	3.06	−1.52	1.14	1.46	0.0049
2.6	3.22	−1.47	1.49	1.91	0.0050
2.8	3.48	−1.59	2.13	2.36	0.0059
3.0	3.84	−1.58	3.27	2.79	0.0058
3.2	4.23	−1.77	4.13	3.33	0.0060
3.4	4.62	−1.88	4.91	3.66	0.0063
3.6	4.99	−2.00	5.61	3.83	0.0067
3.8	5.22	−2.14	5.65	3.64	0.0062
4.0	5.30	−2.25	5.44	3.74	0.0057
4.2	5.63	−2.25	5.46	3.60	0.0058
4.4	5.81	−2.42	5.91	3.42	0.0061
4.6	5.85	−2.32	5.58	3.31	0.0059
4.8	5.90	−2.36	5.30	3.17	0.0054
5.0	5.91	−2.33	4.80	3.12	0.0055

We fit equation (18) to each data set within the range $k < 0.2 \text{ s km}^{-1}$. The behaviour of the best-fitting parameters at $z = 3$ is summarized in Fig. 6. The parameter k_{cross} is always negative, consistent with a Ly α –Si III oscillation amplitude that declines towards smaller scales. It is mainly sensitive to variation in the thermal parameter T_0 , with more rapid damping of the oscillations (less negative k_{cross}) for hotter gas. This happens because the coeval Ly α absorbers – which are typically on the flat part of the curve of

growth (see Fig. 4) – are broadened as the gas temperature increases. This has the effect of shifting the decorrelation between Si III and Ly α to larger scales. The parameter k_{auto} is instead primarily sensitive to the variation in T_0 , and is only weakly dependent on the silicon abundance, $[\text{Si}/\text{H}]$, and the slope of the temperature density relation, γ . Hotter temperatures increase the thermal broadening of the Si III and Ly α absorption features, yielding a smaller k_{auto} (or equivalently, a larger smoothing scale $\sim 1/k_{\text{auto}}$). Note this

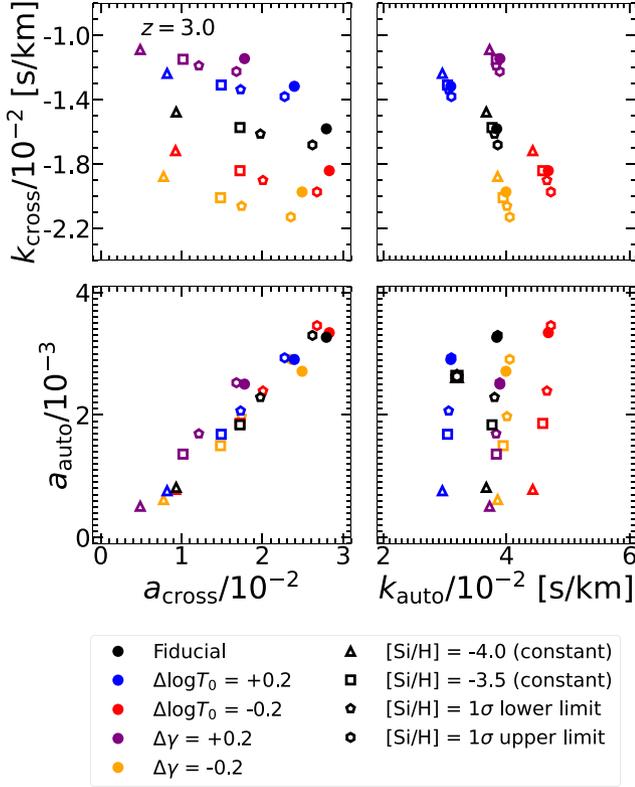


Figure 6. The best-fitting values of k_{auto} , k_{cross} , a_{auto} , and a_{cross} (see equation 18) at $z = 3$. The models are constructed using different silicon abundances and gas temperatures (see the text for details). The temperature variations are indicated by different colours, while the silicon abundance variations are indicated by the different marker styles. Changing the temperature at mean density, T_0 , primarily affects k_{auto} and k_{cross} , while changing the silicon abundance affects a_{auto} and a_{cross} .

produces an *enhancement* in $P_{\text{tot}}(k)/P_{\text{Ly}\alpha}(k)$ at small scales. Colder temperatures have the opposite effect, suppressing the additional small-scale power from the Si III absorbers in $P_{\text{tot}}(k)/P_{\text{Ly}\alpha}(k)$.⁷ The insensitivity of $P_{\text{tot}}(k)/P_{\text{Ly}\alpha}(k)$ to γ is likely because most of the Ly α forest arises from gas close to the mean density, $\Delta = 1$ (see e.g. table 3 in G. D. Becker et al. 2011).

In contrast, variations in silicon abundance primarily affect the amplitude parameters a_{auto} and a_{cross} , while leaving k_{auto} and k_{cross} largely unchanged. The silicon abundance $[\text{Si}/\text{H}]$ directly rescales the Si III optical depths and hence the Si III power spectrum. A higher silicon abundance increases the Si III contribution to the total absorption, thereby raising both the Si III power spectrum and $P_{\text{tot}}(k)/P_{\text{Ly}\alpha}(k)$. The apparent correlation between a_{auto} and a_{cross} in the lower left panel of Fig. 6 also motivated us to explore the possibility of using a simple linear scaling to reduce these two amplitudes to a single parameter. Unfortunately, however, on careful testing we found this linear scaling does not remain fixed on varying the wavenumber range used for the fit. A four parameter model is thus required for generality.

Over the full redshift range considered here, $2.2 \leq z \leq 5.0$, the absolute value of the fractional deviation, $\langle |r(k)| \rangle$, for the best-

⁷Another way to think of this is that hotter (colder) temperatures suppress (enhance) small-scale power for both $P_{\text{Ly}\alpha}(k)$ and $P_{\text{Si III}}(k)$, but $P_{\text{Ly}\alpha}(k)$ is *always suppressed (enhanced)* to a greater degree than $P_{\text{Si III}}$ because of the mass dependence of the thermal broadening kernel (see equation 9).

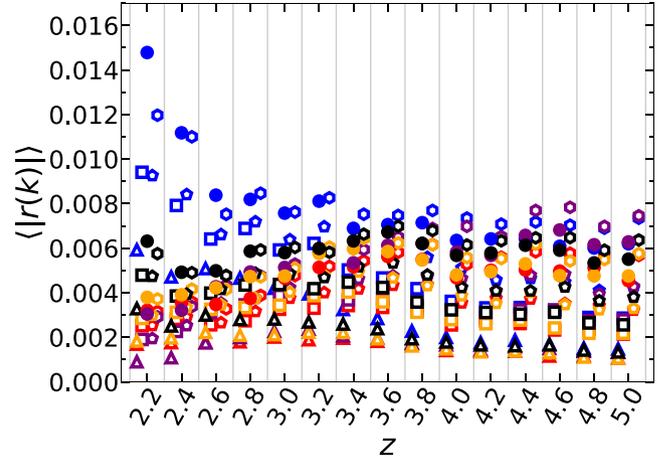


Figure 7. The absolute value of the fractional deviation, $\langle |r(k)| \rangle$, obtained when fitting equation (18) to simulation data at $k < 0.2 \text{ s km}^{-1}$ constructed assuming different silicon abundances and temperatures as a function of redshift (see the text for details). The colour and marker styles are the same as those listed in the legend of Fig. 6.

fitting parameters in equation (18) to each model are displayed in Fig. 7. In general, the fit quality degrades for higher temperatures and larger silicon abundances, most likely because the assumption of $\sim \exp(k/k_{\text{auto}})$ scaling made in Section 3 begins to break down. The fit quality is also poorer at lower redshift as the Ly α forest becomes more transmissive. Nevertheless, $\langle |r(k)| \rangle$ remains below 2 per cent for all models (and below 1 per cent for the fiducial model), indicating that equation (18) continues to provide a very good description of the simulation data across all tested conditions.

4.3 The effect of varying redshift

Finally, in Fig. 8 we examine how the best-fitting parameters in equation (18) vary with redshift. We first consider k_{auto} in the upper left panel. For all models we find k_{auto} increases towards higher redshift. This behaviour is driven by the redshift evolution of the IGM temperature and pixel-to-pixel variations in the Si III absorption. Recall from Section 2 that k_{auto} characterizes the scale dependence of $P_{\text{Si III}}(k)$, which has contributions from both the pixel-dependent optical depth ratio (i.e. equation 10) and thermal broadening. The former depends on the redshift evolution of the silicon abundance, $[\text{Si}/\text{H}]$, and the Si III fraction, $x_{\text{Si III}}$. The latter introduces a redshift dependence according to $k_{\text{auto}}(z) \propto b^{-2}(z) \propto T^{-1}(z)$, or equivalently $k_{\text{auto}}(z) \propto T_0^{-1}(z)$ if the γ redshift dependence is weak. In our fiducial model the pixel dependency has a redshift evolution that approximately follows the behaviour of $a_{\text{auto}}(z)$ in the lower left panel of Fig. 8 (see discussion below) whereas $T_0(z)$ peaks at $z \sim 3$ due to photoheating from He II reionization (see e.g. E. Puchwein et al. 2019; Gaikwad et al. 2021). In contrast, we find k_{cross} decreases towards higher redshift so that the cross-term oscillations are damped less rapidly at earlier times. In our model, this is driven by the increasing abundance of Si III towards higher redshift. This shifts the coeval Ly α absorption lines to lower column densities and pushes the decorrelation between Si III and Ly α to smaller scales.

Next, we consider the amplitude parameters, a_{auto} and a_{cross} . Following the pioneering work by M06, previous studies have typically imposed a redshift dependence $a(z) = f/[1 - \bar{F}(z)]$ where $\bar{F}(z)$ is the mean transmission in the Ly α forest and f is a free parameter. Under this assumption the amplitude of the Si III power

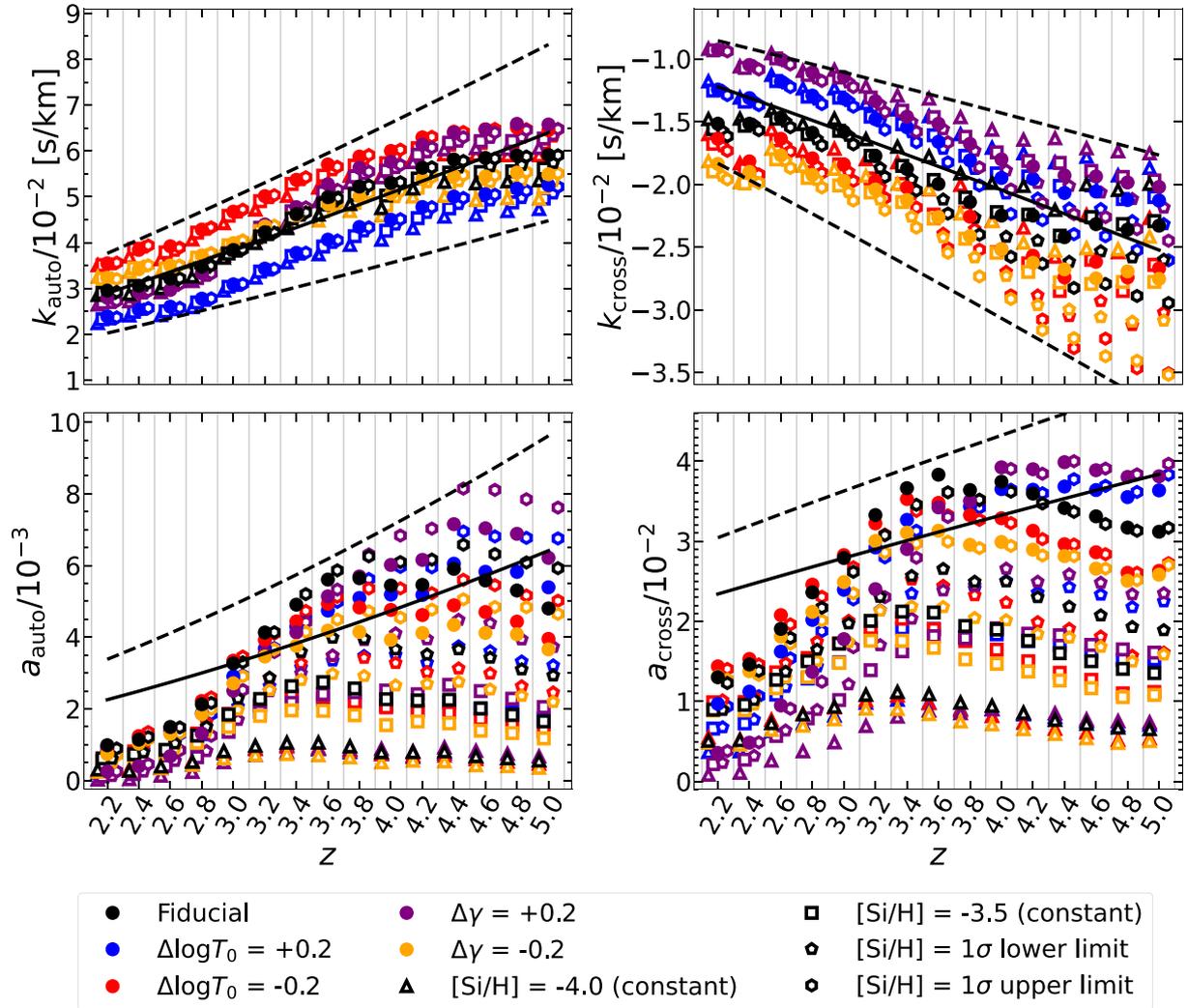


Figure 8. The redshift evolution of the best-fitting parameters for equation (18) to numerical simulations of $P_{\text{tot}}(k)/P_{\text{Ly}\alpha}(k)$ at $k < 0.2 \text{ s km}^{-1}$ that assume different silicon abundances and IGM temperatures. The colour and marker styles follow those in Fig. 7. The solid black lines are power-law approximations to the redshift evolution of the fiducial model, where $k_{\text{auto}}(z) = 3.84 \times 10^{-2}[(1+z)/4]^{1.26} \text{ s km}^{-1}$, $k_{\text{cross}}(z) = -1.58 \times 10^{-2}[(1+z)/4]^{1.15} \text{ s km}^{-1}$, $a_{\text{auto}}(z) = 3.27 \times 10^{-3}[(1+z)/4]^{1.66}$, and $a_{\text{cross}}(z) = 2.79 \times 10^{-2}[(1+z)/4]^{0.79}$. The dashed grey lines show these power laws scaled by a constant to encompass the various models and thus give a rough indication of the prior range, where $0.7 \leq A \leq 1.3$ for $Ak_{\text{auto}}(k)$, $0.7 \leq B \leq 1.5$ for $Bk_{\text{cross}}(k)$, $0 \leq C \leq 1.5$ for $Ca_{\text{auto}}(k)$, and $0 \leq D \leq 1.3$ for $Da_{\text{cross}}(k)$.

spectrum declines towards higher redshift as \bar{F} decreases. However, in this work we find the opposite trend in the simulation data; the amplitude increases with redshift at $z \lesssim 3.5$ and remains relatively flat at higher redshifts, as shown by the redshift evolution of a_{auto} and a_{cross} in the lower panels of Fig. 8. We have further checked this by fitting both our model and the original M06 ansatz to our simulations at $k < 0.02 \text{ s km}^{-1}$. The redshift evolution for the amplitude parameters (not shown here) yielded the same trend, confirming this is not simply a consequence of including small-scale power in the fit. Note again the changes in a_{auto} and a_{cross} are insensitive to thermal variations and their redshift evolution is driven almost entirely by the silicon abundance, $[\text{Si}/\text{H}]$, and/or the Si III fraction, $x_{\text{Si III}}$.

Overall, this implies the redshift evolution of a_{auto} and a_{cross} is primarily driven by $x_{\text{Si III}}$ in our models. This is corroborated by the redshift evolution of $x_{\text{Si III}}$ shown in the right panel of Fig. 1, which exhibits a qualitatively similar trend to a_{auto} and a_{cross} . It is

therefore challenging for the empirically calibrated UV background synthesis model used in this work (E. Puchwein et al. 2019) to produce an Si III power spectrum and cross-term amplitude that scales as $[1 - \bar{F}(z)]^{-1}$ if the silicon abundance does not evolve strongly with redshift. Counteracting the $x_{\text{Si III}}$ evolution would require $[\text{Si}/\text{H}]$ to increase at least as rapidly as the corresponding decrease in $x_{\text{Si III}}$ towards lower redshift as the UV background spectrum hardens (see Appendix A).

It is, nevertheless, useful to parametrize the redshift evolution of k_{auto} , k_{cross} , a_{auto} , and a_{cross} to facilitate marginalization over multiple redshift bins. One possibility is to adopt the redshift scaling of our fiducial model in Table 1. This is model-dependent, however, and may be overly restrictive. A more flexible but slightly less accurate alternative is to approximate the redshift evolution using power laws (see e.g. M. Viel et al. 2013, for a similar approach). We use $k_{\text{auto}}(z) = 3.84 \times 10^{-2}[(1+z)/4]^{1.26} \text{ s km}^{-1}$, $k_{\text{cross}}(z) = -1.58 \times 10^{-2}[(1+z)/4]^{1.15} \text{ s km}^{-1}$, $a_{\text{auto}}(z) = 3.27 \times$

$10^{-3}[(1+z)/4]^{1.66}$, and $a_{\text{cross}}(z) = 2.79 \times 10^{-2}[(1+z)/4]^{0.79}$ to approximate our fiducial model. These are shown as the black solid lines in Fig. 8. The dashed black lines are scaled by a constant to

encompass the various models and thus give a rough indication of the prior range for the amplitude of these relations, where $0.7 \leq A \leq 1.3$ for $Ak_{\text{auto}}(k)$, $0.7 \leq B \leq 1.5$ for $Bk_{\text{cross}}(k)$, $0 \leq C \leq 1.5$ for $Ca_{\text{auto}}(z)$, and $0 \leq D \leq 1.3$ for $Da_{\text{cross}}(z)$. The amplitude priors are derived from the 1σ uncertainties on the observational constraints on the gas temperature and silicon abundance in the IGM (see Section 4.2). Note also that we adopt only upper limits for the priors on $a_{\text{auto}}(z)$ and $a_{\text{cross}}(z)$ to account for the possibility the power spectrum is consistent with no Si III absorption. The slopes of these power laws may also be varied if gas temperature and silicon abundance redshift evolutions that differ from our fiducial model are required. Note, however, that both the redshift evolution of the UV background (which impacts the Si III fraction, $x_{\text{Si III}}$) and IGM temperature at $2 \leq z \leq 5$ are both relatively well constrained by observation (E. Puchwein et al. 2019; P. Gaikwad et al. 2021).

4.4 Recommended implementation

Here, we summarize the implementation of the Si III model presented in this work. The full correction to the Ly α only power spectrum, $P_{\text{Ly}\alpha}(k)$, as a function of wavenumber, for $k \leq 0.2 \text{ s km}^{-1}$, and redshift, for $2.2 \leq z \leq 5.0$, is given by

$$\frac{P_{\text{tot}}(k, z)}{P_{\text{Ly}\alpha}(k, z)} = 1 + Ca_{\text{auto}}(z) \exp\left(\frac{k}{Ak_{\text{auto}}(z)}\right) + Da_{\text{cross}}(z) \exp\left(\frac{k}{Bk_{\text{cross}}(z)}\right) \cos(\Delta v k), \quad (20)$$

where A , B , C , and D scale the redshift dependent parameters $k_{\text{auto}}(z)$, $k_{\text{cross}}(z)$, $a_{\text{auto}}(z)$, and $a_{\text{cross}}(z)$, and $\Delta v = 2271 \text{ km s}^{-1}$ is the velocity separation of coeval Ly α and Si III absorption. We suggest reasonable prior ranges on A , B , C , and D are $0.7 \leq A \leq 1.3$, $0.7 \leq B \leq 1.5$, $0 \leq C \leq 1.5$, and $0 \leq D \leq 1.3$ (where $C, D = 0$ corresponds to no Si III contribution to the power) based on the range of models considered in Fig. 7. Note, however, these prior ranges may be extended if a larger range of IGM temperatures or silicon abundances are of interest. There are then two choices for the redshift-dependent parameters:

(i) **Method I:** Our preferred approach is to adopt the redshift scaling for $k_{\text{auto}}(z)$, $k_{\text{cross}}(z)$, $a_{\text{auto}}(z)$, and $a_{\text{cross}}(z)$ given by the interpolation of the data in Table 1. There are four free parameters in equation (20): A , B , C , and D . For our fiducial IGM model $A = 1$, $B = 1$, $C = 1$, and $D = 1$. The advantage of this approach is that it better captures the redshift evolution in our fiducial IGM model but is less flexible than an approximate power-law redshift scaling.

(ii) **Method II:** Adopt the approximate power-law redshift scalings $k_{\text{auto}}(z) = 3.84 \times 10^{-2}[(1+z)/4]^{1.26} \text{ s km}^{-1}$, $k_{\text{cross}}(z) = -1.58 \times 10^{-2}[(1+z)/4]^{1.15} \text{ s km}^{-1}$, $a_{\text{auto}}(z) = 3.27 \times 10^{-3}[(1+z)/4]^{1.66}$, and $a_{\text{cross}}(z) = 2.79 \times 10^{-2}[(1+z)/4]^{0.79}$, again giving four free parameters in equation (20): A , B , C , and D . For additional flexibility the slopes of the power laws may also be varied to allow for IGM temperatures and silicon abundances that have redshift dependencies that differ significantly from our fiducial model. Note, however, that both the IGM temperature and UV background at $2.2 \leq z \leq 5$ are observationally well-constrained and these parameters should therefore be varied judiciously (see also Appendix A).

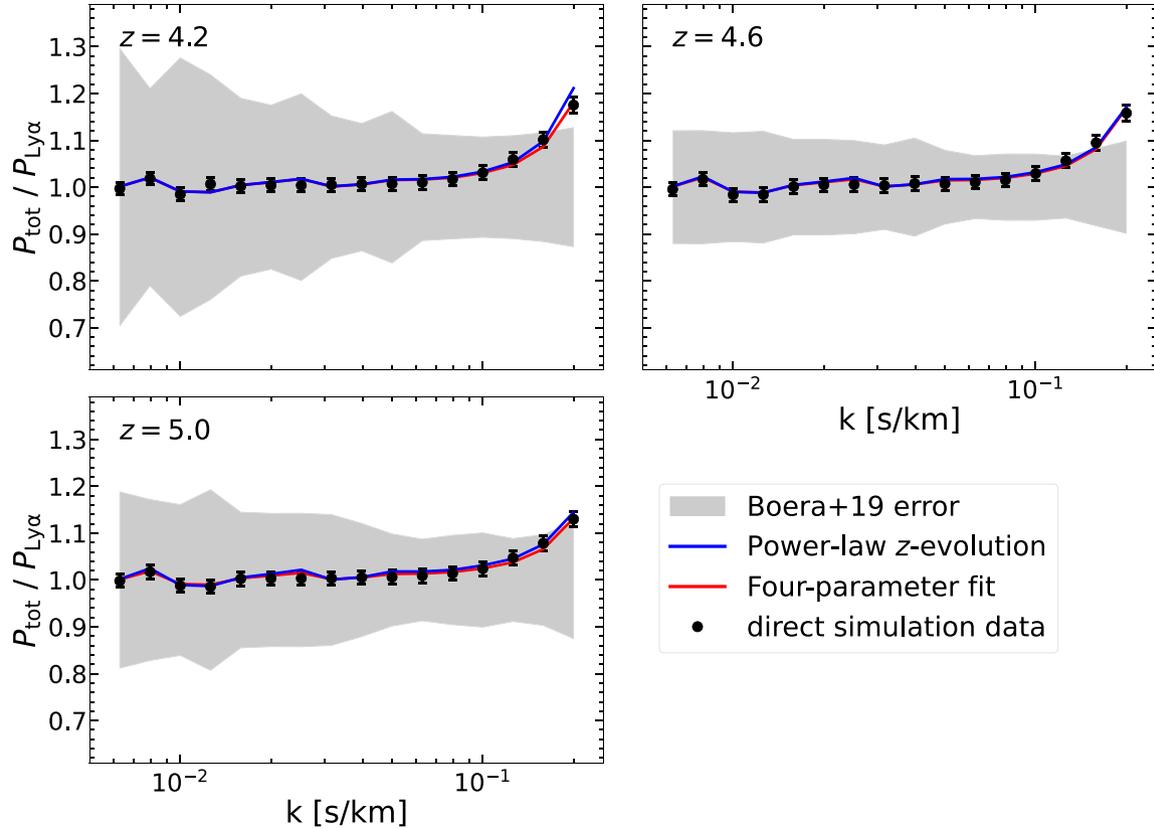


Figure 9. Comparison of our fiducial model with the fractional uncertainties, $\sigma/P_{\text{Ly}\alpha}$, on the one-dimensional power spectrum measurements from E. Boera et al. (2019, grey shaded regions). The red curves represent direct four-parameter fit to equation (18) (**Method I**), while the blue curves represent fits imposing power-law redshift dependencies (**Method II**) described in Section 4.4. We also overplot the direct simulation results using black dots. The error bars represent the 68 per cent confidence interval on the simulation data, matching the results in the bottom panels of Fig. 5. The small-scale power enhancement due to Si III absorption exceeds the E. Boera et al. (2019) uncertainty in the largest k -bins ($k = 0.2 \text{ s km}^{-1}$) for all three redshift bins.

5 IMPLICATIONS FOR WARM DARK MATTER CONSTRAINTS

To assess the implications of our correlated Si III absorption model for existing warm dark matter constraints from the one-dimensional Ly α forest power spectrum, we compare our model to the uncertainties on the E. Boera et al. (2019) power spectrum measurement (grey shading) in Fig. 9. The contribution from Si III becomes increasingly prominent at small scales. The red and blue curves show, respectively, our best-fit fiducial models using **Method I** and **Method II** described in Section 4.4. The black circles show the results from our hydrodynamical simulation with the same error bars displayed in Fig. 5. At scales $k \sim 0.2 \text{ s km}^{-1}$, the small-scale power from the Si III power spectrum – which was not accounted for in earlier models – becomes comparable to the observational error bars.

It is thus interesting to investigate the impact of our model on existing warm dark matter (WDM) constraints from one-dimensional Ly α power spectrum measurements. V. Iršič et al. (2024) recently used the measurement of E. Boera et al. (2019) to set a stringent lower bound on the mass of a WDM thermal relic. Their best-fitting WDM model showed excellent agreement at intermediate scales ($k < 0.1 \text{ s km}^{-1}$; $\chi^2/\text{dof} = 20.4/20$), giving a limit of $m_{\text{WDM}} \geq 4.1 \text{ keV}$ (at 95 per cent confidence). However, extending their analysis to smaller scales ($k < 0.2 \text{ s km}^{-1}$) resulted in a somewhat poorer fit ($\chi^2/\text{dof} = 40.7/34$) with a limit of $m_{\text{WDM}} \geq 5.7 \text{ keV}$. V. Iršič et al.

(2024) noted the larger reduced χ^2 was due to additional small-scale power at $k > 0.1 \text{ s km}^{-1}$ – precisely where our study reveals substantial Si III driven enhancements to the small-scale power.

To quantify the contribution of correlated Si III absorption identified in this study, we incorporate our model (using **Method I**) into the V. Iršič et al. (2024) pipeline and compare the best-fitting models for the one-dimensional Ly α power spectrum with and without our Si III model. We present this comparison in Fig. 10. We find that k_{auto} and a_{auto} are prior-dominated, while k_{cross} and a_{cross} remain unconstrained. This is due to the dominance of the small-scale enhancement from $P_{\text{Si III}}(k)$ over the cross-term oscillations for the wavenumbers considered here. When using physically motivated priors as described in Section 4.4, the χ^2/dof changes from 40.7/34 in V. Iršič et al. (2024) to 40.6/32, shown by the blue curves in Fig. 10.⁸ Consequently, the goodness-of-fit remains similar and the constraint on the WDM mass slightly weakens from $m_{\text{WDM}} \geq 5.71 \text{ keV}$ to $m_{\text{WDM}} \geq 5.65 \text{ keV}$. To better fit the small-scale enhancement, we also try a looser prior range on the silicon abundance such that $0 \leq C \leq 3$ and $0 \leq D \leq 2$, equivalent to allowing $[\text{Si}/\text{H}] \sim -2.5$ (this is within 3σ of our fiducial silicon abundance model). The results are shown by the red curves in Fig. 10. In this case, the χ^2/dof

⁸Note that because k_{cross} and a_{cross} are unimportant for the fit in this example, the degrees of freedom are effectively 32 instead of 30, despite now having four extra free parameters.

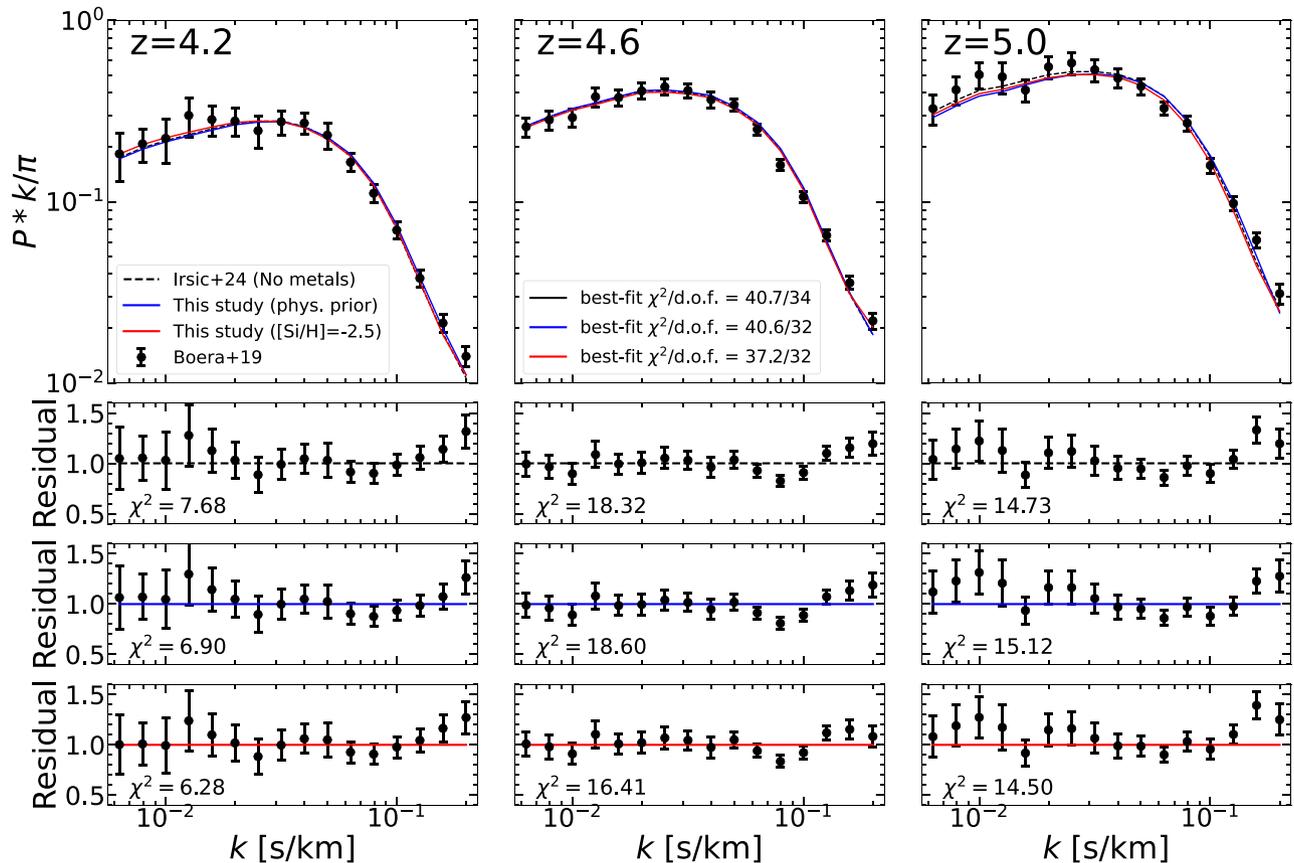


Figure 10. Comparison of the V. Iršič et al. (2024) best-fitting model (black dashed curves) to our Si III model using two different priors (red and blue solid curves). The blue curves show best-fitting model using the physical priors described in Section 4.2, while the red curves use a broader prior range, equivalent to allowing a larger silicon abundance of $[\text{Si}/\text{H}] = -2.5$. The black data points are the one-dimensional power spectrum measurements from E. Boera et al. (2019) in three redshift bins. The lower panels show the residuals of the E. Boera et al. (2019) measurements with respect to the models.

is $37.2/32$, and the constraint on the WDM mass now weakens from $m_{\text{WDM}} \geq 5.71$ keV to $m_{\text{WDM}} \geq 5.58$ keV.

In summary, for physically motivated priors our Si III model slightly reduces the χ^2 for the E. Boera et al. (2019) power spectrum on small scales and does not significantly change the existing constraints on warm dark matter from V. Iršič et al. (2024). Note, however, the goodness-of-fit remains similar due to the additional free parameters. If instead adopting a prior equivalent to $[\text{Si}/\text{H}] \sim -2.5$, the goodness-of-fit improves slightly and the V. Iršič et al. (2024) WDM constraints weakens by 0.07 keV. The silicon abundance is observationally uncertain, however, and it is unclear whether this level of IGM metal enrichment is warranted at $z \sim 5$. Regardless, our results suggest that the small-scale power enhancement observed in the E. Boera et al. (2019) data by V. Iršič et al. (2024) may at least be partially attributable to correlated Si III absorption. We also expect this effect will become more relevant in the future as the precision of constraints on small-scale power at $z > 4$ improve.

6 SUMMARY

We have used a hydrodynamical simulation drawn from the Sherwood–Relics simulation suite (J. S. Bolton et al. 2017; E. Puchwein et al. 2023) to construct an improved model for the impact of correlated Si III absorption on the one-dimensional Ly α forest power spectrum. We have identified a significant enhancement in

the one-dimensional power spectrum due to Si III at small scales ($k \gtrsim 0.06$ s km $^{-1}$) that is not explicitly accounted for in previous work. To address this limitation, we have developed a physically motivated fitting function to extend the canonical framework for modelling the Ly α –Si III correlation first introduced by M06. Our model takes into account two critical effects missed previously – the fact that Si III line profiles are narrower than those of Ly α , and that the ratio of coeval Ly α and Si III optical depths are not constant.

Our improved model yields a four parameter fit (see equation 20) where k_{auto} controls the small-scale power enhancement, k_{cross} controls the damping of the Ly α –Si III cross power spectrum oscillation amplitude, and a_{auto} and a_{cross} independently control the amplitudes of the Si III power spectrum and the Ly α –Si III cross-correlation power spectrum, respectively. We validate this model by comparison to multiple simulated data sets at redshifts $2.2 \leq z \leq 5.0$ for wavenumbers $k \lesssim 0.2$ s km $^{-1}$. The absolute value of the fractional deviation between our fitting function and all tested data sets is within 2 per cent, suggesting good agreement between our fitting function and the simulation data across a variety of IGM thermal and metal enrichment histories. Incorporating this effect will be essential for future cosmological and astrophysical constraints using high resolution Ly α forest data. We provide full guidelines for the implementation of the model within existing parameter inference frameworks in Section 4.4.

We also find the redshift dependence reported in M06 typically assumed for the amplitude of the Si III and Ly α –Si III cross power

spectra – such that $a(z) \propto [1 - \bar{F}(z)]^{-1}$ where $\bar{F}(z)$ is the mean transmission in the Ly α forest and f is a free parameter – is not observed in our fiducial model. Under this assumption the amplitude of the Si III oscillations declines towards higher redshift as \bar{F} decreases. Instead, our empirically constrained simulations return the opposite trend; the amplitude increases with redshift at $2 \leq z \leq 3.5$ and remains approximately constant at higher redshifts. This is primarily due to the hardening of our assumed model for the metagalactic UV background spectrum at $z \lesssim 3.5$ for photon energies around the Si III ionization potential of 33.5 eV, coupled with modest evolution in the intergalactic silicon abundance at $2 < z < 5$ (J. Schaye et al. 2003; A. Aguirre et al. 2004). The origin of this spectral hardening is the increasingly important contribution from quasar emission to the UV background (e.g V. Khaire & R. Srianand 2019; E. Puchwein et al. 2019). Counteracting this trend would require an intergalactic silicon abundance that increases more rapidly towards lower redshift than has been assumed in our fiducial model, and/or a UV background model with a softer ionizing spectrum.

Finally, our Si III absorption model has little impact on existing constraints on WDM free streaming if adopting physically motivated priors on the IGM temperature and silicon abundance, but it may partially account for the otherwise difficult-to-fit small-scale power enhancement at $k > 0.1 \text{ s km}^{-1}$ in the E. Boera et al. (2019) power spectrum measurements noted by V. Iršič et al. (2024). We anticipate that modelling this effect will become more important as the precision of the observational data improves. Our results also present an opportunity to place novel constraints on the metal enrichment of the low density IGM using the one-dimensional power spectrum. We intend to explore this possibility in future work.

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DATA AVAILABILITY

All data and analysis code used in this work are available from the corresponding author on reasonable request. Further guidance on accessing the publicly available Sherwood–Relics simulation data may also be found at <https://www.nottingham.ac.uk/astronomy/sherwood-relics/>.

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THE REDSHIFT EVOLUTION OF THE SILICON ABUNDANCE

In Section 4.3, we demonstrated that the redshift evolution of a_{auto} and a_{cross} is largely driven by the Si III fraction, $x_{\text{Si III}}$, when assuming

a constant intergalactic silicon abundance $[\text{Si}/\text{H}]$. Furthermore, our fiducial model for $[\text{Si}/\text{H}]$ [A. Aguirre et al. 2004, see equation (1); J. Schaye et al. 2003] has a weak redshift dependence. Hence, in both cases, the hardening of the intergalactic radiation field during He II reionization at $z \sim 3$ causes the Si III power spectrum and cross-term amplitudes to *increase* from $z = 2$ to $z \simeq 3.5$ and remain almost constant at $z \gtrsim 3.5$. This contrasts with the usual $[1 - \bar{F}(z)]^{-1}$ scaling used in the literature (e.g. P. McDonald et al. 2006; N. Palanque-Delabrouille et al. 2013; S. Chabanier et al. 2019).

We consider this further in Fig. A1, where we examine the effect of varying the redshift dependence of the silicon abundance, $[\text{Si}/\text{H}]$, on the redshift evolution of the best-fitting parameters in equation (18). We consider our fiducial model (black circles), a constant $[\text{Si}/\text{H}] = -3$ (green points), and a toy model where the silicon abundance decreases by a factor 10^3 from $z = 2$ to $z = 5$, $[\text{Si}/\text{H}] = -3 - (z - 3)$ (red points). Note this is an extreme model and is inconsistent with the pixel optical depth constraints. In contrast, over the same redshift interval the total mass of metals traced by H II absorption systems decreases by a factor of ~ 10 in the Evolution and Assembly of Galaxies and their Environments (EAGLE) hydrodynamical simulation (A. Rahmati & B. D. Oppenheimer 2018, their fig. 5). A similar factor of ~ 10 decrease is observed for gas in the simulations presented by S. Shen, J. Wadsley & G. Stinson (2010, their fig. 6). For the final toy model (blue dots) we therefore decrease the silicon abundance by a factor ~ 10 from $z = 2$ to $z = 5$.

For k_{auto} and k_{cross} , shown in the upper panel of Fig. A1, variations in the $[\text{Si}/\text{H}]$ redshift evolution have a comparatively minor effect, with the most noticeable impact occurring at higher redshifts ($z > 3$). In the lower panels, for constant $[\text{Si}/\text{H}] = -3$ the evolution of a_{auto} and a_{cross} is primarily driven by the Si III fraction $x_{\text{Si III}}$, as already discussed in Section 4.3. Consequently, these parameters increase towards higher redshift (green dots), mirroring the redshift dependence of $x_{\text{Si III}}$ in Fig. 1. Our fiducial model (black dots) coincides with the constant $[\text{Si}/\text{H}] = -3$ case because the redshift dependence in equation (1) is weak ($[\text{Si}/\text{H}] \propto 0.08(z - 3)$). To reverse the redshift evolution of a_{auto} and a_{cross} so that they decrease towards higher redshifts – as required by the canonical scaling, shown by the red solid and dashed curves – requires $[\text{Si}/\text{H}]$ to decrease towards higher redshift.

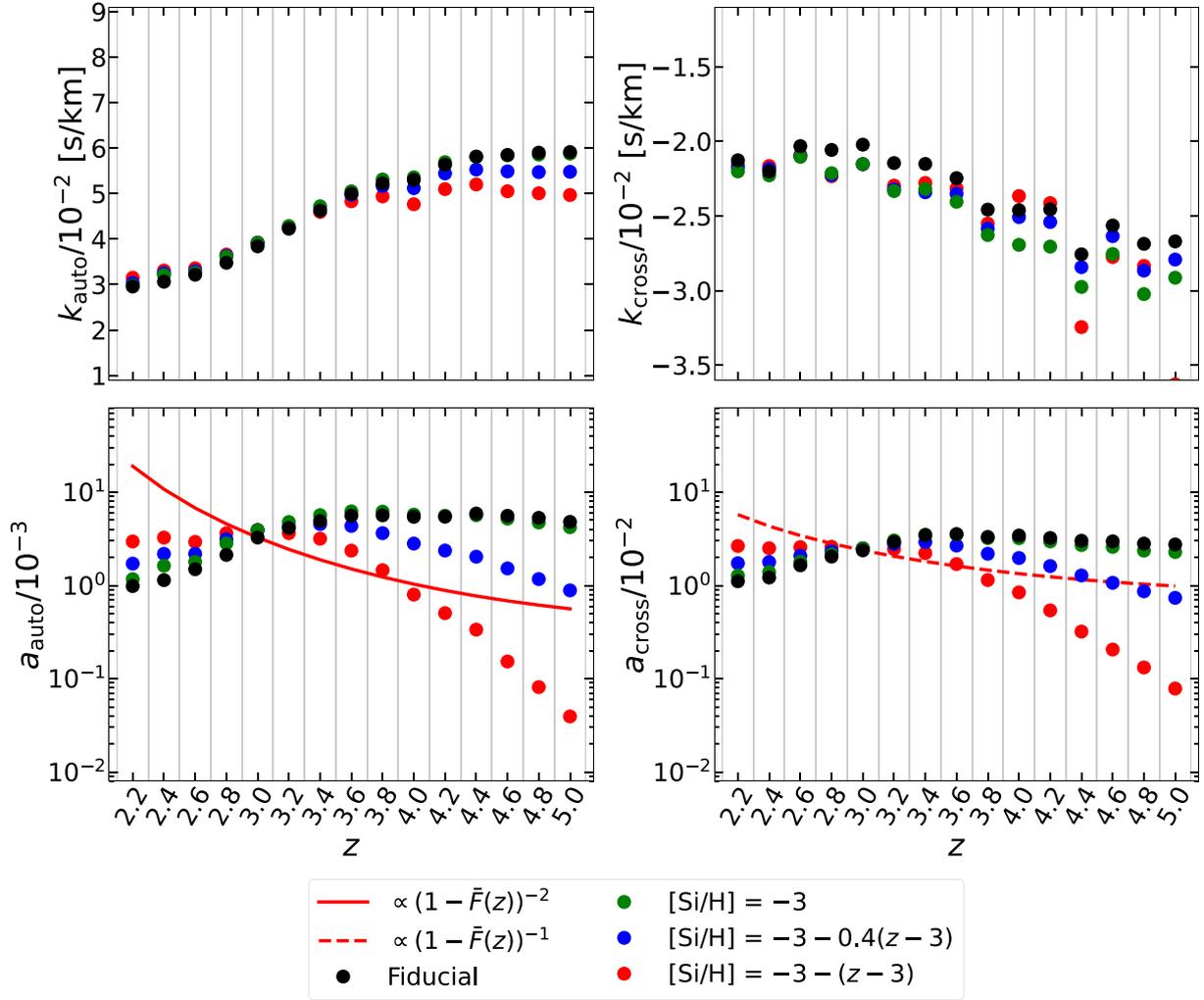


Figure A1. The best-fitting values of k_{auto} , k_{cross} , a_{auto} , and a_{cross} obtained for four different models for the intergalactic silicon abundance [Si/H]. In all cases the Si III fraction, $x_{\text{Si III}}$ is computed using the E. Puchwein et al. (2019) UV background synthesis model. The black points denote our fiducial model for the silicon abundance (equation 1) where [Si/H] $\propto 0.08(z - 3)$, whereas the green points assume a constant silicon abundance, [Si/H] = -3. The blue and red points instead correspond to a [Si/H] that declines respectively by a factor of 10 and 10^3 over $z = 2-5$. For comparison the red solid and dashed curves follow the redshift scaling usually assumed in the literature, $a_{\text{Si III}} \sim a^2(z) \propto [1 - \bar{F}(z)]^{-2}$ (lower left panel) and $a_{\text{Ly}\alpha\text{-Si III}} \sim a(z) \propto [1 - \bar{F}(z)]^{-1}$ (lower right panel), where $\bar{F}(z)$ is the mean Ly α forest transmission.

THE EFFECT OF VARYING THE MAXIMUM WAVENUMBER

In Section 4, we quantified the accuracy of our fit in Fig. 7 for wavenumbers $k \leq 0.2 \text{ s km}^{-1}$. To investigate the impact of varying the maximum wavenumber, k_{max} , of the simulated power spectrum we fit to, in Fig. B1, we additionally show the absolute value of the fractional deviation for the fit as a function of k_{max} across five different redshift bins. The solid curves correspond to the four-parameter fit used in this work (equation 18), while the dashed curves are for the M06 ansatz. Our model remains consistent

with the simulation within 1 per cent regardless of redshift or k_{max} . This demonstrates that our model is applicable across all scales considered and successfully captures both the declining oscillatory behaviour from the cross-power spectrum and the small-scale enhancement from the Si III power spectrum. For comparison, when fitting the M06 model to the simulation data, the relative errors are well below 1 per cent at $k_{\text{max}} < 0.06 \text{ s km}^{-1}$, but increase rapidly above that threshold as the small-scale enhancement due to the Si III–Si III term becomes significant.

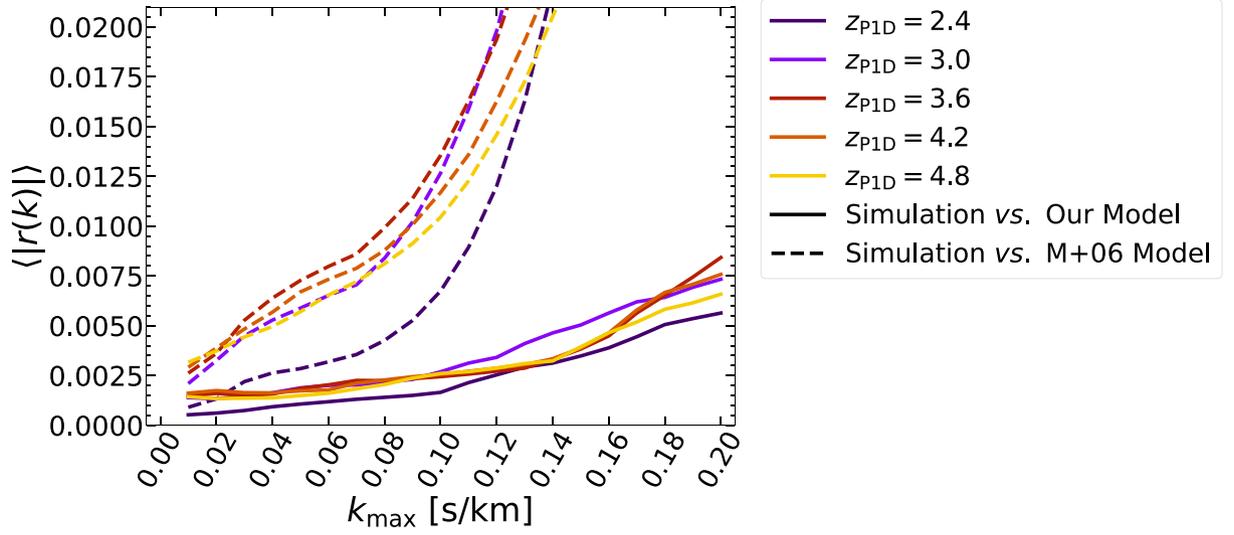


Figure B1. The absolute value of the fractional deviation for fits to the one-dimensional power spectrum from our fiducial simulation using either the improved model proposed in this work (solid curves) or the original M06 ansatz (dashed curves). The results are shown as a function of the maximum wavenumber used in the fit, k_{\max} . The different colours represent the redshift bins shown in the legend. Our model demonstrates good agreement with the simulation within 1 per cent at all redshifts for all k_{\max} values.

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