THE ASTROPHYSICAL S-FACTOR OF THE REACTION ^{7}Be(p,γ)^{8}B IN THE DIRECT CAPTURE MODEL

H. Krauss, K. Grün, T. Rauscher, H. Oberhummer

Institut für Kernphysik, TU Wien, Wiedner Hauptstr. 8–10

A-1040 Wien, Austria

Abstract. The astrophysical S-factor for the reaction ^{7}Be(p,γ)^{8}B up to an energy of 2 MeV (c.m.) and the capture cross section of ^{7}Li(n,γ)^{8}Li up to 1 MeV (c.m.) are calculated using the Direct Capture model (DC). Both calculations are in good agreement with experimental data.

Keywords: Nuclear reactions; ^{7}Be(p,γ)^{8}B; ^{7}Li(n,γ)^{8}Li; Astrophysical S-Factor.

1 Introduction

The reaction ^{7}Be(p,γ)^{8}B plays an important role in the so-called ppIII chain in the hydrogen burning of main-sequence stars. A knowledge of the reaction rate is essential to determine the branching ratios between the ppI, ppII and ppIII chains. The magnitude of the reaction cross section is of special interest for the solar neutrino problem. The reason for this is that in the ^{37}Cl neutrino experiment [1] 77%, in the Kamiokande II experiment [2] 100% and in the gallium experiments [3, 4] 11% of the detected neutrino flux originate from the high-energy neutrinos emitted in the ppIII chain [5]. Therefore, the reaction rate of the above process determines the high-energy solar neutrino flux.

The reaction considered in this paper has been measured by various authors at subCoulomb energies. The most recent data have been obtained in the energy range $E_{\rm c.m.} = 117 - 1230\,{\rm keV}$ [6].

The dominance of the direct interaction (DI) mechanism and the validity of the description with the potential-model approach below the Coulomb barrier has been established in many light-ion reactions ([7] and references therein). In this work we apply this approach to the above reaction. In the next section we introduce the direct capture model. In Section 3 the result for the astrophysical S-factor is given and compared to experimental data. Finally, in Section 4 the results are summarized.

2 Potential Model Approach

Potential models are based on the description of the dynamics of the reaction by a Schrödinger equation with local optical potentials in the entrance and/or exit channels. Such models are the "Distorted Wave Born Approximation" (DWBA) [8, 9, 10] for transfer or the "Direct Capture" model (DC) [11, 12, 13] for capture reactions.

In first order perturbation theory the expression for the differential cross section of a direct capture reaction is [14]:

$$\frac{d\sigma_{\rm DC}}{d\Omega_\gamma} = 2\left(\frac{e^2}{\hbar c}\right)\left(\frac{mc^2}{\hbar c}\right)\left(\frac{k_\gamma}{k_a}\right)^3 \frac{1}{2I_A+1}\,\frac{1}{2S_a+1}\sum_{M_A M_a M_B \sigma} \mid T_{M_A M_a M_B,\sigma} \mid^2 \quad . \tag{1}$$

Here I_A (M_A) and S_a (M_a) are the spins (their projections on the z-axis) of target and projectile, and σ is the polarization of the electromagnetic radiation ($\sigma = \pm 1$). The wave numbers of the emitted γ-rays and of the asymptotic relative wave function in the entrance channel are denoted by k_γ and k_a, and m is the reduced mass. We couple the angular momenta in the spin-orbit representation:

$$\vec{l}_a + \vec{S}_a = \vec{j}_a \quad , \tag{2}$$

$$\vec{l}_b + \vec{S}_a = \vec{j}_b \quad , \tag{3}$$

$$\vec{I}_A + \vec{I}_B = \vec{j}_b \quad . \tag{4}$$

The coupling and notation are the same as in [14].

The transition amplitudes $T_{M_A M_a M_B,\sigma}$ are expanded in terms of rotation matrix elements $d^{\lambda}_{\delta\sigma}(\theta)$ with the electromagnetic multipole λ (λ = E1, E2, M1, ...),

$$T_{M_A M_a M_B,\sigma} = \sum_{\lambda} T^{\lambda}_{M_A M_a M_B,\sigma}\, d^{\lambda}_{\delta\sigma}(\theta) \quad , \tag{5}$$

where $\delta = M_A + M_a - M_B$ and θ is the angle between $\mathbf{k}_a$ and $\mathbf{k}_\gamma$.
The transition amplitudes are proportional to the radial integrals

$$T^{\lambda} \propto I^{\lambda}_{l_b j_b I_B; l_a j_a} = \int dr\, U_{l_b j_b I_B}(r)\, \mathcal{O}^{\lambda}(r)\, \chi_{l_a j_a}(r) \quad , \tag{6}$$

where $U_{l_b j_b I_B}(r)$ and $\chi_{l_a j_a}(r)$ are the radial parts of the bound state wave function and the distorted wave function in the entrance channel, respectively. The functions $\mathcal{O}^{\lambda}(r)$ are the radial parts of the electromagnetic multipole operators, which were taken in their approximated form,

$$\mathcal{O}^{\mathrm{M1}}(r) \simeq 1, \quad \mathcal{O}^{\mathrm{E1}}(r) \simeq r, \quad \mathcal{O}^{\mathrm{E2}}(r) \simeq r^2 \quad . \tag{7}$$

Folding potentials

We solve the radial integral (6) numerically using single folding potentials [15] for the bound state potential and the optical potential,

$$V(r) \;\; = \;\; \lambda \int d\mathbf{r}_A\, \rho_A(\mathbf{r}_A)\, t(E, \rho_A, |\mathbf{s} = \mathbf{r} - \mathbf{r}_A|) \quad , \tag{8}$$

which were assumed to be real. Here r is the separation of the centers of mass of the bound or colliding particles, ρ_A is the nucleon density of the target and λ is a normalization constant. For the effective nucleon-nucleon

interaction t in the entrance and exit channel we chose the density dependent form of the M3Y interaction [16].

The nucleon density ρ_A is derived from the charge density distribution $\rho_{\rm p}(r)$ of the target A with the assumption, that the distribution in the nucleus is the same for protons and neutrons: $\rho_{\rm n} = (N/Z)\rho_{\rm p}$ [9, p. 474]. This leads to

$$\rho_A = \rho_{\rm p} + (N/Z)\,\rho_{\rm p} = (A/Z)\,\rho_{\rm p} \quad . \tag{9}$$

The density $\rho_A(r)$ satisfies

$$4\pi \int \rho_A(r)\, r^2 dr = A, \tag{10}$$

The normalization constant λ of the folding potential ($\lambda_{\rm sc}$ for the potential in the entrance channel and $\lambda_{\rm b}$ for the bound state potential) accounts for the interplay of the Pauli principle and distortion and breakup effects. One of the advantages of the folding procedure lies in the fact that no open geometrical parameters exist, since $\lambda_{\rm b}$ is adjusted to reproduce the experimentally known binding energies and $\lambda_{\rm sc}$ can be adjusted to reproduce scattering phase shifts (if they are available) or the energies of resonant states (which is done for ^{7}Be(p,γ)^{8}B). Therefore the form of the optical and bound state potentials is determined uniquely.

3 Results

^{7}Be(p,γ)^{8}B

The ^{7}Be(p,γ)^{8}B reaction is of crucial importance since it leads to the high-energy ^{8}B neutrinos. Unfortunately, there are still significant experimental uncertainties in the low-energy cross section for this reaction. The reaction was analyzed before in the DC model by using phenomenological potentials [17, 18]. Furthermore, this reaction was also analyzed with the resonating group method (RGM) [19] and the generator coordinate method (GCM) [20, 21]. The numerical calculations in the DI model in this work were performed using the direct capture code TEDCA [22].

The folding potential for ^{7}Be+p

Because of the instability of ^{7}Be there are no experimental charge distributions available to compute the nucleon density with Eq. (9) [23]. Therefore we chose the same distribution as that of ^{7}Li, which should give a suitable approximation. Although the experimental charge distributions of ^{7}Li and ^{7}Be are somewhat different, the resulting potential should be quite similar. We conclude this from calculations comparing the folding potentials of ^{11}B+p and ^{12}C+p.

The folding potential for ^{7}Be+p was calculated using a harmonic oscillator model for the ^{7}Li charge distribution with parameters $a = 1.77$ fm and $\alpha = 0.327$ [23]. The depth of the pure folding potential is 43 MeV ($\lambda = 1$),

the volume integral per nucleon is 660 MeV fm^3.

Calculation of ^{7}Be(p,γ)^{8}B without spin-orbit coupling

All contributions to the transition matrix lead to the ground state of ^{8}B with spin $J^\pi = 2^+$. We use the spectroscopic factors $\mathcal{S}_{1p_{3/2}} = 0.977$ and $\mathcal{S}_{1p_{1/2}} = 0.0561$ from the shell model calculations of [24]. The DC cross section is a sum over the two final state configurations ($j_b = 1/2,\ 3/2$),

$$\sigma_{\rm DC} = \sum_{l_b\, j_b} C^2 \mathcal{S}_{l_b j_b}\, \sigma_{l_b\, j_b} \quad , \tag{11}$$

where C^2 is the isospin Clebsch-Gordan coefficient.

A strength factor of $\lambda_{\rm b} = 1.041$ yields the correct separation energy of 138 keV of the bound proton. The bound state wave function is very diluted due to the very low binding energy. Therefore we had to calculate radial contributions to the transition matrix up to 200 fm.

In Fig. 1 the astrophysical S-factor is shown. The λ-parameter of the scattered wave $\lambda_{\rm sc} = 0.969$ was determined with the condition that the resonance in the p-wave has to be at an energy of 632 keV, which leads to a resonant M1 transition. This value of the normalization parameter determines the potential uniquely and was used for all partial waves in the entrance channel.

In Fig. 2 the radial integrand (6) of the s-wave is plotted at two different energies ($E_{\rm c.m.} = 15$ keV and $E_{\rm c.m.} = 1$ MeV). At low energies there

are contributions to the integral from very far outside the nucleus. Those contributions can be neglected only for distances larger than 200 fm.

As one can see in Fig. 1, the calculated S-factor of ^{7}Be(p,γ)^{8}B agrees with the experiments [6, 25, 26] quite nicely in absolute value over a wide range of energies (100 keV $-$ 2 MeV), except in the resonance energy region near $E_{\rm c.m.} = 632$ keV. In particular there is also a very good agreement with the mean experimental value of $S(0)$ that is used by Bahcall [5] in the standard solar model. A comparison of the parametrization of the potential model calculation with these experimental values is given in Tab. 1. The data of [25] in Fig. 1 were renormalized as suggested by [27].

The experimental width of the 1^+-resonance at 632 keV is not reproduced so well. The calculated width of 100 keV is too large by a factor of 2.5. A possible explanation for this is the neglect of configuration mixing and other channels, such as the channel p $+^7$ Be$_{(1/2)^-}$, which could only be taken into account in a coupled channel calculation.

Calculation with spin-orbit coupling in the entrance channel

Currently another resonance is discussed, which should correspond to a $J^\pi = 1^+$ state of ^{8}B with an excitation energy $E_{\rm x} \approx 1.5$ MeV. This state is derived from comparison with the excitation spectra of the mirror nucleus ^{8}Li [28] (1^+_1 : $E_{\rm x} = 0.9808\,$MeV, 1^+_2 : $E_{\rm x} = 2.255\,$MeV). Up to now, no experimental evidence for the 1^+_2 state in ^{8}B is available. The 1^+_2 state is

predicted by the GCM [20] and also by the potential model, if one takes spin-orbit coupling into account. The two states are assumed to have the configurations ^{7}Be+p($1p_{3/2}$) (1^+_1) and ^{7}Be+p($1p_{1/2}$) (1^+_2).

We are able to include this doublet in our calculation by employing an additional spin-orbit potential in the entrance channel. For this spin-orbit potential we used the usual parametrization [29], with the Woods-Saxon form factor replaced by the form factor of the folding potential. The strength $\lambda_{\rm sc} = 0.94$, together with the spin-orbit term ($V_{\rm so} = 2.44\,$MeV), reproduces the correct energy separation of the two 1^+-resonances at $E_{\rm c.m.} = 632$ keV and $E_{\rm c.m.} = 1.4$ MeV. Fig. 3 displays a plot of the contributions of different partial waves and multipolarities. The influence of the $J^\pi = 1^+_2$-resonance on the S-factor is negligible, especially for thermonuclear energies.

Calculation with spin-orbit coupling in both channels

According to the shell model calculations of [24], the 1^+_1 level in ^{8}B has a better $p_{3/2}$ structure ($\mathcal{S}_{1p_{3/2}} = 0.3215$ and $\mathcal{S}_{1p_{1/2}} = 0.1240$). Nevertheless, we also want to consider the description of the ground and first excited (unbound) state of ^{8}B as a p-wave spin-doublet. The two states should then have the configurations ^{7}Be+p($1p_{3/2}$) (2^+) and ^{7}Be+p($1p_{1/2}$) (1^+_1).

The form factor of the spin-orbit potential stays the same as for the calculation with spin-orbit coupling in the entrance channel. The spin-orbit strength $V_{\rm so} = 2.01\,$MeV was adjusted to describe the correct energy splitting

of the two corresponding states in ^{8}Li. The so determined spin-orbit potential is able to reproduce with $\lambda_{\rm b} = \lambda_{\rm sc} = 1.01733$ both the binding energy of the ground state and the resonance in the entrance channel at 0.632 MeV. The potentials for the three assumptions (no spin-orbit potential, spin-orbit in the entrance channel and spin-orbit in both channels) are shown in Tab. 2.

Fig. 4 displays the S-factor calculated with the additional spin-orbit potential in both channels. Now the height of the resonance at 632 keV is reduced, compared to the previous calculations. At this energy, the dominating M1-contribution is formed by the overlap of the quasi-bound and bound state (Eq. (6)), which are now almost orthogonal. This is due to the assumption of a spin-dublet with a spin-orbit potential. In Fig. 5, the radial wave function of the bound state (full line) is shown together with the scattering wave functions of the p wave at resonance energy (632 keV) for the case of zero spin-orbit potential (dashed line) and spin-orbit potential (dashed dotted line). The overlap of scattering wave and bound state reaches its maximum at 3 fm in both cases, but is much smaller in its absolute value for the case of spin-orbit potential.

As a consequence, the description of the experimental S-factor by the potential model gets worse in the resonance energy region (Fig (4)). The assumption of a spin-dublet does not lead to a good description of the data.

^{7}Li(n,γ)^{8}Li

We now want to calculate the mirror reaction ^{7}Li(n,γ)^{8}Li with the assumption of spin-orbit coupling in the entrance channel only, because this seems to be the most realistic potential. Since the determination of the folding potential is not depending on isospin, the ^{7}Li+n-potential is the same as the ^{7}Be+p-potential of the last section. Therfore the optical potential for the entrance channel can be taken from the analog calculation of ^{7}Be(p,γ)^{8}B (see also Tab. 2). Also the shell model spectroscopic factors are the same for ^{8}Li and ^{8}B.

We consider the capture into the ground state ($J^\pi = 2^+$) and the first excited state ($J^\pi = 1^+$, $E_\mathrm{x} = 0.9808\,\mathrm{MeV}$) of ^{8}Li. The correct binding energies of these states are reproduced with the parameters $\lambda_\mathrm{b} = 1.0361$ for the ground state and $\lambda_\mathrm{b} = 0.9627$ for the first excited state of ^{8}Li.

The result of the calculation is shown in Fig. 6. The experimental data were taken from [30] (filled circles) and [31]. The open circles and triangles result from two different normalization procedures. We did not attempt to reproduce the resonance at $E_\mathrm{c.m.} = 0.222\,\mathrm{MeV}$, which corresponds to a 3^+ state in ^{8}Li, since this state has a bad ^{7}Li+n structure [32].

4 Summary

We have shown that the reactions ^{7}Be(p,γ)^{8}B and ^{7}Li(n,γ)^{8}Li can be described by a potential model at thermonuclear energies. The theoretical values of the low-energy astrophysical S-factor for the different assumptions concerning the single-particle configurations of ^{8}B are in good agreement with the mean experimental values of [5]. It can be concluded that the direct interaction mechanism is dominant at energies well below the Coulomb barrier.

The resonance width and strength of the experimentally known resonance at $E_{\rm c.m.} = 632$ keV are both overestimated by the potential model, if one uses no spin-orbit potential or a spin-orbit potential in the entrance channel only. In these cases no special assumptions concerning the single-particle configurations of the final bound state in ^{8}B are made. If one assumes the configurations ^{7}Be+p($1p_{3/2}$) (2^+) and ^{7}Be+p($1p_{1/2}$) (1^+_1) for this state and introduces an additional spin-orbit potential to describe the bound and first unbound state simultaneously, the DC cross section in the energy region of the resonance is strongly reduced (Fig. 4) and underestimates the data, due to the overlap of relatively orthogonal wavefunctions. But in the non-resonant energy region the reproduction of the cross section data is excellent in all cases.

The reaction ^{7}Li(n,γ)^{8}Li was calculated with the potential in the entrance

channel taken from ^{7}Be(p,γ)^{8}B, since this seems to be the most realistic and therefore favorable potential ansatz. The agreement with the reaction data is good, as can be seen in Fig. 6.

The progress of applying the potential model combined with the folding procedure lies mainly in the fact that no parameter has to be adjusted to the reaction data. This is neither the case in the phenomenological fits of Barker et al. [27] nor Kim et al. [14], where potential parameters have to be fitted to the experimental reaction data.

In our model the strengths of the folding potentials are adjusted to reproduce the energies of the bound and quasi-bound states. This procedure is similar to microscopic models [20], where the correct binding energies are ensured by adjusting a parameter in the nucleon-nucleon interaction.

Acknowledgement: We want to thank the Österreichische Nationalbank (project 3924) and the FWF (project P7838-TEC) for their support.

References

[1] R. Davis Jr., A.K. Mann, L. Wolfenstein: Ann. Rev. Nucl. Part. Sci. 39 (1989) 467

[2] K.S. Hirata *et al.*: Phys. Rev. Lett. 63 (1989) 16

[3] W. Hampel: in Proc. Workshop on Neutrino Physics, ed. by H.V. Klapdor (Heidelberg: Springer 1988) 230

[4] V.N. Gavrin: in Proc. Neutrino '88, ed. by J. Schnepps (Singapore: World Scientific 1989) 317

[5] J.N. Bahcall: Neutrino Astrophysics (Cambridge: Cambridge University Press 1989)

[6] B.W. Filippone, A.J. Elwyn, C.N. Davids, D.D. Koethe: Phys. Rev. C 28 (1983) 2222

[7] H. Oberhummer, G. Staudt: in Nuclei in the Cosmos, ed. by H. Oberhummer (Heidelberg: Springer 1991) 29

[8] N. Austern: Direct Nuclear Reaction Theories (New York: John Wiley & Sons 1970)

[9] G.R. Satchler: Direct Nuclear Reactions (Oxford: Clarendon 1983)

[10] N.K. Glendenning: Direct Nuclear Reactions (New York: Academic Press 1983)

[11] R.F. Christy, I. Duck: Nucl. Phys. A 24 (1961) 89

[12] T.A. Tombrello und P.D. Parker, *Phys. Rev.* 131, 6 (1963) 2582

[13] C. Rolfs: Nucl. Phys. A 217 (1973) 29

[14] K.H. Kim, M.H. Park, B.T. Kim: Phys. Rev. C 35 (1987) 1, 363

[15] G.R. Satchler, W.G. Love: Phys. Rev. 55 (1979) 183

[16] A.M. Kobos, B.A. Brown, R. Lindsay, G.R. Satchler: Nuc. Phys. A 425 (1984) 205

[17] T.A. Tombrello, *Nucl. Phys.* 71 (1965) 459

[18] B.T. Kim, T. Izumoto, K. Nagatani: Phys. Rev. C 23 (1983) 1

[19] C.W. Johnson, E. Kolbe, S.E. Koonin, K. Langanke: Ap. J. (in press)

[20] P. Descouvemont, D. Baye: Nucl. Phys. A 487 (1988) 420

[21] P. Descouvemont, in: *Proceedings of the International Symposium on Nuclear Astrophysics (Nuclei in the Cosmos)* ed. by F. Käppeler, Karlsruhe (1992), Bristol: IOP publishing, in press

[22] H. Krauss: code TEDCA (1990) unpublished

[23] H. de Vries, C.W. de Jager, C. de Vries: At. Data and Nucl. Data Tables 36 (1987) 495

[24] S. Cohen, D. Kurath: Nucl. Phys. A 101 (1967) 1

[25] R.W. Kavanagh, T.A. Tombrello, J.M. Mosher, D.R. Goosman: Bull. Am. Phys. Soc. 14 (1969) 1209

[26] F.J. Vaughn, R.A. Chalmers, D. Kohler and L.F. Chase, Jr.: Phys. Rev. C 2 (1970) 1657

[27] F.C. Barker, R.H. Spear: Ap. J. 307 (1986) 847

[28] F. Ajzenberg-Selove: Nucl. Phys. A 413 (1984) 1

[29] C.M. Perey, F.G. Perey: At. Data Nucl. Data Tabl. 17 (1976) 1

[30] M. Wiescher, R. Steininger and F. Käppeler, *Ap. J.* 344 (1989) 464

[31] W.L. Imhof, R.G. Johnson, F.J. Vaughn, M. Walt: Phys. Rev. 114 (1959) 1037

[32] H. Stöwe und W. Zahn, *Nucl. Phys.* A289 (1977) 317

Figure captions:

Fig. 1: Astrophysical S-factor for the reaction ^{7}Be(p,γ)^{8}B. Full curve: potential model calculation (no spin-orbit coupling). The data points are from [6] (triangles), [25] (circles) and [26] (squares).

Fig. 2: ^{7}Be(p,γ)^{8}B: Radial integrand of Eq. (6) for two different energies in the entrance channel (s-wave only).

Fig. 3: Contributions of different partial waves and multipolarities to the cross section of ^{7}Be(p,γ)^{8}B. Here an additional spin-orbit potential in the entrance channel was used which influences the p-wave contribution near $E_{\rm c.m.} = 1.4$ MeV.

Fig. 4: Astrophysical S-factor of ^{7}Be(p,γ)^{8}B. Full curve: potential model calculation (spin-orbit coupling in both channels).

Fig. 5: Radial wave functions of the ^{7}Be+p bound state (full line) and scattering wave functions of the p wave at resonance energy (0.632 MeV) for zero spin-orbit potential (dashed line) and spin-orbit potential in both channels (dashed dotted line).

Fig. 6: Total cross section for the reaction ^{7}Li(n,γ)^{8}Li. The experimental data are from [30] (filled circles) and [31] (open circles and triangles).

Table captions:

Tab. 1: Comparison of the results of the potential model calculation of ^{7}Be(p,γ)^{8}B for the three different assumptions (no spin-orbit coupling, spin-orbit in the entrance channel and spin-orbit in both channels) with the experimental values of the astrophysical S-factor and its derivative as given in [5].

Tab. 2: Comparison of the potentials used in the calculation of ^{7}Be(p,γ)^{8}B and ^{7}Li(n,γ)^{8}Li.

	Exp. [5]	without SO	$\mathrm{SO}_{(i)}$	$\mathrm{SO}_{(i+f)}$
$S(0)$	0.0243 ± 0.0053	0.0249	0.0249	0.0236
$\dot{S}(0)$	$-3 \cdot 10^{-5}$	$-3.2 \cdot 10^{-5}$	$-3.2 \cdot 10^{-5}$	$-3.1 \cdot 10^{-5}$
$S(18\,\mathrm{keV})$	0.0238 ± 0.0052	0.0243	0.0243	0.0231

	^{7}Be(p,γ)^{8}B			^{7}Li(n,γ)^{8}Li
	without SO	$\mathrm{SO}_{(i)}$	$\mathrm{SO}_{(i+f)}$	$\mathrm{SO}_{(i)}$
λ_b	1.041046	1.041046	1.01733	1.0361 0.9627
V_so	0	0	2.01	0
E_res	-0.138	-0.138	-0.138 —	-2.033 -1.052
j_b	1/2, 3/2	1/2, 3/2	3/2 1/2	1/2, 3/2 1/2, 3/2
λ_sc	0.96895	0.94	1.01733	0.94
V_so	0	2.44	2.01	2.44
E_res	0.632	0.632 1.4	— 0.632	— —
j_a	1/2, 3/2	3/2 1/2	3/2 1/2	3/2 1/2