BER performance analysis of radio over free-space optical systems considering laser phase noise under Gamma-Gamma turbulence channels

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Abstract: This paper analytically investigates a bit error rate (BER) performance of radio over free space optical (FSO) systems considering laser phase noise under Gamma-Gamma turbulence channels. An external modulation using a dual drive Mach-Zehnder modulator (DD-MZM) and a phase shifter is employed because a DD-MZM is robust against a laser chirp and provides high spectral efficiency. We derive a closed form average BER as a function of different turbulence strengths and laser diode (LD) linewidth, and investigate its analytical behavior under practical scenario. As a result, for a given average SNR with normalized perturbation, it is shown that the difference of average BER corresponding to two LDs (with linewidth of 624MHz and 10MHz) under weak turbulence is almost 3 times larger than that under strong turbulence.

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References and links

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1. Introduction

The volume of data traffic continues to increase due to the demand of subscribers for multimedia services that require the access network to support high data rates at any time, in any place inexpensively. Such demands require broadband communication systems. Free space optical (FSO)

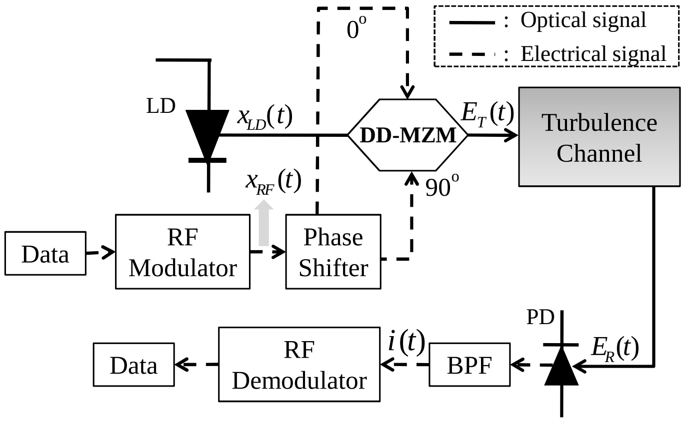

Fig. 1. Overall architecture of the FSO system considering of optical transmitter, turbulence channels, and optical receiver.

systems have been good candidates for next generation broadband services since FSO systems support large bandwidth, unlicensed spectrum, excellent security, and quick and inexpensive setup [1]. In spite of these advantages, the performance of FSO systems can be unreliable due to atmospheric turbulence. The Gamma-Gamma distribution well represents atmospheric turbulence channels with a multiplication of two parameters of small-scale and large-scale irradiance fluctuations–of which pdfs are independent Gamma distributions–and provides excellent agreement between theoretical and simulation results [2].

One of the well-known modulators of FSO systems is the external modulator, with a dual drive Mach-Zehnder modulator (DD-MZM), since it is robust against a laser chirp and provides high spectral efficiency [3]. Fig. 1 represents the FSO system employing a DD-MZM. In this system, the laser phase noise from a laser diode (LD) is one of the decisive factors limiting the performance of FSO systems because the optical system is sensitive to laser phase noise. However, to the best of our knowledge, an analysis of the BER of FSO systems, with a DD-MZM impaired by the laser phase noise under turbulence channels, has not been carried out in the research due to the complexity of the analysis.

Therefore, in this paper, we first represent the optical signal model from a DD-MZM with laser phase noise using the Bessel expansion. We then analyze an average BER according to an average SNR, with normalized perturbation under atmospheric channels, where Gamma-Gamma distribution describes the turbulence-induced fading. Also, numerical results are provided to illustrate the degradation of performance according to the depth of scintillation and LD linewidth.

2. FSO System Architecture and Signal Model

Figure 1 shows the overall architecture of the FSO system. Data is modulated to a binary phase shift keying (BPSK) signal by the RF modulator. A BPSK signal from the RF modulator is split by a $\pi/2$ phase shifter. This BPSK signal is optically modulated by a LD with a DD-MZM. The output signal of the DD-MZM is transmitted via atmospheric turbulence channels between telescopes. The received signals are detected by the photodetector (PD), and the photocurrent corresponding to the transmitted BPSK signal is extracted by the bandpass filter (BPF). Finally, data is extracted by the RF demodulator module. The optical signal, $x_{LD}(t)$ [4] from the laser and the BPSK signal, $x_{RF}(t)$ from the RF modulator are modeled respectively, as follows:

$$
\begin{aligned}
x_{LD}(t) &= V_{LD}\ exp\left[j\left(\omega_c t + \Phi_{LD}(t)\right)\right],\\
x_{RF}(t) &= V_{RF}\ cos\left(\omega_{rf} t + \theta(t)\right).
\end{aligned} \tag{1}
$$

In (1), V_{LD} and V_{RF} are the optical carrier amplitude and the BPSK signal amplitude, respectively, and ω_c and ω_{rf} are angular frequencies of the signals from the LD and the RF modulator. The laser phase noise process $\Phi_{LD}(t)$ is commonly characterized as a Wiener process [6]. $\theta(t) = \sum_{m=-\infty}^{\infty} d_m P(t-mT)$ where $P(t)$ is a unit amplitude pulse of the bit duration T and d_m is the information of the m^{th} bit duration, which takes on $\{0, \pi\}$.

After optically modulating $x_{RF}(t)$ by $x_{LD}(t)$ with a DD-MZM, the output signal of a DD-MZM is represented as [3], [5]

$$
\begin{aligned}
E_T(t) &= \frac{V_{LD}}{\sqrt{2}\,10^{\frac{L}{20}}}\left\{exp\left[j\left(\omega_c t + \frac{\pi}{2} + \Phi_{LD}(t) + \varepsilon\pi\ cos[\omega_{rf} t + \theta(t)]\right)\right]\right.\\
&\left.+exp\left[j\left(\omega_c t + \Phi_{LD}(t) + \varepsilon\pi\ cos[\omega_{rf} t + \theta(t) + \frac{\pi}{2}]\right)\right]\right\}
\end{aligned} \tag{2}
$$

where $\varepsilon(= V_{RF}/V_\pi)$ defines a normalized ac value, V_π is the switching voltage of a DD-MZM, and L is the modulator insertion loss in decibels. Using the Bessel function, (2) can be expanded to

$$
\begin{aligned}
E_T(t) &= \frac{V_{LD}\ exp\left[j\left(\omega_c t + \Phi_{LD}(t)\right)\right]}{\sqrt{2}\,10^{\frac{L}{20}}}\left\{j\sum_{n=-\infty}^{\infty}\left[j^n exp\left[j\,n\left(\omega_c t + \Phi_{LD}(t)\right)\right]J_n(\varepsilon\pi)\right]\right.\\
&\left.+\sum_{n=-\infty}^{\infty}\left[j^n exp\left[j\,n\left(\frac{\pi}{2} + \omega_{RF} t + \theta(t)\right)\right]J_n(\varepsilon\pi)\right]\right\}\\
&\simeq \frac{V_{LD}}{10^{\frac{L}{20}}}\left\{J_0(\varepsilon\pi)\ exp\left[j\left(\omega_c t + \frac{\pi}{4} + \Phi_{LD}(t)\right)\right]\right.\\
&\left.-\sqrt{2}J_1(\varepsilon\pi)\ exp\left[j\left(\omega_c t + \omega_{rf} t + \Phi_{LD}(t) + \theta(t)\right)\right]\right\}.
\end{aligned} \tag{3}
$$

We assume that high-order components of the Bessel function can be negligible since the value of $\varepsilon\pi$ in the Bessel function is very small due to the fact that $V_\pi \gg V_{RF}$ in general. The output signal at a DD-MZM is transmitted via turbulence channels experiencing different group delays due to the chromatic dispersion. After the transmission of turbulence channels, the received optical signal is expressed as

$$
\begin{aligned}
E_R(t) &\simeq \frac{V_{LD}\cdot\sqrt{\delta(t)}}{10^{\frac{L}{20}}}\left\{J_0(\varepsilon\pi)\ exp\left[j\left(\omega_c t + \frac{\pi}{4} + \Phi_{LD}(t-\tau_0) - \psi_0\right)\right]\right.\\
&\left.-\sqrt{2}J_1(\varepsilon\pi)\ exp\left[j\left(\omega_c t + \omega_{rf} t + \Phi_{LD}(t-\tau_1) + \theta(t) - \psi_1\right)\right]\right\}
\end{aligned} \tag{4}
$$

where $\delta(t) = \sum_{m=-\infty}^{\infty} \delta_m P(t-mT)$, and δ_m is the turbulence channel coefficient for the m^{th} data duration, τ_0 and τ_1 are group delays, and ψ_0 and ψ_1 are phase-shift parameters.

Through direct detection, the optical signal can be detected at the PD, and the BPSK signal is extracted by the BPF. After the BPF, the photocurrent $i(t)$ can be obtained as follows:

$$
\begin{aligned}
i(t) &= R|E_R(t)|^2 + n_{th}(t)\\
&\simeq 2\sqrt{2}R\left(\frac{V_{LD}\cdot\sqrt{\delta(t)}}{10^{\frac{L}{20}}}\right)^2 J_0(\varepsilon\pi)J_1(\varepsilon\pi)cos\left(\omega_{rf} t + \Phi_{LD}(t-\tau_1)\right.\\
&\left.-\Phi_{LD}(t-\tau_0) + \theta(t) - \psi_1 + \psi_0\right) + n_{th}(t)
\end{aligned} \tag{5}
$$

where R is the responsivity of the PD and $n_{th}(t)$ is a random fluctuation which has an unit of current due to thermal noise in the load resistance. In general FSO systems, as thermal noise is dominant due to the high operating temperature [7], $n_{th}(t)$ is modeled as a Gaussian process [8]. Also, $\Phi_{LD}(t-\tau_0)$ and $\Phi_{LD}(t-\tau_1)$ are independent–not of t–but of the differential delay $(\tau_1-\tau_0)$. Thus, using a Wiener process property, the difference of two random processes can be simplified as a zero mean Gaussian random variable Ψ with the variance σ^2 of $2\pi\Delta\nu\Delta\tau$ where $\Delta\nu$ is LD linewidth and $\Delta\tau$ is the differential delay [6]. Accordingly, (5) can be represented as

$$i(t)=\delta\cdot\Upsilon\cdot cos\left(\omega_{rf}t+\Psi+\theta(t)+\psi_2\right)+n_{th}(t),\ \ mT\leq t\leq(m+1)T \tag{6}$$

where Υ is $2\sqrt{2}R(V_{LD}/10^{L/20})^2J_0(\varepsilon\pi)J_1(\varepsilon\pi)$, $\psi_2=\psi_0-\psi_1$. When we assume that the coherence time is larger than a bit time duration, $\{\delta_m\}$ can be modeled as independent and identically distributed (i.i.d.) random variables, i.e., $\delta_m=\delta$ with Gamma-Gamma distribution. After the matched filter in the RF demodulator, $\int_0^T i(t)d_t$, thermal noise becomes a random variable, n_{th} with normal distribution, $N[0,\sigma_{th}^2=(4kT/R_L)\Delta f]$ where k is the Boltzmann constant, R_L is the load resistance, and Δf is the effective noise bandwidth [8].

Gamma-Gamma distribution [2] is

$$f_\delta(\delta)=\frac{2(\alpha\beta)^{\frac{\alpha+\beta}{2}}}{\Gamma(\alpha)\Gamma(\beta)}(\delta)^{\frac{\alpha+\beta}{2}-1}K_{\alpha-\beta}\left(2\sqrt{\alpha\beta\delta}\right) \tag{7}$$

where $\delta>0$, α and β are the scintillation parameters, $K_\varepsilon(\cdot)$ is the modified Bessel function of the second kind of order ε, and $\Gamma(\cdot)$ is the Gamma function. According to the atmospheric conditions, α and β are defined as in [2]

$$\alpha=\left(exp\left[\frac{0.49\sigma_R^2}{(1+0.18d^2+0.56\sigma_R^{12/5})^{7/6}}\right]-1\right)^{-1} \tag{8}$$

$$\beta=\left(exp\left[\frac{0.51\sigma_R^2(1+0.69\sigma^{12/5})^{-5/6}}{(1+0.9d^2+0.62d^2\sigma_R^{12/5})^{5/6}}\right]-1\right)^{-1} \tag{9}$$

where, $\sigma_R^2=0.5C_n^2\kappa^{7/6}L^{11/6}$ is Rytov variance which has been used as an estimate of the intensity variance, σ_R is the turbulence strength, $d=(\kappa D^2/4L)^{1/2}$, D is the diameter of the receiver collecting lens aperture, κ is the optical wave number, L is the propagation distance, and C_n^2 stands for the altitude-dependent index of the refractive structure parameter and varies from $10^{-13}m^{-2/3}$ for strong turbulence to $10^{-17}m^{-2/3}$ for weak turbulence. Moreover, according to α and β the scintillation index (SI) is defined as [2]

$$SI=\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\alpha\beta}. \tag{10}$$

3. BER Analysis of FSO systems

In this section, we derive a closed form of the average BER considering turbulence channels and the laser phase noise. We can define the instantaneous electrical SNR as $\Lambda=(\Upsilon\delta)^2/\sigma_{th}^2$. For the convenience of the analysis, we introduce a new measure, $\mu(=\Upsilon^2/\sigma_{th}^2)$ which corresponds to the average SNR with $E[\delta^2]=1$, i.e., the average SNR $E[\Lambda]=\mu\cdot E[\delta^2]=\mu$. When we assume that the random phase noise Ψ is fixed over the bit duration, the conditional BER is represented as [9]

$$P_b(E|\Psi,\Lambda)=\frac{1}{2}erfc(\sqrt{\Lambda\, cos^2[\Psi]}) \tag{11}$$

where $erfc$ is the complementary error function. Consequently, the average BER ($E[P_b]$) can be obtained by using the following integral calculus as

$$E[P_b] = \int_0^{\infty}\int_{-\infty}^{\infty} P_b(E|\Psi,\Lambda) f_{\Psi}(\Psi) f_{\delta}(\delta) d_{\Psi} d_{\delta} \tag{12}$$

Using the change of variable $x = \Psi/\sqrt{2\sigma}$ and Gauss-Hermite quadrature formula, (12) can be simplified to (13)

$$\begin{aligned} E[P_b] &= \frac{1}{2\sqrt{\pi}} \int_0^{\infty}\int_{-\infty}^{\infty} erfc(\sqrt{\mu\delta^2 cos^2[\sqrt{2\sigma}x]})\; exp[-x^2] f_{\delta}(\delta) d_x d_{\delta} \\ &= \frac{1}{2\sqrt{\pi}} \int_0^{\infty} \sum_{i=1}^{N} w_i \; erfc(\sqrt{\mu\delta^2 cos^2[\sqrt{2\sigma}x_i]}) f_{\delta}(\delta) d_{\delta} \end{aligned} \tag{13}$$

where N is the order of approximation, x_i, $i = 1,....,N$ are the zeros of the Nth-order Hermite polynormial, and w_i, $i = 1,....,N$ are weight factors for the Nth-order approximation. Using the following Meijer G functions [10]:

$$erfc(\sqrt{x}) = \frac{1}{\sqrt{\pi}} G_{1,2}^{2,0}\left[x \mid \begin{matrix} 1 \\ 0, \frac{1}{2} \end{matrix}\right] \tag{14}$$

$$K_v(x) = \frac{1}{2} G_{0,2}^{2,0}\left[\frac{x^2}{4} \mid \begin{matrix} - \\ \frac{v}{2}, \frac{-v}{2} \end{matrix}\right] \tag{15}$$

the average BER is represented as

$$\begin{aligned} E[P_b] &= \frac{1}{2\sqrt{\pi}} \sum_{i=1}^{N} w_i \int_0^{\infty} erfc(\sqrt{\mu\delta^2 cos^2[\sqrt{2\sigma}x_i]}) \frac{2(\alpha\beta)^{\frac{\alpha+\beta}{2}}}{\Gamma(\alpha)\Gamma(\beta)} (\delta)^{\frac{\alpha+\beta}{2}-1} K_{\alpha-\beta}\left(2\sqrt{\alpha\beta\delta}\right) d_{\delta} \\ &= \frac{(\alpha\beta)^{\frac{\alpha+\beta}{2}}}{2\pi\Gamma(\alpha)\Gamma(\beta)} \sum_{i=1}^{N} w_i \int_0^{\infty} (\delta)^{\frac{\alpha+\beta}{2}-1} G_{1,2}^{2,0}\left[\mu\delta^2 cos^2[\sqrt{2\sigma}x_i] \mid \begin{matrix} 1 \\ 0, \frac{1}{2} \end{matrix}\right] \\ &\times G_{0,2}^{2,0}\left[\alpha\beta(\delta) \mid \begin{matrix} - \\ \frac{\alpha-\beta}{2}, \frac{\beta-\alpha}{2} \end{matrix}\right] d_{\delta}. \end{aligned} \tag{16}$$

Finally, by using the classic Meijer integral of the two G functions [10, Eq. 07.34.21.0011.01], (16) is simplified as

$$E[P_b] = \frac{2^{\alpha+\beta-3}}{\pi^2\Gamma(\alpha)\Gamma(\beta)} \sum_{i=1}^{N} w_i \; G_{5,2}^{2,4}\left[\frac{2^4\mu cos^2[\sqrt{2\sigma}x_i]}{(\alpha\beta)^2} \mid \begin{matrix} \frac{1-\alpha}{2}, \frac{2-\alpha}{2}, \frac{1-\beta}{2}, \frac{2-\beta}{2}, 1 \\ 0, \frac{1}{2} \end{matrix}\right]. \tag{17}$$

which is as a function of LD linewidth (Δv) and scintillation parameters (α, β).

4. Numerical results

Figure 2 illustrates the results of the average BER according to the average SNR with $E[\delta^2] = 1$. We show two examples of practical LDs with Δv=10MHz and Δv=624MHz, which correspond to typical values of a DFB LD and a FP LD, respectively. We consider that the optical wavelength is 1.55μm, the responsivity is 0.8 A/W, and the differential delay $\Delta\tau$ is 10^{-10} sec.

We numerically evaluate performance under three different turbulence channel conditions:(α,β,SI) $\in${(4,1,1.5),(4,2,0.875),(4,4,0.5625)}. As β decreases and SI increases, the turbulence effects become stronger. The numerical results show that the difference of average BER corresponding to two LDs (with linewidth of 624MHz and 10MHz) under

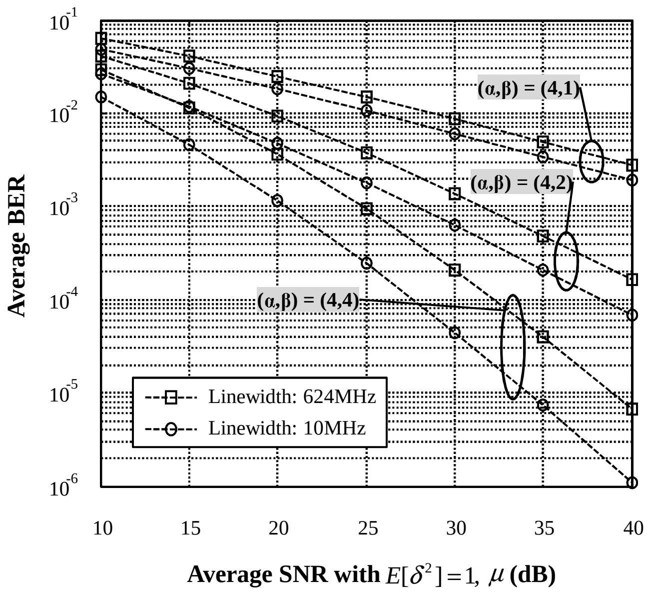

Fig. 2. Average BER according to average SNR when $E[\delta^2] = 1$ according to $(\alpha, \beta, SI) \in \{(4,1,1.5),(4,2,0.875),(4,4,0.5625)\}$ and 624MHz and 10MHz of LD linewidth.

weak turbulence is almost 3 times larger than that under strong turbulence and the effect of the laser phase noise of LD linewidth 624MHz degrades almost 5dB to the SNR than that of LD linewidth 10MHz at all turbulence conditions. It is noteworthy that in (17), we use $N = 10$ for the Gauss-Hermite formula.

5. Conclusions

In this paper, we present an optical signal model from the transmitter output and the receiver input using the Bessel expansion. Also, we derive a close form average BER performance by considering the laser phase noise from LD under atmospheric turbulence channels with Gamma-Gamma distribution using the Gauss-Hermite quadrature formula and Meijer G function. As a result, we can more easily predict BER performance without complicated calculations. In practical terms, when we establish FSO systems, we can make an engineering table by using this derived BER formula according to each LD, which enables us to determine efficient LD under turbulence channel conditions.

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